Title

har — Regression with HAR standard errors

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Syntax

har depvar [varlist1] (varlist2=inst list)[if] [in], kernel(string) [, noconstant level(#)]

options	Description
Model	
kernel(string)	set the type of kernels: Bartlett; Parzen; Quadratic Spectral; Orthonormal Series
noconstant	suppress constant term
Reporting	
level(#)	set confidence level; default is level(95)

kernel(string) is required.
you must tsset your data before using har; see [TS] tsset.
time-series operators are allowed.

Menu

statistics>Time series >Regression with HAR standard errors

Description

The command estimates an IV regression in the presence of heteroskedasticity and autocorrelation. Inferences are based on the fixed-smoothing asymptotics. The command is based on Sun (2013) and Sun (2014). Sun (2013) develops heteroskedasticity and autocorrelation robust F and t tests using an orthonormal series long run variance matrix estimator. The number of orthonormal bases is selected by minimizing the type II error of the associated test while controlling for its type I error. Sun (2014) introduces a new and easy-to-use asymptotic F test (and t test) based on kernel long run variance matrix estimators. The proposed bandwidth selection rule is testing-optimal and is similar to Sun (2013). The command allows for three types of kernels: the Bartlett, Parzen, and Quadratic Spectral kernels.

For the sake of simplicity, we refer to the orthonormal series long run variance estimator in Sun (2013) as the kernel long run variance estimator with kernel “Orthonormal Series.” In total, the command allows for four types of kernels: Bartlett, Parzen, Quadratic Spectral, and Orthonormal Series.

Options

Model

kernel(string)specifies the type of kernels (Bartlett; Parzen; Quadratic Spetral; Orthonormal Series) to be used in the estimation of the covariance matrix.
noconstant; see [R] estimation options.

Reporting

level(#); see [R] estimation options.

Remarks and examples

Sun (2013) and Sun (2014) introduce simple and trustworthy inference procedures that are robust to heteroskedasticity and autocorrelation. The HAR variance estimator in Sun (2013) is based on an orthonormal series long run variance matrix estimator. The optimal number K of orthonormal bases is selected by minimizing the type II error of the associated test subject to a control of its type I error. The tests proposed by Sun (2013) are asymptotic F and t tests in an exact sense: the asymptotic distributions of the adjusted F statistic and t statistic are standard F and t distributions. The HAR variance estimator in Sun (2014) is based on the more conventional kernel long run variance matrix estimators. The F and t tests in Sun (2014) are approximate tests. The asymptotic distributions of the adjusted F and t statistic are not exactly F and t distributions, but they can be well approximated by the standard F and t distributions.

Example 1

```
. webuse idle2

. tsset time
        time variable:  time, 1 to 30
                delta:  1 unit

. har usr idle wio,kernel(bartlett)

Regression with HAR standard errors                 Number of obs   =         30
Kernel: Bartlett                                    F(  2,     17)  =      47.66
Data-driven optimal lag: 2                          Prob > F        =     0.0000
```

usr	Coef.	HAR Std.Err.	t	df	P>|t|	[95% Conf.	Interval]
idle	-.6670978	.0715786	-9.32	22	0.000	-.8155428	-.5186529
wio	-.7792461	.11897	-6.55	13	0.000	-1.036265	-.522227
_cons	66.21805	6.984346	9.48	19	0.000	51.59965	80.83646

Example 2

```
. har usr idle wio,kernel(bartlett) l(99) nocons

Regression with HAR standard errors                Number of obs  =        30
Kernel: Bartlett                                   F(  2,      3)  =      8.88
Data-driven optimal lag: 13                        Prob > F        =    0.0549
```

usr	Coef.	HAR Std.Err.	t	df	P>|t|	[99% Conf.	Interval]
idle	.0186886	.0101968	1.83	5	0.126	-.0224265	.0598037
wio	.2759991	.0954198	2.89	5	0.034	-.1087473	.6607454

Example 3

```
. har usr idle wio,kernel(parzen)

Regression with HAR standard errors                Number of obs  =        30
Kernel: Parzen                                     F(  2,      4)  =     50.87
Data-driven optimal lag: 10                        Prob > F        =    0.0014
```

usr	Coef.	HAR Std.Err.	t	df	P>|t|	[95% Conf.	Interval]
idle	-.6670978	.071317	-9.35	15	0.000	-.8191065	-.5150892
wio	-.7792461	.1143269	-6.82	12	0.000	-1.028343	-.5301492
_cons	66.21805	6.922399	9.57	14	0.000	51.37099	81.06512

Example 4

```
. har usr idle wio,kernel(quadratic)

Regression with HAR standard errors                Number of obs  =        30
Kernel: Quadratic Spectral                         F(  2,      4)  =     46.84
Data-driven optimal lag: 5                         Prob > F        =    0.0017
```

usr	Coef.	HAR Std.Err.	t	df	P>|t|	[95% Conf.	Interval]
idle	-.6670978	.0697384	-9.57	16	0.000	-.8149366	-.5192591
wio	-.7792461	.1131035	-6.89	13	0.000	-1.023591	-.5349009
_cons	66.21805	6.834698	9.69	15	0.000	51.65024	80.78587

Example 5

```
. har usr idle wio,kernel(orthoseries)

Regression with HAR standard errors                  Number of obs  =        30
Kernel: Orthonormal Series                           F(  2,      5)  =     43.17
Data-driven optimal K: 6                             Prob > F        =    0.0007
```

usr	Coef.	HAR Std.Err.	t	df	P>|t|	[95% Conf.	Interval]
idle	-.6670978	.0706388	-9.44	14	0.000	-.8186029	-.5155927
wio	-.7792461	.1122118	-6.94	12	0.000	-1.023735	-.5347576
_cons	66.21805	6.838414	9.68	14	0.000	51.55111	80.88499

Stored results

The command har uses ivregress command to get the estimated coefficient. So, in addition to the standard stored results from ivregress, har also stores the following results in e():

Scalars

e(N)	the number of observations
*# e(fdf)	the first value of degrees of freedom
*e(sF)	the adjusted F statistic
*e(ssdf)	the second value of degrees of freedom
*e(kopt)	the data-driven optimal number k of orthonormal bases
#e(kF)	the adjusted F statistic
#e(ksdf)	the second degrees of freedom
#e(lag)	the data-driven optimal truncation lag

Macros

e(cmd)	har
e(cmdline)	command as typed
e(varline)	variable line as typed
e(carg)	nocons or " " if specified
e(depvar)	name of dependent variable
e(title)	title in estimation output
e(vcetype)	title used to label Std. Err.
e(kerneltype)	kernel in the estimation

Matrices

e(b)	coefficient vector
*e(sstderr)	the adjusted std error for each individual coefficient

*e(sdf)	the degrees of freedom of the approximating t distribution
*e(st)	the t statistic
*e(sbetahat)	the IV coefficient vector
#e(kbetahat)	the IV coefficient vector
#e(kstderr)	the adjusted std error for each individual coefficient
#e(kdf)	the degrees of freedom of the approximating t distribution
#e(kt)	the t statistic

Functions

e(sample)	marks the estimation sample

note： for the other results in ereturn list see[R] ivregress;
* only for Orthonormal Series;
# only for Bartlett, Parzen, Quadratic Spectral.

Methods and formulas

Consider the regression model:

$$Y_t = X_t\theta_0 + e_t, t = 1,2,\cdots,T$$

where $\{e_t\}$ is a zero-mean process that may be correlated with the covariate process $\{X_t \in R^{1\times d}\}$. There are instruments $\{Z_t \in R^{1\times m}\}$ such that the moment conditions:

$$EZ_t'(Y_t - X_t\theta_0) = 0$$

hold if and only if $\theta = \theta_0$. We allow the process $\{Z_t'e_t\}$ to have the autocorrelation of unknown forms. The model may be over-identified with the degree of over-identification $q = m - d \geq 0$.

Define:

$$S_{ZX} = \frac{1}{T}\sum_{t=1}^{T} Z_t' X_t, \quad S_{ZZ} = \frac{1}{T}\sum_{t=1}^{T} Z_t' Z_t, \quad S_{ZY} = \frac{1}{T}\sum_{t=1}^{T} Z_t' Y_t.$$

Then the IV estimator of θ_0 is

$$\hat{\theta}_{IV} = [S_{ZX}'W_{0T}^{-1}S_{ZX}]^{-1}[S_{ZX}'W_{0T}^{-1}S_{ZY}]$$

where $W_{0T} = S_{ZZ} \in R^{m\times m}$, $\underset{T\to\infty}{Plim}\, W_{0T} = W_0$.

We are interested in testing the null $H_0{:}\, R\theta_0 = r$ against the alternative $H_1{:}\, R\theta_0 \neq r$, where $r \in R^{p\times 1}$ and $R \in R^{p\times d}$ is a matrix of full row rank. Nonlinear restrictions can be converted into linear ones via the delta method. Under some standard high-level conditions, we have

$$\sqrt{T}R(\hat{\theta}_{IV} - \theta_0) = \sqrt{T}(R\hat{\theta}_{IV} - r) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T} u_t + o_p(1)$$

where $G_0 = ES_{ZX} \in R^{m\times d}$ and $u_t = R(G_0'W_0^{-1}G_0)^{-1}G_0'W_0^{-1}Z_t'e_t$ is the transformed moment process. It then follows that $\sqrt{T}R(\hat{\theta}_{IV} - \theta_0) \xrightarrow{d} N(0,\Omega)$, where $\Omega = \sum_{j=-\infty}^{j=+\infty} E\, u_t u_{t-j}'$ is the long run variance of $\{u_t\}$. The Wald statistic for testing H_0 against H_1 is

$$F_{IV} = \sqrt{T}\left(R\hat{\theta}_{IV} - r\right)'\left(\hat{\Omega}\right)^{-1}\sqrt{T}\left(R\hat{\theta}_{IV} - r\right)/p.$$

Let $G_T = S_{ZX}$, $\hat{u}_t = R(G_T' W_{0T}^{-1} G_T)^{-1} G_T' W_{0T}^{-1} Z_t'$, and $\hat{u}^{ave} = T^{-1} \sum_{s=1}^{T} \hat{u}_s$. We consider the estimator $\widehat{\Omega}$ of the form

$$\widehat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \sum_{s=1}^{T} Q_h\left(\frac{s}{T}, \frac{t}{T}\right) (\hat{u}_t - \hat{u}^{ave})(\hat{u}_s - \hat{u}^{ave})',$$

where $Q_h(r, s)$ is a weighting function, and h is the smoothing parameter. The above estimator includes the kernel HAR variance estimators and the orthonormal series HAR variance estimator as special cases.

For the kernel HAR variance estimator, we let $Q_h(r, s) = k((r - s)/b)$ and $h = 1/b$ for a kernel function $k(\cdot)$ with $M_T = bT$ being the so-called truncation lag. Define

$$c_1 = \int_{-\infty}^{+\infty} k(x)dx, \ c_2 = \int_{-\infty}^{+\infty} k^2(x)dx.$$

For the Bartlett kernel, $c_1 = 1$, $c_2 = 2/3$; For the Parzen kernel, $c_1 = 3/4$, $c_2 = 0.539285$; For the Quadratic Spectral kernel, $c_1 = 1.25$, $c_2 = 1$. Let

$$K = \max\left(\left\lceil \frac{1}{bc_2} \right\rceil, p\right) - p + 1,$$

where $\lceil\cdot\rceil$ is the ceiling function, and

$$\kappa = \frac{1}{2}(\exp(b[c_1 + (p-1)c_2]) + (1 + b[c_1 + (p-1)c_2])).$$

Based on the kernel estimator $\widehat{\Omega}$, Sun (2014) shows that

$$P(F_{IV} > \kappa F_{p,K}^{\alpha}) = \alpha + o(b) + O((bT)^{-q}) + O\left(\frac{logT}{\sqrt{T}}\right),$$

where $F_{p,K}^{\alpha}$ is the $100(1-\alpha)\%$ quantile of the standard $F_{p,K}$ distribution.

Sun (2014) obtains the testing-optimal bandwidth b_{opt}:

$$b_{opt} = \begin{cases} \left[\dfrac{2qG'_{p,\delta^2}(\chi_p^{1-\alpha})|\bar{B}|}{\delta^2 G'_{(p+2),\delta^2}(\chi_p^{1-\alpha})c_2}\right]^{\frac{1}{q+1}} T^{\frac{-q}{q+1}}, \bar{B} > 0 \\ \left[\dfrac{G'_p(\chi_p^{1-\alpha})\chi_p^{1-\alpha}|\bar{B}|}{(\tau-1)\alpha}\right]^{\frac{1}{q}} \dfrac{1}{T}, \bar{B} \le 0 \end{cases}$$

where $\tau > 1$ is the permitted tolerance, $G'_{p,\delta^2}(z)$ is the pdf of the noncentral χ^2 distribution with degrees of freedom p and noncentrality parameter δ^2. In the above formula, $\chi_p^{1-\alpha}$ is the $1-\alpha$ quantile of the χ^2 distribution with p degrees of freedom and δ^2 is chosen to satisfy $P\{\varkappa > \chi_p^{1-\alpha}\} = 75\%$, where $\varkappa \sim \chi_p^2(\delta^2)$. In addition,

$$\bar{B} = tr(B\Omega^{-1})/p, B = -\rho_q \sum_{j=-\infty}^{\infty} |j|^q Eu_t u_{t-j}'$$

where q is the order of the kernel used, and ρ_q is the Parzen characteristic exponent of the kernel. For the Bartlett kernel, $q = 1, \rho_q = 1$; For the Parzen kernel, $q = 2, \rho_q = 6$; For the QS kernel, $q = 2, \rho_q = 1.421223$.

The parameter $\bar{B}$ is estimated by a standard VAR(1) plug-in procedure. This is what we opt for in the new command. Plugging the estimate of B into b_{opt} yields b_{temp}. The data-driven choice of b is then given by $\hat{b}_{opt} = \min(b_{temp}, 0.5)$. We do not use a *b* larger than 0.5 in order to avoid large power loss.

For the OS HAR variance estimator, we let $Q_h(r,s) = K^{-1}\sum_{j=1}^{K}\phi_j(r)\phi_j(s)$, and $h = K$, where $\{\phi_j(\cdot)\}_{j=1}^{K}$ are orthonormal basis functions on $L^2[0,1]$ satisfying $\int_0^1 \phi_j(r)\,dr = 0$ for $j = 1,2,\cdots,K$. Sun (2013) shows that the usual Wald statistic F_{IV} satisfies

$$\frac{K-p+1}{K}F_{IV} \overset{d}{\rightarrow} F_{p,K-p+1} = \frac{\chi_p^2/p}{\chi_{K-p+1}^2/(K-p+1)}$$

where $F_{p,K-p+1}$ is the F distribution with degrees of freedom$(p, K-p+1)$.

Sun (2013) obtains the testing-optimal K_{opt} as follows

$$K_{opt} = \begin{cases} \left[\dfrac{\delta^2 G'_{(p+2),\delta^2}\left(\chi_p^{1-\alpha}\right)}{4G'_{p,\delta^2}\left(\chi_p^{1-\alpha}\right)|\bar{B}|}\right]^{\frac{1}{3}} T^{\frac{2}{3}}, \bar{B} > 0 \\ \left[\dfrac{(\tau-1)\alpha}{G'_p\left(\chi_p^{1-\alpha}\right)\chi_p^{1-\alpha}|\bar{B}|}\right]^{\frac{1}{2}} T, \bar{B} \le 0 \end{cases}$$

As before, the parameter $\bar{B}$ is estimated by a standard VAR(1) plug-in procedure. Plugging the estimate of $\bar{B}$ into K_{opt} yields $\hat{K}_{temp}$.We truncate $\hat{K}_{temp}$ to be between $p+4$ and *T*. That is, we take

$$\tilde{K}_{temp} = \begin{cases} p+4, & if\ \hat{K}_{temp} \le p+4 \\ \hat{K}_{temp}, & if\ \hat{K}_{temp} \in (p+4, T] \\ T, & if\ \hat{K}_{temp} > T \end{cases}$$

Imposing the lower bound $p+4$ ensures that the variance of the approximating distribution $F_{p,K-p+1}$ is finite and that power loss is not very large. Finally, we round $\tilde{K}_{temp}$to the greatest even number less than $\tilde{K}_{temp}$. We take this greatest even number, denoted by $\hat{K}_{opt}$to be our data-driven and testing-optimal choice for *K*.

References

Sun, Y., 2013. A heteroskedasticity and autocorrelation robust F test using an orthonormal series variance estimator. Econometrics Journal, 16: 1-26.

Sun, Y., 2014. Let's fix it: Fixed-b asymptotics versus small-b asymptotics in heteroskedasticity and autocorrelation robust inference. Journal of Econometrics, 178: 660-677.

Also see

[TS]tsset—Declare data to be time-series data.