The Power of Upstream Contracting over Downstream Collusion

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Abstract

Collusion by downstream firms can be detrimental to the upstream suppliers. I show that a monopolist supplier can use nonlinear pricing contracts to weaken downstream firms’ ability to engage in collusive behavior, while also generating a positive welfare effect. Regulatory policy targeting upstream nonlinear pricing may weaken downstream competition. Because upstream pricing behavior differs with and without downstream collusion, the model also provides authorities with a new tool for detecting collusion.

Keywords: Vertical Contracting, Downstream Collusion, Antitrust, Cartel, Collusion Detection

JEL Classification: D43, K21, L41, L42

1 Introduction

Many collusion cases involve downstream firms. While downstream firms’ collusive behavior hurts consumers, it also injures upstream suppliers. Yet existing theories of collusion do not address upstream suppliers’ incentives to influence collusion between downstream firms. In this paper, I examine strategic contracting as one way for upstream suppliers to restrict downstream collusion, and I ask the following questions: Do some types of supply contracts make it more difficult for downstream firms to collude than other types of contracts? If so, how does policy regulating pricing of upstream

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suppliers impact competition in a downstream market? And can we infer collusion in a downstream market from upstream suppliers’ pricing behavior?

Antitrust enforcers are interested in how collusion is affected by vertical relations. Researchers have shown that vertical restraints and vertical mergers can facilitate upstream collusion. But less is known about the effects of vertical relations on downstream collusion; work in this area typically assumes downstream firms have the power to make contract offers to their suppliers. This paper reverses this assumption, investigating how an upstream supplier can strategically design supply contracts to influence collusion between downstream firms. The main finding of this paper is that an upstream supplier can use nonlinear pricing to restrict downstream collusion. I use two-part tariff contracts to demonstrate this result.

I consider an industry with an upstream monopolist supplier and two identical downstream retailers. First, the supplier offers the same two-part tariff contract to the two retailers. The contract is stationary, which means it applies to trade in all future periods. Then the two retailers engage in infinitely-repeated market interaction, by each choosing quantities to purchase from the supplier for resale to consumers in each period. I focus my analysis on collusion between the retailers, who strive to collectively obtain the retail monopoly profit. I will show that by utilizing a two-part tariff with the retailers, the supplier can induce colluding retailers to choose a total quantity larger than what they would have chosen under optimal linear pricing. It is worth noting that in this model, two-part tariff “restricts” retail collusion not in the sense that it makes retailers less likely to collude, but in the sense that it forces retailers to collude at a larger quantity than they would otherwise be able to achieve. The key idea is that collusion relies on repeated-game punishments (Nash reversion, Friedman (1971)).

With a two-part tariff contract, both the per-unit price and the fixed fee have an effect on how retail collusion can be maintained. The model explores the interaction between these two effects. As a starting point, suppose the supplier offers the optimal linear price. If the supplier chooses to decrease the price, then the colluding retailers’ incentive constraints will be tightened, leading to a higher quantity, and a greater surplus. The supplier would want to extract the greater surplus using a fixed fee. The addition of a fixed fee leads to an interesting trade-off: on the one hand, although the supplier would like to charge a large fixed fee, the fixed fee cannot be so large that no retailer will participate in trading; on the other hand, a large enough fixed fee can affect...
the retailers’ punishment profile, further tightening the retailers’ collusion constraints.

To understand how the fixed fee can affect the retailers’ punishment profile, consider first that without a fixed fee, the retailers engage in quantity competition in a punishment period. Adding a large enough fixed fee can make quantity competition unprofitable, in which case in a punishment period, the retailers would rather purchase nothing and stay out of the market than compete and obtain negative profits. Consequently, non-participation replaces competition as the retailers’ “Nash reversion”. Therefore, for any quantity agreement, a retailer choosing to collude expects to pay a fixed fee in every period, while a retailer choosing to deviate expects to pay a fixed fee only once. This effect on retail punishment reduces a retailer’s net gain from colluding relative to deviating for any quantity agreement. As such, some quantities become unsustainable in collusion. Particularly, the supplier can strategically choose a fixed fee such that only a large quantity is sustainable. In equilibrium, this large collusive quantity not only exceeds the retail collusive quantity under linear pricing, but, in fact, it also exceeds the retail competition quantity under linear pricing. This result means that as far as welfare is concerned, the positive effect of nonlinear pricing more than offsets the negative effect of downstream collusion.

Thus, the model predicts that downstream firms have incentives to collude, and that a two-part tariff contract is more limiting to downstream firms’ ability to collude than a linear pricing contract. Although a two-part tariff is a particularly attractive limiting case of nonlinear pricing, the key property is discount pricing (decreasing per-unit price for a larger quantity). For example, car-rental companies typically receive quantity discounts when purchasing fleet vehicles from auto makers. Moreover, cartel cases are relatively rare in the car-rental industry. This example is consistent with the model’s predictions: While the existence of car-rental cartels manifests an incentive to collude in a downstream market, the relatively restrained collusion may be explained by nonlinear pricing upstream. For some examples of existing cases and probes in the car-rental industry, see *Alice Springs Car Rental Cartel*[^5], *Shames et al. v. Hertz Corp. et al.*[^6],

[^4]: Two-part tariff is discount pricing in the extreme – it is equivalent to charging a very high price for the first unit, and a low price for additional units.


The model has several policy implications. First, regulation affecting upstream firms’ pricing behavior may affect downstream competition. Second, I show that nonlinear pricing differs in a systematic way with and without downstream collusion. In particular, the supplier charges a lower per-unit price and a higher fixed fee (or offers a steeper discount for large quantities and demands a higher payment for small quantities) when threatened by downstream collusion than otherwise. This suggests that in establishing downstream collusion, courts could permit evidence of changes in upstream pricing as a new “plus factor” (to use a term in the antitrust literature). This means that circumstantial evidence to establish collusion does not need to be confined to horizontal actions: Looking upstream may yield evidence as well. Third, upstream collusion in pricing may have a benign motivation, which is to restrict downstream collusion. Fourth, damages caused to upstream firms by downstream collusion may be less severe if nonlinear pricing is used in supply contracts than if linear pricing is used. Fifth, an upstream monopolist facing downstream collusion may price low for large quantities to restrict downstream collusion, but such pricing behavior may be misinterpreted as predatory.

This paper fits in the literature of collusion studies in vertical settings. This literature has largely focused on vertical restraints and vertical integration. Discussion mostly centers around the trade-off between the procompetitive and anticompetitive effects of vertical practices, with some focused on upstream collusion (e.g. Jullien and Rey (2007), Piccolo and Reisinger (2011), Nocke and White (2007), Normann (2009)). A less rich line of research studies downstream collusion induced by buyer power. These studies investigate the collusive effects of buyer power, but leave out of consideration upstream firms’ incentives to influence downstream collusion. It remains unexplored how upstream firms can affect downstream collusion without vertical coordination. To the best of my knowledge, this paper is the first to venture into this terrain.

This paper also adds to the literature of nonlinear pricing. A number of existing studies in the marketing and management literature analyze the channel-coordinating effects of nonlinear pricing in a competitive environment. Ingene and Parry (1995a) establish that in a retail competitive environment, a manufacturer can use the right

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8See http://www.franceinfo.fr/actu/economie/article/six-loueurs-de-voitures-soupconnes-de-pratiques-anticomcurrentielles-683007 (accessed May 19, 2016).

9Abstracting away from the “countervailing power” of large buyers shown by Snyder (1996), a few authors worked on showing buyer power has collusive effects. These efforts include: theoretical work by Doyle and Han (2014), Piccolo and Miklós-Thal (2012), and experimental work by Normann et al. (2015).
nonlinear pricing contract to fully coordinate the channel\textsuperscript{10}. Further, with downstream competition, a monopolist upstream supplier is able to use nonlinear pricing (e.g. two-part tariff) to capture a large share of, if not all, surplus of a coordinated channel. Because nonlinear pricing was created for the upstream tier to increase channel profits and capture surplus better, rarely has attention been paid to other potential effects of using such contracts. But retailers are often in repeated interaction with each other. By taking into consideration retailers’ repeated interaction, this paper finds another useful aspect of nonlinear pricing: the use of nonlinear pricing to influence downstream competition.

The rest of the paper is organized as follows. In Section 2, I set up the model. In Section 3, I provide equilibrium analysis, characterizing the supplier’s optimal per-unit price and fixed fee when retailers collude in a symmetric fashion, as well as the market outcome. I then explore how retailers’ patience affects the total quantity, and how the supplier’s optimal per-unit price and fixed fee differ with and without retail collusion. Finally, I provide welfare comparison between different contracts in different competition environments. In Section 4, I discuss some policy implications of the results. In Section 5, I extend the model to allow for asymmetric collusion between the retailers, where only one retailer trades in each period. Some robustness results are provided. Section 6 concludes.

2 The Model

I consider a two-tier vertical model with an upstream monopolist supplier $S$ and two independent, identical downstream retailers $R_1$ and $R_2$. The retailers purchase a homogeneous good from the supplier, for resale to the consumers\textsuperscript{11}. For the supplier, the marginal cost of production is $c$.

The timing of the game is as follows. First, the supplier offers the same wholesale contract to both retailers, which is stationary across all periods. Then, an infinitely repeated game is played by the retailers, with the following happening in every period:

1. The two retailers independently and simultaneously decide whether to accept the contract, and if they do, purchase some quantities of the product from the supplier

\textsuperscript{10}Here fully coordinating the channel means that the system obtains the maximum possible surplus achievable by a single integrated entity. Related to this theme, Ingene and Parry (1995b) show that nonlinear pricing can achieve channel coordination with non-competing retailers. Going back further, Jeuland and Shugan (1983), Jeuland and Shugan (1988), Shugan (1985) and Moorthy (1987) studied coordinating mechanisms in a single dyadic channel where there is one manufacturer and one independent retailer.

\textsuperscript{11}We can also think of the transactions as downstream firms purchasing an input good from the supplier, then transforming it into an output good to sell to the consumers.
for sale in the current period. Denote these quantities \( q_1, q_2 \geq 0 \). Rejection of the contract offer leads to zero profit for a retailer.

2. The market clears, with the price determined by inverse linear demand \( P = a - bQ \), and all players’ profits are realized.

The game is of common knowledge and perfect monitoring. Each player maximizes his or her stream of discounted payoffs over an infinite horizon. The retailers share a common discount factor \( \delta \in (0,1) \). For illustration purposes, I will use the female pronoun when referring to the supplier, and the male pronoun to refer to a retailer.

Since the focus of this paper is on downstream collusion, I assume that the retailers engage in horizontal collusion whenever collusion is profit-enhancing and sustainable with infinite “Nash reversion”. It is also assumed that the downstream firms cannot resell the upstream good to one another\(^{12}\).

The model uses two-part tariff contracts. Contract terms are as follows: A retailer purchasing a positive quantity pays a fixed fee \( f \), plus a per-unit price of \( w \); a retailer pays nothing if no quantity is purchased. As a result, in any period \( t \), retailer \( i \) gets payoff

\[
\pi_{it} = \begin{cases} 
0, & \text{if } q_{it} = 0; \\
q_{it}(a - bq_{it} - bq_{jt} - w) - f, & \text{if } q_{it} > 0.
\end{cases}
\]

### 3 Equilibrium Analysis

#### 3.1 Symmetric Collusive Equilibrium

The retailers consider collusion when collusion is more profitable than competing. In this subsection, I derive all players’ equilibrium behavior when retailers collude symmetrically. By symmetric collusion, I refer to a collusive scheme where both retailers are expected to adopt the same behavior in every period. Such a collusive scheme prescribes that each retailer purchase quantity \( q \) from the supplier in each period. The solution concept is formally defined as follows:

**Definition 1.** A Symmetric Collusive Equilibrium is a subgame perfect Nash equilibrium given by the supplier’s strategy \((w, f)\), and each retailer’s strategy \( q_{it} \) such that:

1. given \((w, f)\), \( Q_t = q_{1t} + q_{2t} \) maximizes the retailers’ collective profits in each period \( t \);
2. \( q_{it} = q \).

\(^{12}\)We can think of resale downstream being prohibited by issues like warranty, trademark, and sales tax.
Using standard infinite Nash reversion as punishment for deviation, I denote firm \(i\)'s profit in a collusive period by \(\pi_{Coll}^i\), his deviation profit by \(\pi_{Dev}^i\), and his profit in a punishment period by \(\pi_{Pun}^i\), respectively. Given the symmetric nature of the collusive scheme, I will omit the subscript \(i\). The condition to sustain retail collusion is:

\[
\pi_{Coll}^i \geq (1 - \delta)\pi_{Dev}^i + \delta\pi_{Pun}^i,
\]
or

\[
(a - 2bq - w)q - f \geq (1 - \delta)\frac{1}{4b}(a - bq - w)^2 - f(1 - \delta)
+ \delta \max \left\{ \frac{(a - w)^2}{9b} - f, 0 \right\} .
\]

Note the punishment profit above. \(f_0(w) = \frac{(a-w)^2}{9b}\) is a retailer’s profit in Cournot competition when he faces a unit price \(w\) and no fixed fee. If the fixed fee \(f\) exceeds this profit, then “Nash reversion” becomes no purchase, and the punishment profit becomes zero.

Let collusion condition (1) be rewritten as \(h(q, w, f) \geq 0\). That is, let \(h(q, w, f) \equiv (a - 2bq - w)q - (1 - \delta)\frac{1}{4b}(a - bq - w)^2 - \delta f - \delta \max \left\{ \frac{(a-w)^2}{9b} - f, 0 \right\} \). And let \(\Theta(w, f)\) denote the set of collusive quantities that can be sustained by the retailers under contract terms \((w, f)\), so \(\Theta(w, f) \equiv \{ q : h(q, w, f) \geq 0 \}\).

After observing \(w\) and \(f\), the retailers’ problem is

\[
\max_{q} \pi_{Coll} = (a - 2bq - w)q - f \\
\text{subject to } h(q, w, f) \geq 0.
\]

Notice that when \(\frac{(a-w)^2}{9b} - f > 0\), or \(f < \frac{(a-w)^2}{9b}\), slightly increasing \(f\) without changing \(w\) does not have any impact on collusion condition (1), or the constraint \(h(q, w, f) \geq 0\). In other words, the supplier can always increase her profit by increasing \(f\) without changing the total quantity purchased by colluding retailers. This implies that strategies satisfying \(\frac{(a-w)^2}{9b} - f > 0\) will never be played in equilibrium.

Lemma 1. In symmetric collusive equilibrium, \(f \geq f_0(w)\).

The supplier’s problem is:

\[
\max_{w,f} \pi_{S} = 2[q(w - c) + f] \\
\text{subject to: } q \text{ solves } (R).
\]

Lemma 1 implies that the retailers’ problem may not have an interior solution. Intuitively, a fixed fee exceeding a retailer’s profit in competition leads the retailers to
stay out of the market in a punishment period. So retailers expect a larger burden of fixed fees from colluding than from deviating. As a result, there may only be a small set of quantities sustainable in collusion ($\Theta(w, f)$ being a small set), thus an interior solution may not be attainable. This also has consequences for the supplier’s problem. To deal with these complications, I will parse the problem into two steps: (1) I show that in equilibrium, every $w$ maps to a unique optimal $f$ for the supplier, as well as a unique optimal $q$ for the retailers. (2) The supplier’s problem then reduces to choosing an optimal $w$, which can be solved mathematically.

To analyze the equilibrium, it will be useful to have the following glossary of notation:

- $q^N(w) = \frac{a-w}{\delta b}$, a retailer’s quantity in Cournot competition when facing per-unit price $w$ and no fixed fee;
- $\hat{q}(w) = \frac{a-w}{b} \cdot \frac{3-\delta}{9-\delta}$, the unconstrained maximizer for $h(q, w, f)$;
- $\check{q}(w, f)$ is the solution to the retailers’ problem ($R$) when contract terms are $(w, f)$;
- $f^*(w) = \text{the supplier’s optimal fixed fee for a given } w$;
- $q^*(w) = \text{the solution to the retailers’ problem } (R) \text{ when the contract terms are } (w, f^*(w))$.

**Step (1):** Given a per-unit price $w$, consider the supplier’s choice of a corresponding fixed fee $f$. That is, find $f^*(w)$.

In equilibrium, $h(q, w, f) = 0$. To see this, notice that when $h(q, w, f) > 0$, the supplier can always do better by slightly increasing the fixed fee $f$ without affecting the retailers’ collusive quantity. Therefore, the supplier would keep increasing $f$, at least until $h(q, w, f) = 0$. Hence, all strategies by the supplier that would lead to $h(q, w, f) > 0$ will never be played in equilibrium. This implies that the equilibrium collusive quantity $q$ is a corner solution to the retailers’ problem. The retailers want to choose a collusive quantity as close to $\hat{q}(w)$ as possible, but are constrained by the condition $h(q, w, f) = 0$. As long as $\Theta(w, f)$ is not empty, the larger the fixed fee $f$ is, the larger the collusive quantity $q$ is. Knowing this, the supplier would keep raising $f$ until there is only one point left in $\Theta(w, f)$. The result is the following proposition.

**Proposition 1.** For any $w$, the supplier’s optimal fixed fee is $f^*(w) = \frac{1}{\delta}[(a - 2b\check{q}(w) - w)\hat{q}(w) - (1 - \delta)\frac{1}{3}\{(a - b\check{q}(w) - w)^2\} > 0$, and the retailers’ corresponding optimal individual collusive quantity is $q^*(w) = \hat{q}(w) = \frac{a-w}{b} \cdot \frac{3-\delta}{9-\delta} > \check{q}(w)$.

**Proof.** See Appendix A. 

Proposition 1 shows that using a fixed fee, a monopolist supplier does not technically prevent the retailers from colluding, but in fact makes retailers collude at a quantity larger than what they would have colluded on under linear pricing.
Figures 1 - 3 graph the retailers’ individual collusive profit $\pi^{Coll}(q, w, f)$ and their collusion condition $h(q, w, f)$ against the collusive quantity $q$. They illustrate how the retailers’ optimizing $q$ changes as only $f$ (not $w$) changes. Progressing from Figure 1 to Figure 3, $f$ increases: $f_1 < f_2 < f^*$.\(^{13}\) Figure 1 depicts a case where $\delta$ is large enough to sustain retail collusion at the downstream monopoly quantity when there is no fixed fee. However, the assumption of a large $\delta$ is not needed for the equilibrium analysis. If $\delta$ is otherwise small, it simply means $\inf \Theta(w, f) > q^*(w)$ for all $f \geq 0$. It does not change the optimality of $f^*(w)$ for the supplier, and $q^*(w)$ for the retailers.

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\(^{13}\) All notations in the graphs represent functions of $w$. I omit the arguments in the graphs for clarity of presentation.
Next, I examine how $\delta$ affects the retailers’ choice of collusive quantity.

**Lemma 2.** (i) $\hat{q}(w) \to \hat{q}(w)$ as $\delta \to 1$. (ii) $\hat{q}(w) \to q^N(w)$ as $\delta \to 0$.

Despite the fixed fee, the most patient retailers can sustain collusion at the downstream monopoly quantity under linear pricing, and the most impatient retailers choose the competitive quantity. The fixed fee has a restrictive effect on collusion when $\delta \in (0, 1)$.

**Step (2):** With the result in Proposition 1, it is now clear that the supplier’s problem is:

$$\max_w \pi_S = \frac{2[q(w-c)+f]}{1-\delta},$$

where $q = q^*(w) = \hat{q}(w) = \frac{a-w}{b} \cdot \frac{3-\delta}{9-\delta}$, and $f = f^*(w) = \frac{1}{\delta}[(a-2bq-w)q - (1-\delta)\frac{1}{4b}(a-bq-w)^2]$.

The following is a closed-form solution to the supplier’s problem:

**Proposition 2.** In equilibrium, the supplier sets a unit price $w^{**} = c + (a-c)\frac{1-\delta}{4-2\delta} \in (c, \frac{a+c}{2})$, and a positive fixed fee: $f^{**} = \frac{(a-c)^2}{4b} \cdot \frac{(3-\delta)^2}{(2-\delta)^2(9-\delta)} > 0$.

**Proof.** See Appendix A.

Denote the equilibrium total quantity of the model $Q^{**}$, i.e., $Q^{**} = 2\hat{q}(w^{**})$. And denote with $Q^L$ the equilibrium total quantity under linear pricing with retail competition. I compare these two quantities, and have the following result:

**Proposition 3.** The total quantity under two-part tariff when retailers collude is larger than the total quantity under optimal linear pricing when retailers compete. That is, $Q^{**} > Q^RC$.

**Proof.** See Appendix A.
Figures 4 - 9 in Appendix C provide a summary of the results in Propositions 1, 2 and 3. They show that when retailers collude, contracting with two-part tariff leads to a larger total quantity than contracting with linear pricing. Further, it is also larger than the total quantity with linear pricing when retailers compete. Figure 4 exhibits the standard double marginalization problem under linear pricing: Point A represents the market price and quantity when retailers collude\textsuperscript{14}, while point B represents the market price and quantity when retailers engage in Cournot competition. \(Q^M\) denotes the industry monopoly quantity. Figure 5 shows the retailers’ set of sustainable collusive quantities. If this set brackets the retail monopoly quantity under linear pricing \(Q^{RM}\), the retailers would optimize by colluding at \(Q^{RM}\). However, according to the results obtained above, if the supplier simply adds a fixed fee to the contract (while keeping the wholesale unit price at the industry monopoly level \(P^M\)), she can in fact already restrict the retailers’ set of sustainable collusive quantities down to a single point: point C in Figure 7. Point C represents a quantity larger than the retail monopoly level under linear pricing. In addition, as shown in Figure 8, the supplier will also adjust the wholesale unit price down, which results in a quantity even larger than that at point C. Finally, equilibrium is shown as point C in Figure 9.

Essentially, the model shows that an upstream monopolist supplier can use a two-part tariff to effectively force the downstream retailers to collude at a large quantity. The resulting quantity is larger than the retail monopoly quantity under optimal linear pricing, and close to the industry monopoly quantity.

3.2 Comparative Statics

**Lemma 3.** As \(\delta\) changes, total quantity \(Q^{**} = 2\hat{g}(w^{**})\) changes in the following way, all else held constant: \(\frac{\partial Q^{**}}{\partial \delta} < 0\) for \(\delta < \frac{3}{5}\), \(\frac{\partial Q^{**}}{\partial \delta} = 0\) for \(\delta = \frac{3}{5}\), and \(\frac{\partial Q^{**}}{\partial \delta} > 0\) for \(\delta > \frac{3}{5}\).

**Proof.** See Appendix A. \qed

In a nutshell, Lemma 3 shows that under wholesale two-part tariff, the change in retailers’ total collusive quantity is not monotonic in \(\delta\). This is in contrast to wholesale linear pricing, under which an increase in \(\delta\) from 0 to 1 monotonically decreases the retailers’ total collusive quantity, until the total quantity is pushed down to the retail monopoly level.

To understand this result, note that when \(\delta = 0\), retailers compete, and the supplier is able to achieve the industry monopoly quantity with the right pair of \((w, f)\). As \(\delta\) exceeds zero, retail cartelization takes effect, resulting in a total quantity smaller than the monopoly quantity. When a small \(\delta\) increases, the retail cartel gets stronger

\textsuperscript{14}Here, I am again making the harmless assumption that retailers are patient enough to cooperate at the retail monopoly quantity when facing linear pricing.
and is able to sustain collusion to a larger extent, pushing down the total quantity.\textsuperscript{15} However, as retailers become even more patient, the trend reverses: The total quantity increases, approaching the monopoly quantity again. The reason is that \( \delta \) being close enough to 1 indicates the retailers’ willingness to sustain collusion for even just a small amount of benefits, which enables the supplier to treat them as close to a single entity. When there is a single entity downstream, the supplier can again achieve the industry monopoly quantity and obtain all channel profits using two-part tariff.

Next, I explore the effects of changing \( \delta \) on the supplier’s optimal choice of two-part tariff terms. An examination of Proposition 2 reveals the following results:

**Proposition 4.** \( \frac{\partial w^*}{\partial \delta} < 0, \frac{\partial f^*}{\partial \delta} > 0. \)

*Proof.* See Appendix A.

More patient retailers can sustain collusion more easily, so the supplier would need to use a lower per-unit price and a larger fixed fee to tighten the retailers’ collusion conditions.

### 3.3 Comparison with No Collusion

Now that we know the players’ equilibrium behavior, I juxtapose the supplier’s optimal strategy \((w^*, f^*)\) under retail collusion and her optimal strategy \((w_c^*, f_c^*)\) when retailers play the static Nash equilibrium. I find that the supplier offers a lower per-unit price and a higher fixed fee under retail collusion than under retail competition:

**Proposition 5.** \( w^* < w_c^* \), and \( f^* > f_c^* \).

*Proof.* When the two retailers compete, the supplier using a two-part tariff obtains surplus amount \( \frac{(a-c)^2}{4b} \), by imposing \( w_c^* = \frac{a+3c}{4} \), and \( f_c^* = \frac{(a-c)^2}{16b} \). \textsuperscript{16}

To show that \( w^* < w_c^* \), we use the result \( \frac{\partial w^*}{\partial \delta} < 0 \) from Proposition 4. Combine \( \frac{\partial w^*}{\partial \delta} < 0 \) with the fact that when \( \delta = 0, w^* = w_c^* = \frac{a+3c}{4} > c \), and when \( \delta = 1, w^* = c \), and it follows that \( w^* \in (c, w_c^*) \).

To show that \( f^* > f_c^* \), we use the result \( \frac{\partial f^*}{\partial \delta} > 0 \) from Proposition 4. Combine \( \frac{\partial w^*}{\partial \delta} > 0 \) with the fact that when \( \delta = 0, f^* = f_c^* = \frac{(a-c)^2}{16b} \), and when \( \delta = 1, f^* = \frac{(a-c)^2}{8b} \), and it follows that \( f^* \in (f_c^*, \frac{(a-c)^2}{4b}) \).

\textsuperscript{15}Note, however, that the use of two-part tariff makes sure this small total quantity is not as small as it would have been, had linear pricing been in place. This is the result of Proposition 1.

\textsuperscript{16}These are the one-shot equilibrium price and fixed fee. They confirm our earlier calculations in the repeated game with retail collusion: Letting \( \delta = 0 \) (no collusion) in both \( w^* = c + \frac{(a-c)^2}{16b} \) and \( f^* = \frac{(a-c)^2}{4b} \cdot \frac{(3-\delta)^2}{(2-\delta)(9-\delta)} \) gives precisely \( w_c^* = \frac{a+3c}{4} \) and \( f_c^* = \frac{(a-c)^2}{16b} \).
Compared to retail competition, retail collusion causes the supplier to charge a higher fixed fee in order to restrict the collusive quantity. This enables the supplier to induce a larger total quantity using a lower per-unit price.

For direct comparison, Table 1 in Appendix B lists equilibrium prices/tariffs, along with total quantities under linear pricing and two-part tariff, in settings with and without retail collusion. Note that in equilibrium, the total quantity exceeds the retail competition quantity under linear pricing, and is very close to the industry monopoly quantity.

3.4 Supplier’s Incentive for Choosing Two-Part Tariff

Under retail collusion, two-part tariff works better than linear pricing for the supplier, not only because two-part tariff affords the supplier the ability to capture retailers’ surplus\(^{17}\), but also because a supplier using two-part tariff can adjust the wholesale marginal price to mitigate her profit loss due to retail collusion. With linear pricing, the supplier does not have the ability to limit retail collusion, and thus has to absorb all the resulting profit loss.

The result in Proposition 1 applies to any \(w\), and has the following implication. Suppose a supplier is currently adopting a linear pricing rule with her downstream retailers, and is experiencing a less-than-expected profit due to retail collusion. Proposition 1 suggests that the supplier can in fact lift her profit simply by adding a fixed fee to the wholesale contract, without even changing the wholesale per-unit price.

In summary, Proposition 1 implies the following:

**Corollary 1.** When facing retail collusion, the supplier prefers contracting with two-part tariff to contracting with linear pricing.

**Proof.** This is an immediate result from \(f^*(w) > 0\) for any \(w\).

Importantly, as will be shown in the next part, when there is retail collusion, the supplier’s self-incentivized choice of two-part tariff over linear pricing is also beneficial to consumers and society.

3.5 Welfare Analysis

In a one-shot scenario, two-part tariff is superior to linear pricing in terms of supplier surplus, consumer surplus, and total surplus for society. In a setting with retail collusion, we have established that two-part tariff rewards the supplier with higher surplus than linear pricing (from Corollary 1). In fact, with retail collusion, consumers also obtain a higher surplus under two-part tariff, so does society.

\(^{17}\)This is the standard one-shot result.
Proposition 6. With retail collusion, consumer surplus is higher under two-part tariff than under linear pricing.

Proposition 7. With retail collusion, total surplus for society is higher under two-part tariff than under linear pricing.

Table 2 in Appendix B lists all surpluses under linear pricing and two-part tariff for comparison. It shows that when faced with retail collusion, the supplier has an incentive to adopt two-part tariff. In addition to helping the supplier capture downstream surplus, two-part tariff restricts downstream collusion, leading to large consumer surplus and total surplus.

4 Policy Discussion

The relation between upstream contracting and downstream collusion has implications for a variety of policy issues on antitrust and regulation. In this section, I discuss some of these policy implications.

4.1 Relation Between Upstream and Downstream Collusion Cases

In the United States, public antitrust enforcers are the Antitrust Division of the Department of Justice (DoJ), the Federal Trade Commission (FTC), and the State Attorneys General. According to Antitrust Guidelines for Collaborations Among Competitors, which were jointly issued by FTC and DoJ in 2000, the Supreme Court uses two types of analysis to determine the lawfulness of an agreement among competitors: per se and rule of reason. Some types of agreements have been determined to be highly likely to harm competition while providing no significant procompetitive benefit. These agreements, once identified, can be challenged as per se illegal, and other types of agreements are evaluated under the rule of reason:

Types of agreements that have been held per se illegal include agreements among competitors to fix prices or output, rig bids, or share or divide markets by allocating customers, suppliers, territories, or lines of commerce. The courts conclusively presume such agreements, once identified, to be illegal, without inquiring into their claimed business purposes, anticompetitive harms, procompetitive benefits, or overall competitive effects. Agreements not challenged as per se illegal are analyzed under the rule of reason to determine their overall competitive effect. These include agreements of a type that otherwise might be considered per se illegal, provided
they are reasonably related to, and reasonably necessary to achieve pro-
competitive benefits from, an efficiency-enhancing integration of economic
activity.\textsuperscript{18}

In designing these guidelines, the Agencies’ goal is to protect consumers and society
from harmful agreements. But from a practical standpoint, perhaps it is fair to say
that the guidelines (or at least the implementation of them) mainly focus on the impact
of agreements on direct purchasers. In long supply chains, where there are many tiers
of intermediaries, agreement among firms within one tier affects not only their direct
customers, but also their indirect customers (and their direct and indirect suppliers). In
this case, the true overall impact on society is almost impossible to quantify in practice,
and remains a distant unknown.

Furthermore, it is possible that customers of colluding firms are intermediaries who
can also form agreements among themselves. When two adjacent tiers of a supply chain
separately but simultaneously collude, it would be wrong to dismiss the effects of the
two instances of collusion on each other. Since collusion in one layer is likely to injure
firms in an adjacent tier, one can reasonably assume that the two simultaneous cases
of collusion are at odds with each other. If the two tiers of collusion undermine each
other, then eliminating only one of them can in some cases leave the society worse off
than eliminating neither. Yet current implementation of the Guidelines does not seem
to address or recognize this issue. It seems that the courts have only been interested
in tackling cartel cases one by one, without considering the possible relation between
some of them which may happen in the same supply chain. This paper finds that an
upper tier has both the incentive and the means (contracting) to restrict collusion of its
direct purchasers, pushing up the quantity and lowering the price for consumers. It then
follows that upstream collusion (upstream firms collaborating with each other on writing
contracts with downstream firms) should not be dismissed as downright harmful in all
cases. If downstream collusion exists but is not caught, then catching and eliminating
upstream collusion can actually do a disservice to consumers and society. To make things
worse, downstream colluding firms have an absolute incentive to sue upstream firms for
collusion, while hiding their own colluding behavior. Given the current antitrust legal
standing, unless the upstream firms can produce sufficient evidence to counter sue the
downstream firms, the court is likely to award the downstream colluding firms treble
damages when they are in fact the ones doing the most damaging to society, and the
treble damages come from fining the relatively benign upstream agreement. This type
of surplus reallocation would undermine our sense of fairness. There may not be an easy
solution to this problem, since private plaintiffs carry the burden of proof, but perhaps
an initial improvement can be achieved if the courts start considering upstream and

downstream collusion cases together whenever relevant and possible.

The Guidelines hold that price fixing is per se unlawful. In light of the finding of this paper, perhaps regulators should adopt a more granular approach if an upstream group of firms are accused of price fixing when the court is uncertain about the competitive environment downstream.

The Guidelines also stipulate that agreements that are not challenged per se unlawful be judged based on the court’s inquiry into their overall competitive effects - the combination of anticompetitive harm and procompetitive benefit. This paper suggests broadening the interpretation of “overall competitive effects” to include possible effects on competition in other tiers of the same supply chain.

The finding of this paper could potentially also be applied to mergers. Although mergers are different from collusion, they sometimes impose effects on competition similar to those imposed by collusion. According to Federal Trade Commission and U.S. Department of Justice (2000),

The Agencies treat a competitor collaboration as a horizontal merger in a relevant market and analyze the collaboration pursuant to the Horizontal Merger Guidelines if appropriate, which ordinarily is when: (a) the participants are competitors in that relevant market; (b) the formation of the collaboration involves an efficiency-enhancing integration of economic activity in the relevant market; (c) the integration eliminates all competition among the participants in the relevant market; and (d) the collaboration does not terminate within a sufficiently limited period by its own specific and express terms.

Moreover, as acknowledged by U.S. Department of Justice and Federal Trade Commission (2010), a merger may diminish competition by enabling or encouraging post-merger coordinated interaction among firms that harms customers. At least in some cases, the merging of upstream firms facilitates upstream efforts to coordinate contracting terms, and the model predicts a procompetitive effect on the downstream market analogous to that in the case of upstream collusion. This implication may be of interest to antitrust enforcers who have been busy policing the recent boom in mergers-and-acquisition activity involving U.S. companies.

4.2 Collusion Detection

A cartel raises profits of its members by artificially restricting output and increasing prices at the expense of customers and suppliers, reducing the total surplus for society. Thus to public enforcers, the value of correctly detecting and deterring collusion is self-evident. To private plaintiffs, it is also vital to be able to prove collusive behavior when
such behavior exists.

Needless to say, even with explicit collusion, evidence of communication between cartel members is hard to come by, because communication of this nature is illegal and thus is always surreptitious if carried out. Hence besides finding evidence of explicit communication, other methods are needed to detect collusion. There are corporate leniency programs in place in both the U.S.\textsuperscript{19} and Europe\textsuperscript{20}, encouraging whistle blowing. The U.S. Department of Justice also uses an individual leniency policy\textsuperscript{21}. By offering leniency on legal sanctions, these programs incentivize a “race to the courthouse” among cartel participants. These programs are helpful in catching collusion, but they are far from being sufficient.

In order for antitrust laws concerning collusion to operate effectively, it is crucial to develop rigorous methods to identify cartel behavior using economic evidence. In practice, this is not an easy task because many actions taken by firms to ensure successful operation of a cartel can also have legitimate noncollusive grounds. For example, simultaneous increases in prices can simply be oligopolistic behavior after an industry-wide cost increase. Therefore, it is important for antitrust practitioners to infer collusion from economic evidence using reliable empirical methods.

Currently, courts accept inference of collusion based on strong enough circumstantial evidence. Firms that make collusive arrangements violate Section 1 of the Sherman Act\textsuperscript{22}. To reach the conclusion of violation without direct evidence of collusion, courts require presentation of sufficient economic circumstantial evidence that goes beyond the parallel movement of prices by firms. The collection of such economic circumstantial evidence is referred to as “plus factors”.\textsuperscript{23} Kovacic et al. (2011) and Marshall and Marx (2012) provide detailed discussion on collusion detection using plus factors. Examples of cartel actions that can act as plus factors include: price elevation, quantity restriction, allocation of collusive gain, redistributions, enforcement and punishment, dominant-firm conduct, to name a few. Evidence of individual or joint appearances of plus factors can help with collusion diagnosis.

One result of the model in this paper identifies systematically different pricing behavior by the upstream tier with and without downstream cartelization. Particularly, the model suggests that downstream collusion causes the optimizing upstream supplier to offer a steeper discount for large quantities but demand a higher payment for small quantities (in two-part tariff, a lower per-unit price and a higher fixed fee), compared to a market with no downstream collusion. This result suggests an interesting new plus factor that is associated with upstream pricing behavior. Strategically, it makes

\textsuperscript{19}See U.S. Department of Justice (1993).
\textsuperscript{20}See http://ec.europa.eu/competition/cartels/leniency/leniency.html.
\textsuperscript{21}See U.S. Department of Justice (1994).
\textsuperscript{23}ABA Section of Antitrust Law, Antitrust Law Developments (2007):11-16.
economic sense for suppliers to adjust prices when their profits are threatened by the collusive conduct of their customers, who are resellers (or intermediaries) of their products. How these price adjustments are made depends on the types of supply contracts. In general, linear pricing contracts do not give suppliers much scope to work with to contain profit loss due to downstream collusion, because optimizing suppliers charge the same linear price with and without downstream collusion. But nonlinear pricing contracts do, and many industries traditionally adopt nonlinear pricing to enhance total profits of the supply chain. Many nonlinear pricing contracts are simply various forms of quantity discount (two-part tariff is quantity discount in the extreme). With these contracts, price adjustments in response to downstream collusion could follow a similar pattern. If this pattern is identified, then courts may add it to the basket of plus factors permitted as circumstantial evidence of collusion. The novelty lies in the fact that this potential new plus factor would be upstream behavior used as circumstantial evidence of downstream collusion.

4.3 Antitrust Damages

The supplier’s ability to use supply contracts to restrict downstream collusion renews our understanding of collusion damages.

In the United States, The Clayton Act of 1914 awards treble damages and the cost of suit to “any person who shall be injured in his business or property by reason of anything forbidden in the antitrust laws”\(^\text{24}\), providing incentives for private enforcement efforts. In cartel cases, courts are willing to award overcharge to direct purchasers, but are reluctant to restore suppliers’ lost profits due to anticompetitively reduced demand [see \textit{Associated General Contractors}\(^\text{25}\)]. While the flaw of direct purchaser overcharge as a measure of antitrust harm is recognized [see, for example, \textit{Fisher} (2006) and \textit{Han et al.} (2009)], it remains difficult, if not impossible, to accurately calculate damages to direct and indirect suppliers of cartel members. One difficulty in measuring harm to direct suppliers is that suppliers can adjust pricing in response to decreased demand. Thus, for antitrust practitioners who want to correctly measure upstream damages, it is crucial to soundly understand suppliers’ responses to downstream cartel behavior.

It would be an oversimplification of the problem to assume that suppliers respond to decreased demand the same way regardless of the cause of the demand decrease. Particularly, if the cutback in demand has a collusive motive, then suppliers can adjust pricing to rein in downstream collusion (as this paper shows); but if the demand reduction is not caused by anticompetitive activity, then suppliers would adjust pricing in a


Admittedly, in practice, suppliers may not know whether downstream firms are colluding when deciding on pricing, and even if they do know, they may not be aware that using nonlinear pricing can constrict downstream collusion and mitigate their profit loss compared to using linear pricing. So how can the finding of this paper be used to improve upstream damage calculation in practice? My answer to this question is twofold:

(1) If suppliers have only offered the same linear pricing contracts in both collusion and non-collusion periods, then it is a sign that their pricing strategies have been taken advantage of by the downstream firms in forming a cartel. In this case, it is reasonable to compare the actual demand with the demand that would have realized without cartelization (the “but-for” consideration), and multiply the difference in demand by the upstream per-unit profit to obtain an estimation of upstream damages.26

(2) If suppliers have offered nonlinear pricing contracts during periods of collusion, then there is reason to believe that the negative impact of downstream collusion is minimal due to the limited scope of collusion allowed for by upstream nonlinear pricing. This information could also have ramifications for overcharge measurement when the plaintiffs are direct purchasers, but arguments would be more delicate on that front: Given upstream nonlinear pricing contracts, colluding may be the only way for the downstream firms to survive. In this situation, one could argue that it may actually be unfair to slap antitrust fines all on the downstream cartel, because suppliers’ nonlinear pricing behavior alone could have left these firms with no other choice but to collude. But one could also argue that suppliers would not have needed to price this way if there had not been any threat of downstream cartelization to begin with. In this sense, there seems to be no simple way to cleanly isolate the amount of responsibility borne by the downstream firms alone in creation of the cartel. Thus, decisions to transfer the amount of “overcharge” from cartel members to their direct customers simply based on the so-called “but-for” transactions between the two groups may be misguided.

The above is far from suggestion of a perfect solution to the problem of measuring upstream damages. Rather, it proposes one possible way to partially reconcile the theoretical ideal of awarding all injured parties their respective antitrust damages, and the Court’s concern that an overly complex apportionment of damages would undermine the effectiveness of treble-damages suits. The Court is willing to forgo the benefit of appropriately compensating injured parties who are not direct purchasers, in exchange for a simple, low-cost judicial process, in order to preserve private litigation efforts. The above discussion does not speak for all injured parties, but it suggests that allowing suppliers legal standing to sue for collusion damages could actually be viable (and not too complex to implement) if upstream pricing is linear. Denying standing to suppliers

26Strategically, suppliers who only offer linear pricing optimally maintain the same unit price with and without downstream collusion.
who price nonlinearly could be less costly to society than denying standing to those who
price linearly.

4.4 Predatory Pricing Diagnosis

In this part, I dabble in the territory of Section 2 of the Sherman Act\textsuperscript{27}, which penalizes
monopolization behavior and attempts to monopolize. Predatory pricing cases fall into
this category.

In their seminal article, Areeda and Turner (1975) proposed a test for proving preda-
tory pricing (strategically pricing low temporarily to force out competition), which has
since had a significant amount of legal impact. As stated by Hovenkamp (2015), every
federal circuit court except the Eleventh has embraced some variation of the test, and
the Supreme Court has also come very close to adopting it.

The Areeda-Turner Test in its original form has two components: (1) test for “re-
coupment”, and (2) the AVC (average variable cost) test. In essence, to prove that a
firm’s pricing behavior is predatory, the plaintiff must show two things: (1) the preda-
tor can reasonably expect future monopoly profits to more than cover the initial cost of
pricing low temporarily, (2) the predator prices below average variable cost.

But for many years since Areeda and Turner’s article was popularized in 1975, the
courts mostly used only the second part of the test to rule on predatory pricing cases.
This practice has drawn criticisms from many economists. One criticism is that in mar-
kets with high fixed costs, pricing low but above average variable cost can still have
exclusionary effects. Thus the AVC test alone could produce too many false negatives,
and under-deter predation. This problem is only exacerbated by the fact that markets
with high fixed costs are precisely the ones most susceptible to predatory pricing. It
is then perhaps not surprising that predatory pricing claims have often been difficult
to win. The Areeda-Turner AVC test, though still in use, has much room for improve-
ment.\textsuperscript{28} In particular, finding a good way to determine above-cost predatory pricing is
difficult, but it would be of great value.

There are two other popular criticisms of the Areeda-Turner AVC test. One of them
is the inadequacy of average variable cost as a proxy for short-run marginal cost. The
other one is the more fundamental flaw of using short-run measures to infer predation.

The recoupment requirement has been brought back into discussion in the courtroom
since Matsushita\textsuperscript{29}, but the AVC assessment still remains an important part of the
Areeda-Turner test.

\textsuperscript{28}One attempt to improve the test is by Areeda and Hovenkamp (2015), who refine the definition
of “variable” cost. However, this refinement hugely complicates fact finding, and has so far not seen
much success in case law.
\textsuperscript{29}Matsushita Electric Industrial Co., Ltd. v. Zenith Radio Corp., 475 U.S. 574 (1986).
In order to improve the test on predatory pricing, we need to have a solid understanding of all underlying economic factors that could possibly cause a defendant to price low. Traditional places to look for these factors are the defendant’s cost structure and competitive landscape. But the results of this paper suggest that antitrust practitioners should also examine competition in the downstream market, provided that the downstream market is not the end user market, and the defendant prices nonlinearly. This is because an upstream supplier with market power who prices nonlinearly can and will optimally adjust pricing in response to changes in downstream competition, which means competition in the downstream market should also influence a defendant’s pricing decision. A supplier raising the fixed fee and lowering the unit price (or offering a steeper discount for large quantities while demanding a higher payment for small quantities) may simply be responding to downstream cartelization, though this pricing behavior could have the appearance of predation.

4.5 Regulation

Different regulations aim to achieve different goals, not all of which are economic. Regulations with economic goals may also try to achieve different outcomes. One regulation may be in place to increase economic efficiency, while another may be enforced for redistribution.

A large part of economic regulation, with various goals, has to do with pricing. The government may want to regulate a natural monopoly not because it is inefficient in production, but because it is inefficient in allocation. By regulating how a monopolist prices, we can potentially achieve increases in both quantity and total surplus.

Ideally, all regulations should result from full analyses of benefits and costs. A regulation based on only partial benefit-cost analysis could have unintended consequences. This paper aims to shed some new light on the relationship between pricing by an upstream supplier and competition in the direct downstream market. The main conclusion of the model is that downstream competition can be influenced by upstream pricing. Thus it becomes an issue that price regulation on an upstream industry may unintentionally affect downstream competition. It seems that price regulations are rarely made with a holistic consideration of their competitive effects on all related markets. This could be a potential area for improvement for regulators.

4.6 Implication for Antitrust Enforcement

In view of the above policy implications, the following scenario reflects some potential weaknesses of antitrust enforcement in its current state.

Consider Supplier A, a natural monopolist who manufactures and sells a product to
retailers, who then compete in reselling to consumers. Supplier A has acquired monopoly power through production efficiency, and offers quantity discounts to all retailers. Using quantity discounts, Supplier A is able to coordinate the supply chain and capture a large portion of the total profit. But at some point, the retailers form a cartel, and enter into a secret agreement to all cut back on their orders from Supplier A. By colluding to reduce quantities available in the consumer market, the retailers are able to increase the price of the product, and obtain higher profits than before. As a result, consumer surplus is reduced, and Supplier A’s profit falls. Unsuspecting consumers have not taken any legal action against the colluding retailers, and Supplier A cannot take the retailers to court because as a supplier, not customer of the retailers, it will not be given antitrust standing to sue for damages. However, Supplier A can change the terms of the supply contracts to influence the colluding retailers’ behavior. It starts offering retailers steeper discounts for large quantities and charging higher prices for small quantities.\footnote{Refer to Proposition 5.} In response, the retailers increase their collusive quantities.\footnote{Now the retailers have technically lost their ability to compete, but are forced to collude at a large quantity.} Consequently, Supplier A’s profit loss is reduced, and consumer surplus is restored to almost the same level as before retail collusion took place.

Supplier A’s execution of price change is noticed by Supplier B, a company that has been eyeing the industry but has not been successful in entering the market, because it owns outdated technology, and simply cannot produce a product as good as Supplier A’s, or offer a price as low as Supplier A’s. Supplier B now sees an opportunity, and sues Supplier A for predatory pricing, based on the steeper discounts Supplier A recently started to offer to retailers. Supplier B’s claim is that it is not able to enter the market because Supplier A has decided to price low anticompetitively in order to keep out potential competitors. The judge examines Supplier A’s costs, and applies the Areeda-Turner AVC test. Evidence shows that Supplier A’s average variable cost (AVC), used as a proxy for its short-run marginal cost (SRMC), is higher than the discounted price it currently offers. And the judge concludes that Supplier A can reasonably anticipate future monopoly profits to more than cover its “investment” of temporarily pricing at a loss. Supplier A defends its pricing strategy by explaining the need to restrict retail collusion. But the court dismisses Supplier A’s defense as baseless and irrelevant, and rules in Supplier B’s favor. (In this case, AVC is a poor proxy for SRMC. Areeda and Turner themselves noted that only in a competitive market in equilibrium, and with modest fixed costs, AVC and SRMC are close together. Supplier A owns a superior technology. Its fixed cost is not modest. It is also likely not operating in equilibrium at this point, as quantity has just been artificially reduced anticompetitively, and it is responding to this change. In such a response, Supplier A may be pricing below AVC,
but it is not pricing below SRMC, thus it is not incurring a loss in the short run.)

With its pricing behavior being “disciplined” by the court, Supplier A now has no way to reverse the profit loss caused by retail collusion. So it turns to cost cutting, and in doing so, sacrifices the quality of its once superior product. This creates an opening for Supplier B to enter the market with a subpar product, and share some of the industry profit. Meanwhile, retailers continue to collude, because collusion still gives them higher profits than competing. After a while, Supplier A and Supplier B decide that it would help both of them if they collude in pricing in order to restrict retail collusion. Retailers soon find out about the two suppliers’ collusive arrangement, and sue them for damages as direct customers. Retailers win the case and receive compensation, yet their own colluding behavior stays unpunished.

5 Extension: Asymmetric Collusion

In this section, I show that Propositions 1 is robust to retailers adopting a common asymmetric collusive scheme, in the sense that when confronted with asymmetric downstream collusion, two-part tariff still induces a larger total quantity than linear pricing, and it is still preferred by the supplier (as well as consumers and society) to linear pricing.\(^{32}\)

I analyze an asymmetric equilibrium where the two retailers collude in the following way under two-part tariff: in each period, only one retailer purchases a positive amount for resale, while the other stays off the market. The idea of the collusive agreement is to maximize the retailers’ collective profit in each period by having only one of them pay the fixed fee. The two retailers take turns staying off the market.\(^{33}\)

In the asymmetric formulation, after observing \(w\) and \(f\), the retailers’ problem is

\[
\text{maximize } \Pi^{\text{Coll}} = \Pi^{\text{Coll}} = (a - bq - w)q - f \\
\text{subject to } h_{\text{In}}(q, w, f) \geq 0 \\
\text{and } h_{\text{Out}}(q, w, f) \geq 0 ,
\]

where \(h_{\text{In}}(q, w, f)\) is the collusion condition for the retailer in the market, and \(h_{\text{Out}}(q, w, f)\) is the collusion condition for the retailer outside of the market. \(h_{\text{In}}(q, w, f)\) and \(h_{\text{Out}}(q, w, f)\) together constitute the retailers’ individual rationality constraint.

The supplier wants to maximize \(\pi_S = q(w - c) + f\) by choosing \(w\) and \(f\). We will be able to backward induct how these choices are made by the supplier after analyzing

\(^{32}\)I am writing another paper to examine how upstream contracting influences downstream collusion when there are \(N \geq 2\) retailers in the downstream market. In that paper, I provide more robustness results for asymmetric collusion.

\(^{33}\)In practice, this phenomenon may take the form of retailers reselling the good to one another.
the retailers’ best responses.

Suppose such an asymmetric collusive equilibrium exists. Then in equilibrium, two collusion conditions must hold simultaneously for any one period: a retailer who is in the market finds it at least as profitable to adhere to the collusive agreement as to deviate, and the same goes for a retailer who is outside of the market. As such, we have the following two conditions:

\[
(1 - \delta)[(a - bq - w)q - f] + \frac{1}{2} \delta[(a - bq - w)q - f] \\
\geq (1 - \delta)\frac{(a - w)^2}{4b} - (1 - \delta)f + \delta \max \left\{ \frac{(a - w)^2}{9b} - f, 0 \right\} \tag{In}
\]

for the active retailer, and

\[
\frac{1}{2} \delta[(a - bq - w)q - f] \\
\geq (1 - \delta)\frac{1}{4b}(a - bq - w)^2 - (1 - \delta)f + \delta \max \left\{ \frac{(a - w)^2}{9b} - f, 0 \right\} \tag{Out}
\]

for the inactive retailer.\(^\text{34}\)

Let collusion condition (In) be rewritten as \(h_{I_n}(q, w, f) \geq 0\). In other words, let

\[
h_{I_n}(q, w, f) \equiv (1 - \delta)[(a - bq - w)q - f] + \frac{1}{2} \delta[(a - bq - w)q - f] - (1 - \delta)\frac{(a - w)^2}{4b} + (1 - \delta)f - \delta \max \left\{ \frac{(a - w)^2}{9b} - f, 0 \right\}.
\]

Let collusion condition (Out) be rewritten as \(h_{O_{out}}(q, w, f) \geq 0\). In other words, let

\[
h_{O_{out}}(q, w, f) \equiv \frac{1}{2} \delta[(a - bq - w)q - f] - (1 - \delta)\frac{1}{4b}(a - bq - w)^2 + (1 - \delta)f - \delta \max \left\{ \frac{(a - w)^2}{9b} - f, 0 \right\}.
\]

First the supplier chooses a pair \((w, f)\) as the contract offer, then the two retailers agree on \(q\), the quantity purchased by an active retailer in a period. Each player maximizes the value of a discounted stream of profits going into the future. The supplier makes her decision at only one point in time, whereas the retailers first make a collective decision on the choice of \(q\), then each retailer individually purchases his own quantity in each period.

Similarly to the previous case of retail symmetric collusion, complications arise in solving the supplier’s optimization problem, due to the non-interior nature of the solution. However, we can still reach some meaningful conclusions by analyzing how the supplier would choose the fixed fee \(f\) for any per-unit price \(w\). In what follows, I show that the main result is robust to asymmetric collusion.

\(^{34}\)Profits are a discounted stream of values, counting from the current period. For example, an active retailer could choose to collude or deviate in a period. If he chooses to collude, then he gets a collusive profit in the current period, and expects to get half of the total collusive profit for all future periods, conditional on the other retailer colluding.
For a given \( w \), Lemmas 4 - 7 show the dynamics of the collusion conditions caused by a changing fixed fee. Understanding these dynamics will be helpful to analyzing both the supplier and the retailers’ incentives in the asymmetric collusion setting. With this understanding, I will then go as far as I can to characterize the players’ equilibrium behavior, as relevant to the robustness result. Lemmas 4 - 7 are presentations of the dynamics of the individual collusion conditions in response to a changing fixed fee, while holding \( w \) constant.

**Lemma 4.** For any given \( w \), when \( f < f_0 = \frac{(a-w)^2}{9b} \), increasing \( f \) loosens individual collusion condition \((\text{In})\), by enlarging the set of sustainable collusive quantities \( \Theta_{\text{In}}(w, f) \equiv \{q : h_{\text{In}}(q, w, f) \geq 0\} \).

**Proof.** When \( f < f_0 = \frac{(a-w)^2}{9b} \), \( \max \left\{ \left(\frac{(a-w)^2}{9b}\right) - f, 0 \right\} = \frac{(a-w)^2}{9b} - f \). Thus, condition \((\text{In})\) can be written as

\[
\begin{align*}
h_{\text{In}}(q) &= \kappa_1 + \frac{1}{2} \delta f \geq 0,
\end{align*}
\]

where \( \kappa_1 = (1 - \frac{1}{2} \delta)(a - bq - w)q - (1 - \delta)\frac{1}{4b} (a - w)^2 - \delta\frac{(a-w)^2}{9b} \) is independent of \( f \). \( h_{\text{In}}(q) \) is a quadratic in \( q \), whose graph opens downward. Since \( \frac{1}{2} \delta > 0 \), increasing \( f \) loosens the individual collusion condition.

**Lemma 5.** For any given \( w \), when \( f \geq f_0 = \frac{(a-w)^2}{9b} \), increasing \( f \) tightens individual collusion condition \((\text{In})\), by shrinking the set of sustainable collusive quantities \( \Theta_{\text{In}}(w, f) \equiv \{q : h_{\text{In}}(q, w, f) \geq 0\} \).

**Proof.** When \( f \geq f_0 = \frac{(a-w)^2}{9b} \), \( \max \left\{ \left(\frac{(a-w)^2}{9b}\right) - f, 0 \right\} = 0 \). Thus, condition \((\text{In})\) can be written as

\[
\begin{align*}
h_{\text{In}}(q) &= \kappa_2 - \frac{1}{2} \delta f \geq 0,
\end{align*}
\]

where \( \kappa_2 = (1 - \frac{1}{2} \delta)(a - bq - w)q - (1 - \delta)\frac{1}{4b} (a - w)^2 \) is independent of \( f \). \( h_{\text{In}}(q) \) is a quadratic in \( q \), whose graph opens downward. Since \( -\frac{1}{2} \delta < 0 \), increasing \( f \) tightens the individual collusion condition.

**Lemma 6.** For any given \( w \), when \( f < f_0 = \frac{(a-w)^2}{9b} \), increasing \( f \) loosens individual collusion condition \((\text{Out})\), by enlarging the set of supportable collusive quantities \( \Theta_{\text{Out}}(w, f) \equiv \{q : h_{\text{In}}(q, w, f) \geq 0\} \).

**Proof.** When \( f < f_0 = \frac{(a-w)^2}{9b} \), \( \max \left\{ \left(\frac{(a-w)^2}{9b}\right) - f, 0 \right\} = \frac{(a-w)^2}{9b} - f \). Thus, condition \((\text{Out})\) can be written as

\[
\begin{align*}
h_{\text{Out}}(q) &= \kappa_3 + \left(1 - \frac{1}{2} \delta\right) f \geq 0,
\end{align*}
\]

where \( \kappa_3 = \frac{1}{2} \delta(a - bq - w)q - (1 - \delta)\frac{1}{4b} (a - bq - w)^2 - \delta\frac{(a-w)^2}{9b} \) is independent of \( f \). \( h_{\text{Out}}(q) \) is a quadratic in \( q \), whose graph opens downward. Since \( 1 - \frac{1}{2} \delta > 0 \), increasing \( f \) loosens the individual collusion condition.
Lemma 7. For any given \( w \), when \( f \geq f_0 = \frac{(a-w)^2}{9b} \), increasing \( f \) would:

- loosen individual collusion condition (Out), by enlarging the set of sustainable collusive quantities \( \Theta_{\text{Out}}(w,f) \equiv \{ q : h_{\text{In}}(q,w,f) \geq 0 \} \), if \( \delta < \frac{2}{3} \);

- tighten individual collusion condition (Out), by shrinking the set of sustainable collusive quantities \( \Theta_{\text{Out}}(w,f) \equiv \{ q : h_{\text{In}}(q,w,f) \geq 0 \} \), if \( \delta > \frac{2}{3} \); or

- not affect individual collusion condition (Out), if \( \delta = \frac{2}{3} \).

Proof. When \( f \geq f_0 = \frac{(a-w)^2}{9b} \), max \( \left\{ \frac{(a-w)^2}{9b} - f, \ 0 \right\} = 0 \). Thus, condition (Out) can be written as

\[
h_{\text{Out}}(q) = \kappa_4 + \left( 1 - \frac{3}{2} \delta \right) f \geq 0,
\]

where \( \kappa_4 = \frac{1}{2} \delta (a - bq - w) q - (1 - \delta) \frac{1}{9b} (a - bq - w)^2 \) is independent of \( f \). \( h_{\text{Out}}(q) \) is a quadratic in \( q \), whose graph opens downward. The sign of the coefficient \( (1 - \frac{3}{2} \delta) \) depends on the value of \( \delta \). If \( \delta < \frac{2}{3} \), then increasing \( f \) loosens the individual collusion condition; if \( \delta > \frac{2}{3} \), then increasing \( f \) tightens the individual collusion condition; if \( \delta = \frac{2}{3} \), then changing \( f \) does not affect the individual collusion condition. \( \Box \)

The dynamics of changing the fixed fee \( f \) goes as follows: Increasing the fixed fee from zero to the Cournot-Nash profit \( f_0 \) makes both types of retailers (the active and the inactive) better able to collude\(^{35}\), where the individual collusion condition for the inactive retailer loosens at a faster rate than that for the active retailer\(^{36}\). To understand this effect, note that in a cooperative period, an inactive retailer would only have to pay the fixed fee if he chooses to deviate. Thus, increasing a small fixed fee makes an inactive retailer more willing to collude (thereby avoiding a larger fee). As for an active retailer, increasing a small fixed fee also makes him more willing to collude, because an active retailer who chooses to collude also expects to save the fixed fee in some future periods.

Once \( f \) exceeds \( f_0 \), the collusion condition for the active retailer begins to tighten, for the same reason we previously discussed in the symmetric case: A fixed fee exceeding \( f_0 \) wipes out the entirety of the Cournot-Nash profit, in which case the punishment profit is zero because exiting the market would serve a retailer better than staying and paying a hefty fixed fee. On the other hand, when \( f \) exceeds \( f_0 \), the individual collusion condition for the inactive retailer could either loosen or tighten, depending on the value of \( \delta \). Three possible scenarios:

1. If \( \delta < \frac{2}{3} \), then further increasing the fixed fee above the threshold \( f_0 \) only works to further loosen the individual collusion condition for the inactive retailer. In this

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\(^{35}\)The expansion of the set of sustainable collusive quantity indicates a group better able to collude. 

\(^{36}\)This can be seen with a contrast of conditions (In:1) and (Out:1): the coefficient of \( f \) in (Out:1) is larger than the coefficient of \( f \) in (In:1).
case, the binding individual collusion condition would be $h_{In}$, and the supplier would keep increasing $f$ until $h_{In} = 0$.

2. If $\delta = \frac{2}{3}$, then further increasing the fixed fee above the threshold $f_0$ does not affect the individual collusion condition for the inactive retailer. In this case, the binding individual collusion condition would still be $h_{In}$, and the supplier would keep increasing $f$ until $h_{In} = 0$.

3. If $\delta > \frac{2}{3}$, then further increasing the fixed fee above the threshold $f_0$ tightens the individual collusion condition for the inactive retailer. In this case, it is not immediately clear whether (In) or (Out) would be the binding individual collusion condition. But note that for the purpose of proving robustness of the main result, it is sufficient to know that one of (In) and (Out) would bind.

The above three scenarios can be explained by two opposing forces. On the one hand, there is a strictly positive benefit of colluding: Retailers can expect to save some fixed fees in at least some periods. This consideration makes an inactive retailer more willing to collude when the fixed fee increases, because when the fixed fee increases, the benefit of colluding becomes higher. This is the force that expands the retailers’ set of sustainable collusive quantities when the fixed fee increases. On the other hand, similarly to the case of symmetric collusion, increasing the fixed fee above the Cournot-Nash profit impacts the retailers’ punishment profile, and thus reduces the set of sustainable collusive quantities. Which effect is stronger depends on how big $\delta$ is. If $\delta$ is small, then the fixed-fee saving effect dominates (Scenario 1); if $\delta$ is large, then the collusion restriction effect dominates (Scenario 3).

**Assumption 1.** For each per-unit price $w$, we only study $\delta(w)$ sufficiently large to support the retail monopoly quantity under the wholesale linear pricing rule $\Phi(q_i) = wq_i$, where $\Phi(q_i)$ is the required payment to the supplier from retailer $i$ who purchases quantity $q_i$.

Assumption 1 does not undermine the robustness result in a meaningful way, because from a welfare standpoint, it is when retailers are patient enough to sustain a retail monopoly quantity that methods to restrict collusion are of the most interest. If retailers are rather impatient, then the downstream market would be rather competitive under all types of supply contracts. In that case, the supplier would be less concerned about retail collusion, and can easily coordinate the supply channel, so any constraint that can be put on downstream collusion in theory would be less interesting and useful for practice.

If $\delta$ satisfies Assumption 1, then Proposition 1 is robust to retailers adopting the asymmetric collusive scheme. This result is summarized below in Propositions 8 and 9.
Proposition 8. When confronted with asymmetric retail collusion, a monopolist supplier still prefers offering a two-part tariff contract to offering a linear pricing contract.

Proof. It suffices to show that for any per-unit price $w$, $f^*(w) > 0$ is true. To do that, I argue that $f(w) = 0$ cannot be in equilibrium.

Suppose $f(w) = 0$. Then optimizing retailers must choose $q$ such that $q = Q_{RM}(w) = \frac{a-w}{2b}$, where $Q_{RM}(w)$ is the retail monopoly quantity for a given $w$.\(^{37}\) For any given $w$, the individual Cournot-Nash profit is $f_0(w) = \frac{(a-w)^2}{9b} > 0$. Since $f(w) = 0$, each retailer obtains $f_0(w)$ in any punishment period. Due to Lemmas 4 and 6, the supplier could deviate to charging a positive fixed fee $f^*(w) \in (0, f_0(w))$ without affecting the retail cartel’s choice of $q$. Such deviation would strictly increase the supplier’s profit. Therefore, $f(w) = 0$ cannot be in equilibrium.

We can again go further than Proposition 8 to investigate finer characteristics of $f^*(w)$ under asymmetric retail collusion, which would be essential for examination of the collusion-restricting effect of a two-part tariff contract.

Proposition 9. Under asymmetric retail collusion, a monopolist supplier offering a two-part tariff chooses $(w, f^*(w))$ such that $f^*(w) > f_0(w) = \frac{(a-w)^2}{9b}$. As a result, when $\delta > \frac{2}{3}$, the total quantity in equilibrium exceeds the retail monopoly quantity under linear pricing $Q_{RM}(w) = \frac{a-w}{2b}$.

Proof. Under Assumption 1, $f(w) \in [0, f_0(w))$ cannot be in equilibrium, since the supplier can always do better by adding $\epsilon > 0$ to $f(w) \in [0, f_0(w))$.

For $f(w) \in [f_0(w), \infty)$, the punishment profile is no longer the Cournot-Nash profile, but a profile where all retailers exit the market and obtain zero profit. This is the mechanism through which an appropriately set fixed fee that is high enough can restrict retail collusion. The supplier would keep increasing $f$ beyond the Cournot-Nash profit $f_0(w)$, until the set of sustainable collusive quantities, $\Theta_{Asym}(w, f) \equiv \{q : h_{In}(q, w, f) \geq 0 \text{ and } h_{Out}(q, w, f) \geq 0\}$, becomes a singleton. This is because when $\Theta_{Asym}(w, f)$ becomes a singleton, either $h_{In}(q, w, f) = 0$ with a single quantity $\hat{q}_{A,In}^{38}$, or $h_{Out}(q, w, f) = 0$ with a single quantity $\hat{q}_{A,Out}^{39}$. When $h_{In}(q, w, f) = 0$ at $q = \hat{q}_{A,In}$, each participating retailer purchases

$$\hat{q}_{A,In} = \frac{a-w}{2b};$$

\(^{37}\)Assumption 1 guarantees that the retailers are patient enough to sustain this collusive quantity.

\(^{38}\)\(q = \hat{q}_{A,In}\) is the unconstrained maximizer of $h_{In}(q, w, f)$.

\(^{39}\)\(q = \hat{q}_{A,Out}\) is the unconstrained maximizer of $h_{Out}(q, w, f)$.
when \( h_{Out}(q, w, f, M) = 0 \) at \( q = \hat{q}_{A,Out} \), each participating retailer purchases

\[
\hat{q}_{A,Out} = \frac{a - w}{b} \cdot \frac{1}{\delta + 1}.
\]

Since \( Q_{RM}(w) = \frac{a - w}{2b} \), we have the following:

\[
Q_{RM}(w) = \hat{q}_{A,In} < \hat{q}_{A,Out}, \tag{2}
\]

for any \( \delta \in (0, 1) \). Lemmas 4 - 7 imply that when \( \delta > \frac{2}{3} \), \( \hat{q}_{A,Out} \) will be reached. This result says that patient retailers’ ability to sustain the downstream monopoly quantity with an asymmetric collusive scheme can be restricted by a supplier demanding a fixed fee higher than the Nash threshold. The supplier is self-incentivized to pick such a high fixed fee, since this high fee also induces a large total quantity in equilibrium.

\[ \square \]

6 Conclusion

Collusion increases cartel members’ profits at the expense of not only their customers, but also their suppliers. While not having much legal standing to sue for damages caused by downstream collusion, a supplier with market power can nonetheless adjust nonlinear pricing (e.g., two-part tariff) in supply contracts to contain profit loss. The supplier adjusts pricing in a way that pushes downstream firms to collude at a large quantity. This adjustment of price by suppliers also increases the welfare of consumers and of society overall.

Because this effect of price adjustment on downstream collusion is only possible with nonlinear pricing, but not linear pricing, there is a new incentive for suppliers to contract with nonlinear pricing.

When upstream contracts are nonlinear, the presence of downstream collusion changes upstream pricing in a systematic way. This points to a potential new place for competition authorities to look for circumstantial evidence of collusion. Examination of changes in upstream pricing behavior may assist with detection of downstream collusion.

Regulation on upstream suppliers’ pricing behavior may impact competition in the downstream market.

Upstream collusion or mergers may facilitate the type of nonlinear pricing that restricts downstream collusion. Examination on upstream cartelization should not be isolated from examination on the downstream competitive environment.

Downstream collusion can cause damages to upstream suppliers. Such damages are less severe with upstream nonlinear pricing than with upstream linear pricing.
An upstream monopolist supplier can offer steep quantity discounts to restrict down-
stream collusion, but such an offering could have the appearance of predatory pricing.

Appendix A Some Proofs

PROOF OF PROPOSITION 1:

Both \( \pi^{\text{Coll}}(q, w, f) \) and \( h(q, w, f) \) are quadratic functions in \( q \) that open downwards, as shown in Figures 1 - 3.

The solution to the retailers’ problem \((R)\) is \( \hat{q}(w, f) = \arg \max_{q \in \Theta(w, f)} \pi^{\text{Coll}}(q, w, f) \).

If \( \hat{q}(w) \in \Theta(w, f) \), then \( \hat{q}(w, f) = \hat{q}(w) \). If \( \hat{q}(w) \notin \Theta(w, f) \), then \( \hat{q}(w, f) = \arg \min_{q \in \Theta(w, f)} \abs{q - \hat{q}(w)} \).

\( \hat{q}(w) = a - w \cdot \frac{1}{b} \), and \( \hat{q}(w) = a - w \cdot \frac{3-\delta}{3-\delta} \). Because \( \delta \in (0, 1) \), we have \( \hat{q}(w) < \hat{q}(w) \).

Note that \( \pi^{\text{Coll}}(q, w, f) \geq h(q, w, f) \) for all \( q \in \Theta(w, f) \). This is because \( h(q, w, f) \) represents the per-period average gain in profit from colluding rather than deviating, and thus cannot be larger than \( \pi^{\text{Coll}}(q, w, f) \), the per-period average profit from colluding, for sustainable collusive quantities.

Further, we know that in equilibrium, \( \frac{\partial h(q, w, f)}{\partial f} = -\delta < 0 \). So as \( f \) increases, \( \Theta(w, f) \) becomes a smaller set, and \( \inf \Theta(w, f) \) becomes larger (exceeding \( \hat{q}(w) \)). This means that before \( \Theta(w, f) \) reduces to a singleton, the supplier always has an incentive to increase \( f \), since doing that not only brings in additional revenue from a higher fixed fee, but also increases colluding retailers’ purchase quantities.

When \( \Theta(w, f) \) becomes a singleton, \( \tilde{q}(w) = \hat{q}(w) \), and \( h(\tilde{q}(w), w, f^*(w)) = 0 \), from which \( f^*(w) \) can be solved.

We can verify that the supplier does not have any incentive to further raise \( f \) above \( f^*(w) \): If she does, then \( \Theta(w, f) \) becomes empty, which means retail collusion breaks down. Since \( f^*(w) > f_0(w) \), the Cournot-Nash profit threshold, the retailers will simply exit the market in response. Retailers leaving the market would lead to zero profit for the supplier. Knowing this, the supplier would not want to deviate from \( f = f^*(w) \).

By definition, \( q^*(w) \) solves \((R)\) when the contract terms are \((w, f^*(w))\). So \( q^*(w) = \tilde{q}(w) \).

PROOF OF PROPOSITION 2:

\[
\frac{\partial \pi_S}{\partial w} = \frac{2}{1 - \delta} \left[ \frac{\partial q}{\partial w} \cdot (w - c) + q + \frac{\partial f}{\partial w} \right] = 0
\]

\[\implies w^* = c + (a - c) \frac{1 - \delta}{4 - 2\delta} \in \left( c, \frac{a + c}{2} \right).\]
Correspondingly,

\[ f^{**} = \frac{(a - w^{**})^2}{4b[2 + \frac{1-\delta}{4}]} = \frac{(a - c)^2}{4b} \cdot \frac{1}{2 - \delta^2}(9 - \delta) > 0. \]

**PROOF OF PROPOSITION 3:**

\[ Q^{**} = 2\hat{q}(w^{**}) \]
\[ = \frac{2(a - w^{**})}{b} \cdot \frac{3 - \delta}{9 - \delta} \]
\[ = \frac{a - c}{b} \cdot \frac{(3 - \delta)^2}{(2 - \delta)(9 - \delta)}. \]

When the supplier uses linear pricing, and the two retailers compete, in equilibrium, the supplier sets the industry monopoly price \( P_M = \frac{a+c}{2} \), and the resulting total quantity is \( Q^{RC} = \frac{a-c}{3b} \). Because \( \frac{(3-\delta)^2}{(2-\delta)(9-\delta)} > \frac{1}{3} \), it follows that \( Q^{**} > Q^{RC} \).

**PROOF OF LEMMA 3:**

\[ Q^{**} = \frac{a - c}{b} \cdot \frac{(3 - \delta)^2}{(2 - \delta)(9 - \delta)}. \]

Hence,

\[ \frac{\partial Q^{**}}{\partial \delta} = \frac{a - c}{b} \cdot \frac{(3 - \delta)(5\delta - 3)}{(2 - \delta)^2(9 - \delta)^2}. \]

Thus, when \( \delta < \frac{3}{5} \), we have \( \frac{\partial Q^{**}}{\partial \delta} < 0 \); when \( \delta = \frac{3}{5} \), \( \frac{\partial Q^{**}}{\partial \delta} = 0 \); and when \( \delta > \frac{3}{5} \), \( \frac{\partial Q^{**}}{\partial \delta} > 0 \).

**PROOF OF PROPOSITION 4:**

\[ \frac{\partial w^{**}}{\partial \delta} = \frac{-(a - c)}{2(2 - \delta)^2} < 0, \]

\[ \frac{\partial f^{**}}{\partial \delta} = \frac{-2(a - w^{**})}{4b[2 + \frac{1-\delta}{4}]^2} \frac{\partial w^{**}}{\partial \delta} [2 + \frac{1-\delta}{4}] - (a - w^{**})^2(-\frac{1}{4}) > 0. \]

**Appendix B  Tables**
<table>
<thead>
<tr>
<th>Contract</th>
<th>Setting</th>
<th>$w$</th>
<th>$f$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Pricing</td>
<td>one-shot</td>
<td>$\frac{a+c}{2}$</td>
<td>-</td>
<td>$\frac{a-c}{3b}$</td>
</tr>
<tr>
<td></td>
<td>repeated/collusion</td>
<td>$\frac{a+2c}{2}$</td>
<td>-</td>
<td>$\frac{a-c}{4b}$</td>
</tr>
<tr>
<td>Two-Part Tariff</td>
<td>one-shot</td>
<td>$\frac{a+3c}{4}$</td>
<td>$\frac{(a-c)^2}{16b}$</td>
<td>$\frac{a-c}{2b}$</td>
</tr>
<tr>
<td></td>
<td>repeated/collusion</td>
<td>$c + (a - c)\frac{1-\delta}{1-2\delta}$</td>
<td>$\frac{(a-c)^2}{4b} \cdot \frac{(3-\delta)^2}{(2-\delta)(9-\delta)}$</td>
<td>$\frac{a-c}{b} \cdot \frac{(3-\delta)^2}{(2-\delta)(9-\delta)}$†</td>
</tr>
</tbody>
</table>

†The numbers in brackets reflect the relative sizes of the values in each column: they are order numbers from smallest to largest in each column. For example, two-part tariff in a repeated game setting gives the smallest equilibrium price $w$ (denoted with [1]), while linear pricing gives the highest equilibrium $w$ in both one-shot and repeated game settings (denoted with [3]).
†This result shows that restricted collusion with two-part tariff leads to a larger quantity than competition with linear pricing. In fact, $\frac{(3-\delta)^2}{(2-\delta)(9-\delta)}$ is very close to $\frac{1}{2}$ for all $\delta \in (0, 1)$.
Table 2: Welfare Comparison

<table>
<thead>
<tr>
<th>Contract Setting</th>
<th>Retailers</th>
<th>Supplier</th>
<th>R + S</th>
<th>Consumers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Pricing</td>
<td>one-shot</td>
<td>[3] $\frac{(a-c)^2}{b} \cdot \frac{1}{15}$</td>
<td>[2] $\frac{(a-c)^2}{b} \cdot \frac{1}{6}$</td>
<td>[2] $\frac{(a-c)^2}{b} \cdot \frac{2}{9}$</td>
<td>[2] $\frac{(a-c)^2}{b} \cdot \frac{1}{18}$</td>
</tr>
<tr>
<td></td>
<td>repeated/collusion</td>
<td>[4] $\frac{(a-c)^2}{b} \cdot \frac{1}{16}$</td>
<td>[1] $\frac{(a-c)^2}{b} \cdot \frac{1}{8}$</td>
<td>[1] $\frac{(a-c)^2}{b} \cdot \frac{3}{16}$</td>
<td>[1] $\frac{(a-c)^2}{b} \cdot \frac{1}{32}$</td>
</tr>
<tr>
<td>Two-Part Tariff</td>
<td>one-shot</td>
<td>[1] 0</td>
<td>[4] $\frac{(a-c)^2}{b} \cdot \frac{1}{4}$</td>
<td>[4] $\frac{(a-c)^2}{b} \cdot \frac{1}{4}$</td>
<td>[4] $\frac{(a-c)^2}{b} \cdot \frac{1}{8}$</td>
</tr>
<tr>
<td></td>
<td>repeated/collusion</td>
<td>[2] $\frac{(a-c)^2}{b} \cdot \lambda_R^\ddagger$</td>
<td>[3] $\frac{(a-c)^2}{b} \cdot \lambda_S^⋆$</td>
<td>[3] $\frac{(a-c)^2}{b} \cdot (\lambda_R + \lambda_S)^⦵$</td>
<td>[3] $\frac{(a-c)^2}{b} \cdot \lambda_C^*$</td>
</tr>
</tbody>
</table>

1These are surplus values.
2The column “Retailers” shows the total surplus obtained by the two retailers.
3The column “R + S” shows the total surplus obtained by all three players.
4The numbers in brackets reflect the relative sizes of the values in each column: they are order numbers from smallest to largest in each column, the same way as in Table 1.

$\lambda_R = \frac{3(3-\delta^2)(1-\delta)}{2(3-\delta)^2(1-\delta^2)}, 0 < \lambda_R < 0.005$, $\lambda_R$ is very close to zero for all $\delta \in (0, 1)$.

$\lambda_S = \frac{3(3-\delta)^2(1-\delta)}{2(3-\delta)^2(1-\delta^2)}, \frac{1}{12} \leq \lambda_S < \frac{1}{4}$, $\lambda_S$ is very close to $\frac{1}{4}$ for all $\delta \in (0, 1)$.

$\lambda_R + \lambda_S = \frac{(3-\delta)(9-5\delta)}{12(1-\delta^2)(1-\delta)}$, $\frac{1}{12} < \lambda_R + \lambda_S < \frac{1}{4}$, $\lambda_R + \lambda_S$ is very close to $\frac{1}{4}$ for all $\delta \in (0, 1)$.

$\lambda_C^*$ is very close to $\frac{1}{4}$. This can be implied from the result in Table 1 that the equilibrium total quantity in the model is very close to the industry monopoly quantity.

$\lambda_R + \lambda_S + \lambda_C^*$ is very close to $\frac{3}{8}$. This can be implied from the result in Table 1 that the equilibrium total quantity in the model is very close to the industry monopoly quantity.
Appendix C  Graphs

Upstream Two-Part Tariff Restricts Downstream Collusion.

Figure 4: Restricting collusion (1).

Figure 5: Restricting collusion (2).

Figure 6: Restricting collusion (3).

Figure 7: Restricting collusion (4).
References


