Not-for-Publication Appendix to
Neville Francis & Valerie Ramey "Is the Technology-Driven Real Business Cycle Dead?"

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This appendix shows the steps used to derive equations (2)-(7) in the text.

Consider the following model of the economy:

$Y_t = (A_t N_t)^\alpha K_t^{1-\alpha}$	Production Function	
$A_t = \mu^t A_0, \quad \mu > 1$	Technology Growth	
$K_{t+1} = (1-\delta)K_t + I_t$	Capital Accumulation	(A-1)
$C_t + I_t + G_t \le Y_t$	Resource Constraint	
$U(C_t, N_t) = \ln(C_t) + \phi_t \ln(1-N_t)$	Utility	
$C_t + I_t = (1-\tau_{nt})W_t N_t + (1-\tau_{kt})r_t K_t + \delta\tau_{kt}K_t - \psi_t$	Household Budget Constraint	
$G_t = \tau_{nt}W_t N_t + \tau_{kt}(r_t - \delta)K_t + \psi_t$	Government Budget Constraint	

Y is output, *A* is an exogenous process for labor augmenting technical change, *K* is capital, *N* is labor input, δ is the depreciation rate, *I* is investment, *C* is consumption, *G* is government purchases, ϕ_t is a preference shifter, *W* is the real wage, r_t is the pre-tax return on capital, τ_n is the tax on labor income, τ_k is the tax on capital income, and ψ is a lump-sum tax. The representative consumer chooses capital, consumption and labor to maximize the expected present discounted value of utility, with discount factor β. Consumers own the capital and rent it to firms. The government finances its spending through a combination of lump-sum taxes and distortionary labor and capital income taxes.

Following standard practice, we transform the economy to eliminate the nonstationarity arising from technology by dividing Y_t, K_t, I_t, C_t, G_t, W_t and ψ_t by A_t. Let lower case letters denote variables divided by A_t, and lower case letters with tildes denote variables divided by output Y, i.e., $k_t = K_t/A_t$ and $\tilde{k}_t = K_t/Y_t$. We have:

(A-2) $y_t = N_t^{\alpha} k_t^{1-\alpha}$

(A-3) $\mu \cdot k_{t+1} = (1-\delta)k_t + i_t$

(A-4) $c_t + i_t + g_t \leq y_t$

(A-5) $U(c_t, N_t) = \ln(c_t) + \phi_t \ln(1-N_t) + \ln(A_t)$

(A-6) $c_t + \mu \cdot k_{t+1} - (1-\delta)k_t = (1-\tau_{nt})w_t N_t + (1-\tau_{kt})r_t k_t + \delta\tau_{kt}k_t - \varphi_t$

(A-7) $g_t = \tau_{nt} w_t N_t + \tau_{kt}(r_t - \delta)k_t + \varphi_t$

Consumer optimization problem and FOCs:

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \phi_t \ln(1-N_t) + \ln A_t + \lambda_t [(1-\tau_{nt})w_t N_t + (1-\tau_{kt})r_t k_t + \delta\tau_{kt}k_t - \varphi_t - c_t - \mu \cdot k_{t+1} + (1-\delta)k_t] \right\}$$

(A-8) $\frac{\partial L}{\partial c_t} = 0 \rightarrow \frac{1}{c_t} = \lambda_t$

(A-9) $\frac{\partial L}{\partial N_t} = 0 \rightarrow \frac{\phi_t}{1-N_t} = \lambda_t (1-\tau_{Nt}) \cdot w_t$

(A-10) $\frac{\partial L}{\partial k_{t+1}} = 0 \rightarrow \beta \cdot \lambda_{t+1}(1-\tau_{kt+1})r_{t+1} + \beta \cdot \lambda_{t+1} \cdot \delta \cdot \tau_{kt+1} + \beta \cdot \lambda_{t+1}(1-\delta) = \mu \cdot \lambda_t$

Firm optimization problem and FOCs:

$$Max \quad \pi = N_t^{\alpha} k_t^{1-\alpha} - w_t N_t - r_t k_t$$

(A-11) $\frac{\partial \pi}{\partial k_t} = 0 \rightarrow \ r_t = (1-\alpha)\left(\frac{k_t}{N_t}\right)^{-\alpha}$

(A-12) $\frac{\partial \pi}{\partial N_t} = 0 \rightarrow \ w_t = \alpha\left(\frac{k_t}{N_t}\right)^{1-\alpha}$

Calculating the steady-state of the transformed economy:

1. Derivation of the Marginal Rate of Substitution Equation (text equation (4)):

Imposing steady-state, equations (A-8), (A-9) and (A-11) imply:

$$\frac{\phi}{1-N} = \frac{(1-\tau_N)\alpha N^{\alpha-1} k^{1-\alpha}}{c}$$

which with substitutions from (A-2) and reorganization implies:

$$\frac{\phi N}{1-N} = \frac{(1-\tau_N)\alpha y}{c}$$

Combining this with (A-4) and recalling that tildes denote upper case variables dividing by Y, we have equation (4) from the text:

$$\frac{1-N}{N} = \frac{\phi}{\alpha(1-\tau_N)}\left[1 - \tilde{i} - \tilde{g}\right]$$

2. Derivation of Marginal Product of Capital condition (text equation (2)):

Rewrite (A-10) with elimination of time-subscripts (since steady-state):

$$\beta[(1-\tau_k)r+\delta\cdot\tau_k+1-\delta]=\mu$$

Combine this equation with (A-11) we obtain equation (2) from the text:

$$1+(1-\tau_k)[(1-\alpha)\left(\frac{k}{N}\right)^{-\alpha}-\delta]=\frac{\mu}{\beta}$$

Equation (5) from the text comes from imposing steady-state on equation (A-3) and dividing by Y. The derivation of the rest of the equations is obvious.

The following are the list of balanced growth equations from the text:

$1+(1-\tau_k)[(1-\alpha)\left(\frac{k}{N}\right)^{-\alpha}-\delta]=\frac{\mu}{\beta}$	MP of Capital-Time Preference Link	(2)
$\tilde{c}+\tilde{g}+\tilde{i}=1$	Resource Constraint	(3)
$\frac{1-N}{N}=\frac{\phi}{\alpha(1-\tau_n)}[1-\tilde{i}-\tilde{g}]$	Marginal Rate of Substitution	(4)
$\tilde{i}=(\mu+\delta-1)\left(\frac{k}{N}\right)^{\alpha}$	Investment Rate	(5)
$Y=AN\left(\frac{k}{N}\right)^{1-\alpha}$	Production Function	(6)
$\alpha\frac{Y}{N}=W=\alpha A\left(\frac{k}{N}\right)^{1-\alpha}$	Labor Productivity	(7)