Public investment in a medium scale DSGE model featuring ROT consumers

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Abstract

This note presents the structure of the medium-scale New Keynesian model used both by Valerie A. Ramey and Sarah Zubairy "Short-Run Government Spending Multipliers in DSGE Models: The Crucial Role of Investment Adjustment Costs and Driving Processes" and Ramey (forthcoming). It is an accompaniment to the Dynare programs.

These notes combine material from Uribe's (2007) rule of thumb notes for Galí et al. (2007) with wage stickiness and other rigidities following Colciago (2011) and Schmitt-Grohé and Uribe (2005). In addition, they incorporate public investment.

1 Households

There is a continuum of households indexed by $i \in [0,1]$. As in GLV (2004) and GLV (2007), households in the interval $[0,\gamma]$ cannot access financial markets and do not have an initial capital endowment. These agents simply consume their available labor income in each period. The rest of the households in the interval $(\gamma,1]$ are standard Ricardian households that have access to the market for physical capital and to a full set of state-contingent securities.

We assume a continuum of differentiated labor inputs indexed by $j \in [0,1]$. As in Schmitt-Grohe and Uribe (2005), agent i supplies each possible type of labor input. Wage-setting decisions are made by labor type specific unions indexed by $j \in [0,1]$. Given the wage W_t^j, fixed by union j, agents stand ready to supply as many hours to the labor market j, h_t^j, as required by firms, that is

$$h_t^j = \left(\frac{w_t^j}{w_t}\right)^{-\eta_w} h_t^d$$

where $\eta_w > 1$ is the elasticity of substitution between labor inputs. Here h_t^d aggregate labor demand and $w_t \equiv W_t/P_t$, where W_t is an index of the wages prevailing in the economy at time t. Agents are distributed uniformly across unions; hence, aggregate demand for labor type j is spread uniformly across the households. It follows that the individual quantity of hours worked, h_t^i, is common across households, and we denote it as h_t. This must satisfy the time resource constraint $h_t = \int_0^1 h_t^j dj$ Combining them, we get,

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\eta_w} dj$$

The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by $h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\eta_w} dj$.

Optimizing Households

Optimizing households maximize

$$E_0 \sum_{t=1}^{\infty} \beta^t \left[\ln c_{ot} - \nu \frac{h_{ot}^{1+\phi}}{1+\phi} \right]$$

subject to

$$c_{ot} + i_{ot} + \frac{E_t r_{t,t+1} A_{t+1}}{P_t} + \tau_{ot} = \frac{A_t}{P_t} + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj + r_t^k u_t k_{ot} + \Phi_t$$

(1)
$$k_{ot+1} = (1 - a(u_t))k_{ot} + i_{ot} \left(1 - S(\frac{i_{ot}}{i_{o,t-1}}) \right)$$

and to a no-Ponzi game constraint of the form $\lim_{t\to\infty} E_t r_{t,t+j} A_{t+j} \geq 0$, where $r_{t,t+j}$ is a stochastic pricing factor such that the period-t value of a stochastic nominal payment A_{t+1} in $t+j$ is given by $E_t r_{t,t+j} A_{t+j}$. where the Lagrangian is:

$$\begin{aligned} L &= E_0 \sum_{t=1}^{\infty} \beta^t \left[\frac{c_{ot}^{1-\sigma}}{1-\sigma} + \xi \frac{c_{gt}^{1-\sigma}}{1-\sigma} - \nu \frac{h_{ot}^{1+\phi}}{1+\phi} \right] \\ &+ \beta^t \lambda_t \left[\frac{A_t}{P_t} + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj + r_t^k u_t k_{ot} + \Phi_t - c_{ot} - i_{ot} - \frac{E_t r_{t,t+1} A_{t+1}}{P_t} - \tau_{ot} \right] \\ &+ \beta^t \lambda_t q_t \left[(1 - a(u_t))k_{ot} + i_{ot} \left(1 - S(\frac{i_{ot}}{i_{o,t-1}}) \right) - k_{ot+1} \right] \end{aligned}$$

The optimality conditions are:

(2)
$$\frac{1}{c_{ot}} = \lambda_t$$

$$\lambda_t r_{t,t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}}$$

(3)
$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^k u_{t+1} + q_{t+1}(1 - a(u_{t+1})) \right]$$

(4)
$$r_t^k = q_t a'(u_t)$$

$$(5) \qquad \lambda_t = \lambda_t q_t \left(1 - S\left(\frac{i_{ot}}{i_{o,t-1}}\right) - i_{ot} S'\left(\frac{i_{ot}}{i_{o,t-1}}\right)\right) - \beta E_t \lambda_{t+1} q_{t+1} S'\left(\frac{i_{o,t+1}}{i_{o,t}}\right) i_{o,t+1}$$

Rule-of-Thumb Households

Rule-of-thumb households maximize

$$E_0 \sum_{t=1}^{\infty} \beta^t \left[\ln c_{rt} - \nu \frac{h_{rt}^{1+\phi}}{1+\phi}\right]$$

subject to

$$c_{rt} = h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\eta_w} dj - \tau_{rt}$$

Wage Setting Union

We extend the analysis in GLV (2007) and following Colciago (2011) assume that the nominal wage newly reset at t, $\tilde{W}_t$, is chosen to maximize a weighted average of agents' lifetime utilities. The weights attached to the utilities of Ricardian and non-Ricardian agents are $(1-\gamma)$ and γ, respectively. Notice that, in writing down the problem, we have assumed that the union takes into account the fact that firms allocate labor demand uniformly across different types workers of type, independently of their household type. It follows that, in the aggregate, we will have

$$(6) \qquad h_t^r = h_t^o = h_t$$

for all t.

The union problem is to maximize the following

$$E_0 \sum_{t=1}^{\infty} \beta^t \left[\left[(1-\gamma)\ln c_{ot} + \gamma \ln c_{rt}\right] - \nu \frac{h_t^{1+\phi}}{1+\phi}\right]$$

and relevant parts of the Lagrangian is

$$
\begin{aligned}
L &= E_0 \sum_{t=1}^{\infty} \beta^t \left[[(1-\gamma)\ln c_{ot} + \gamma \ln c_{rt}] - \nu \frac{h_t^{1+\phi}}{1+\phi} \right] \\
&+ \beta^t \lambda_t^o \left[\frac{A_t}{P_t} + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj + r_t^k u_t k_{ot} + \Phi_t - c_{ot} - i_{ot} - \frac{E_t r_{t,t+1} A_{t+1}}{P_t} - \tau_{ot} \right] \\
&+ \beta^t \lambda_t^r \left[h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj - \tau_{rt} - c_{rt} \right] \\
&+ \beta^t \lambda_t^o \frac{w_t}{\mu_t} \left[h_t - h_t^d \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj \right] \\
&+ \beta^t \lambda_t^r \frac{w_t}{\mu_t} \left[h_t - h_t^d \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj \right]
\end{aligned}
$$

The optimality condition for labor is given by,

$$
\nu h_t^{\phi} = \lambda_t^o \frac{w_t}{\mu_t} + \lambda_t^r \frac{w_t}{\mu_t} = \left[\frac{(1-\gamma)}{c_{ot}} + \frac{\gamma}{c_{rt}} \right] \frac{w_t}{\mu_t}
$$

where we substitute for the relevant expression for marginal utility of consumption for each type of household. This can be rewritten as,

$$
\frac{\mu_t}{w_t} = \left[\frac{(1-\gamma)}{\nu c_{ot} h_t^{\phi}} + \frac{\gamma}{\nu c_{rt} h_t^{\phi}} \right] \tag{7}
$$

Note that because the labor demand curve faced by the union is identical across all labor markets, and because the cost of supplying labor is the same for all markets, one can assume that wage rates, $\tilde{w}_t$, and employment, $\tilde{h}_t$, will be identical across all labor markets updating wages in a given period. We introduce wage stickiness in the model by assuming that each period the household (or union) cannot set the nominal wage optimally in a fraction $\theta^w \in [0,1)$ of randomly chosen labor markets.

Note that in these markets, the wage rate is indexed to the previous period's consumer price inflation according to the rule $W_t^j = W_{t-1}^j \pi_{t-1}^{\chi}$, where χ is a parameter measuring the degree of wage indexation. When χ equals 0, there is no wage indexation. When χ equals 1, there is full wage indexation to past consumer price inflation. In general, χ can take any value between 0 and 1. So that $w_t^i = \tilde{w}_t$ if set optimally, and $w_t^i = w_{t-1}^i \pi_{t-1}^{\chi} \pi_t$ otherwise. This also means that, if in period $t+1$ wages are not reoptimized in that market, the real wage is $\tilde{w}_t \pi_t^{\chi} \pi_{t+1}$. This is because the nominal wage is indexed by χ percent of past price inflation. In general, s period after the last reoptimization, the real wage is $\tilde{w}_t \prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}$.

In order to derive the household's first-order condition with respect to the wage rate in those markets where the wage rate is set optimally in the current period, it is convenient to reproduce the parts of the Lagrangian given above that are relevant for this purpose,

$$L^w = E_t \sum_{s=0}^{\infty} (\theta^w \beta)^s (\lambda_{t+s}^o + \lambda_{t+s}^r) h_{t+s}^d w_{t+s}^{\eta_w} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{\eta_w} \left[\tilde{w}_t^{1-\eta_w} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{-1} - \frac{w_{t+s}}{\mu_{t+s}} \tilde{w}_t^{-\eta_w} \right]$$

The first order condition with respect to $\tilde{w}_t$ is:

$$0 = E_t \sum_{s=0}^{\infty} (\theta^w \beta)^s (\lambda_{t+s}^o + \lambda_{t+s}^r) h_{t+s}^d w_{t+s}^{\eta_w} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{\eta_w} \left[\frac{(1-\eta_w)\tilde{w}_t^{-\eta_w}}{\prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)} - (-\eta_w) \frac{w_{t+s}}{\mu_{t+s}} \tilde{w}_t^{-\eta_w - 1} \right]$$

$$0 = E_t \sum_{s=0}^{\infty} (\theta^w \beta)^s (\lambda_{t+s}^o + \lambda_{t+s}^r) h_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\eta_w} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{\eta_w} \left[\frac{\frac{(\eta_w - 1)}{\eta_w} \tilde{w}_t}{\prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)} - \frac{w_{t+s}}{\mu_{t+s}} \right]$$

$$0 = E_t \sum_{s=0}^{\infty} (\theta^w \beta)^s \left[\frac{(1-\gamma)}{c_{o,t+s}} + \frac{\gamma}{c_{r,t+s}} \right] h_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\eta_w} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{\eta_w} \left[\frac{\frac{(\eta_w - 1)}{\eta_w} \tilde{w}_t}{\prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)} - \left[\frac{(1-\gamma)}{\nu c_{o,t+s} h_{t+s}^{\phi}} + \frac{\gamma}{\nu c_{r,t+s} h_{t+s}^{\phi}} \right]^{-1} \right]$$

We can write the wage setting equation in recursive form. To this end, define

$$f_t^1 = \frac{(\eta_w - 1)}{\eta_w} \tilde{w}_t E_t \sum_{s=0}^{\infty} (\theta^w \beta)^s \left[\frac{(1-\gamma)}{c_{o,t+s}} + \frac{\gamma}{c_{r,t+s}} \right] h_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\eta_w} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{\eta_w - 1}$$

and

$$f_t^2 = -\tilde{w}_t^{-\eta_w} E_t \sum_{s=0}^{\infty} (\theta^w \beta)^s h_{t+s}^d w_{t+s}^{\eta_w} \nu h_{t+s}^{\phi} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{\eta_w}$$

One can express these recursively as,

$$(8) \quad f_t^1 = \frac{(\eta_w - 1)}{\eta_w} \tilde{w}_t \left[\frac{(1-\gamma)}{c_{ot}} + \frac{\gamma}{c_{rt}} \right] h_t^d \left(\frac{w_t}{\tilde{w}_t} \right)^{\eta_w} + \theta^w \beta E_t \left(\frac{\pi_{t+1}}{\pi_t^{\chi}} \right)^{\eta_w - 1} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\eta_w - 1} f_{t+1}^1$$

$$f_t^2 = \nu h_t^{\phi} h_t^d \left(\frac{w_t}{\tilde{w}_t}\right)^{\eta_w} + \theta^w \beta E_t \left(\frac{\pi_{t+1}}{\pi_t^{\chi}}\right)^{\eta_w} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\eta_w} f_{t+1}^2 \tag{9}$$

With these definitions at hand, the wage-setting equation becomes

$$f_t^1 = f_t^2 \tag{10}$$

2 Firms

Firms Producing Final Goods

The final good, y_t, is produced with a continuum of intermediate goods, y_{it}, $i \in [0,1]$, with the technology

$$y_t = \left[\int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

Firms in this market operate under perfectly competitive conditions. Profits are given by

$$P_t y_t - \int_0^1 P_{it} y_{it} di$$

Firms maximize profits subject to the above production technology. The implied demand functions for intermediate goods are

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} y_t$$

Perfect competition drives profits to zero. As a consequence, the price level is given by

$$P_t = \left[\int_0^1 P_{it}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

Firms Producing Intermediate Goods

Intermediate good i is produced with capital and labor services with a Cobb-Douglas technology. Formally,

$$y_{it} = k_{gt}^{\alpha_g} k_{it}^{\alpha} h_{it}^{1-\alpha}$$

Given the output level y_{it} chosen in period t, firm i hires capital and labor services to minimize total cost, given by

$$r_t^k k_{it} + w_t h_{it}$$

subject to the production technology. The optimality conditions of this problem are the technological constraint and

$$\frac{k_{it}}{h_{it}} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

The associated marginal cost is given by

$$mc_{it} = k_{gt}^{-\alpha_g} r_{k,t}^{\alpha} w_t^{1-\alpha} \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$$

Assume that prices are sticky a la Calvo (1983). Each period, firm i has the opportunity to adjust prices with probability $1-\theta$. Suppose firm i has the chance to adjust the price in period t. Let P_{it}^* be the chosen price. Then, P_{it}^* is set so as to maximize

$$E_t \sum_{j=0}^{\infty} \theta^j r_{t,t+j} y_{it+j}[P_{it}^* - mc_{it+j}P_{t+j}]$$

subject to

$$y_{it+j} = \left(\frac{P_{it}^*}{P_{t+j}}\right)^{-\epsilon} y_{t+j}$$

$$E_t \sum_{j=0}^{\infty} \theta^j r_{t,t+j} \frac{P_{t+j}}{P_t} y_{it+j}[\frac{P_{it}^*}{P_{t+j}} - \mu mc_{it+j}]$$

where $\mu \equiv \epsilon/\epsilon - 1$. Note that $r_{t,t+1}P_{t+1}/P_t = \lambda_{t+1}/\lambda_t$. Let $p_{it}^* \equiv P_{it}^*/P_t$

$$x_1^t \equiv \mu E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} y_{it+j} mc_{it+j}$$

$$x_2^t \equiv \mu E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} y_{it+j} p_{it}^* \frac{P_t}{P_{t+j}}$$

Then we can write x_1^t and x_t^2 recursively as

$$x_t^1 = \mu y_{it} mc_{it} + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} x_{t+1}^1$$

$$x_t^2 = y_{it} p_{it}^* + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{p_{it}^*}{p_{it+1}^*} \frac{P_t}{P_{t+1}} x_{t+1}^2$$
$$x_t^2 = x_t^1$$

3 Symmetric Equilibrium, Market Clearing and Aggregation

We assume that all firms adjusting prices in period t set the same price, or $P_{it}^* = P_{jt}^*$ We can then write the price level as

$$P_t^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} di = (1-\theta) P_t^* + \theta P_{t-1}^{1-\epsilon}$$

Thus, we can write

$$1 = (1-\theta) p_t^{*1-\epsilon} + \theta \pi_t^{\epsilon-1} \tag{11}$$

where $\pi_t \equiv P_t / P_{t-1}$ denotes the gross rate of inflation between $t-1$ and t and $p_t^* \equiv P_t^* / P_t$ denotes the relative price of the varieties whose price are adjusted in t relative to the final good.

Aggregation also yields the following, where $mc_{it} = mc_t$ across all firms, and $\int_0^1 y_{it} di = \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} y_t di = y_t p_t^{*-\epsilon}$,

$$x_t^1 = \mu p_t^* y_t mc_t + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} x_{t+1}^1 \tag{12}$$

$$x_t^2 = y_t p_t^{*1-\epsilon} + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{p_t^*}{p_{t+1}^*} \pi_{t+1}^{-1} x_{t+1}^2 \tag{13}$$

$$x_t^2 = x_t^1 \tag{14}$$

Let γ denote the fraction of rule-of-thumb households. Then, letting c_t, i_t, k_t, and τ_t denote aggregate consumption, investment, capital, and lump-sum taxes, respectively, we have

$$c_t = \gamma c_{rt} + (1-\gamma)c_{ot} \tag{15}$$

$$i_t = (1-\gamma)i_{ot} \tag{16}$$

$$k_t = (1-\gamma)k_{ot} \tag{17}$$

$$\tau_t = \gamma\tau_{rt} + (1-\gamma)\tau_{ot} \tag{18}$$

Letting g denote government spending, we have

$$y_t = c_t + i_t + g_t \tag{19}$$

where

$$g_t = c_{gt} + i_{gt} \tag{20}$$

and

$$k_{gt+1} = (1-\delta^g)k_{gt} + i_{gt} \tag{21}$$

We now need a relationship linking y_t to production.

$$k_{gt}^{\alpha_g} k_{it}^{\alpha} h_{it}^{1-\alpha} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} y_t$$

Noting that the capital labor ratio is common across firms, and letting $u_t k_t = \int_0^1 k_{it} di$ and $h_t^d = \int_0^1 h_{it} di$.

$$k_{gt}^{\alpha_g} h_{it}\left(\frac{k_{it}}{h_{it}}\right)^{\alpha} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} y_t$$

Integrating,

$$k_{gt}^{\alpha_g} {h_t^d}^{1-\alpha}(u_t k_t)^{\alpha} = s_t y_t \tag{22}$$

where $s_t \equiv \int_0^1 (\frac{P_{it}}{P_t})^{-\epsilon} di$. We can write s_t recursively as

$$s_t = (1-\theta)p_t^{*-\epsilon} + \theta \pi_t^{\epsilon} s_{t-1} \tag{23}$$

Aggregating over capital and labor for the following equations

$$\frac{k_{it}}{h_{it}} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

and the associated marginal cost,

$$mc_{it} = k_{gt}^{-\alpha_g} r_{k,t}^{\alpha} w_t^{1-\alpha} \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$$

yields,

$$\frac{u_t k_t}{h_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \tag{24}$$

and

$$mc_t = k_{gt}^{-\alpha_g} r_{k,t}^{\alpha} w_t^{1-\alpha} \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \tag{25}$$

Now considering the market clearing in the labor markets, recall, that the aggregate demand for labor of type $j \in [0,1]$, given by

$$h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\eta_w} h_t^d$$

where $h_t^d = \int_0^1 h_{it} di$ is the aggregate demand for the composite labor input. Since nominal wage rate is identical across all labor markets at which wages are allowed to change optimally, which we denote as $\tilde{w}_t$. Since,

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\eta_w} dj$$

we can write is as follows,

$$h_t = (1-\theta^w)h_t^d \sum_{s=0}^{\infty}(\theta^w)^s \left(\frac{\tilde{W}_{t-s}\prod_{k=1}^{s}\pi_{t+k-s}^{\chi}}{W_t}\right)^{-\eta_w}$$

Let s_t^w be the coefficient on h_t^d in the above expression, which measures the degree of wage dispersion across different types of labor. The above expression can be written as,

$$h_t = s_t^w h_t^d \tag{26}$$

and the evolution of this term is given as

$$s_t^w = (1-\theta^w)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\eta_w} + \theta^w \left(\frac{w_{t-1}}{w_t}\right)^{-\eta_w}\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta_w} s_{t-1}^w \tag{27}$$

The nominal wage index W_t is given by

$$W_t \equiv \left[\int_0^1 W_t^{j\,1-\eta_w} dj\right]^{\frac{1}{1-\eta_w}}$$

which leads to the expression for the real wage rate

$$w_t^{1-\eta} = (1-\theta^w)\tilde{w}_t^{1-\eta_w} + \theta^w \left(\frac{w_{t-1}\pi_{t-1}^{\chi}}{\pi_t}\right)^{(1-\eta_w)} \tag{28}$$

Another consequence of this aggregation is that the rule-of-thumb consumer budget constraint,

$$c_{rt} = h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\eta_w} dj - \tau_{rt}$$

can be rewritten as follows since with the unions, all the workers earn the same wage and work the same number of hours.

$$c_{rt} = w_t h_t^r - \tau_{rt} \tag{29}$$

4 Monetary and Fiscal Policy

Taking conditional expectations on the expression $\frac{\lambda_t}{P_t} r_{t,t+1} = \beta \frac{\lambda_{t+1}}{P_{t+1}}$, we obtain

$$\lambda_t = R_t \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{30}$$

where R_t denotes the gross nominal interest rate. Monetary policy takes the form of a simple Taylor rule

$$R_t - R = \phi_\pi (\pi_t - \pi) \tag{31}$$

where π is an inflation target pursued by the monetary authority and R is the associated steady state value of the nominal interest rate.

The government budget constraint is given by

$$\frac{B_t}{R_t} - B_{t-1} + P_t \tau_t = P_t g_t$$

where B_t denotes nominally risk-free bonds issue in period $t-1$. Let $b_t \equiv B_{t-1}/P_t$. Then,

$$\frac{b_t}{R_t} - \frac{b_{t-1}}{\pi_t} + \tau_t = g_t \tag{32}$$

Fiscal policy is given by

$$\tau_t - \tau = \phi_b (b_{t-1} - b) + \phi_g (g_t - g) \tag{33}$$

where g, y, and b denote the steady-state values of g_t, y_t, and b_t, respectively. Finally, we impose:

$$\tau_{ot} - \tau_o = \tau_{rt} - \tau_r \tag{34}$$

Government spending investment is assumed to follow an exogenous AR(1) process of the form

$$\frac{i_{gt} - i_g}{y} = \rho_g^i \frac{i_{gt-1} - i_g}{y} + \epsilon_t^{ig}$$

where $\epsilon_t^i g$ is an i.i.d. shock with mean zero and variance σ^{ig}. And similarly for government consumption,

$$\frac{c_{gt} - c_g}{y} = \rho_g^c \frac{c_{gt-1} - c_g}{y} + \epsilon_t^{cg}$$

5 Complete Set of Equilibrium Conditions

$$k_{ot+1} = (1 - a(u_t))k_{ot} + i_{ot}\left(1 - S(\frac{i_{ot}}{i_{o,t-1}})\right) \tag{1}$$

$$\frac{1}{c_{ot}} = \lambda_t \tag{2}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1}\left[r_{t+1}^k u_{t+1} + q_{t+1}(1 - a(u_{t+1}))\right] \tag{3}$$

$$\lambda_t = \lambda_t q_t \left(1 - S\left(\frac{i_{ot}}{i_{o,t-1}}\right) - i_{ot} S'\left(\frac{i_{ot}}{i_{o,t-1}}\right)\right) - \beta E_t \lambda_{t+1} q_{t+1} S'\left(\frac{i_{o,t+1}}{i_{o,t}}\right) i_{o,t+1} \tag{4}$$

$$r_t^k = q_t a'(u_t) \tag{5}$$

$$c_{rt} = w_t h_t^r - \tau_{rt} \tag{6}$$

$$\frac{\mu_t}{w_t} = \left[\frac{(1-\gamma)}{\nu c_{ot} h_t^{\phi}} + \frac{\gamma}{\nu c_{rt} h_t^{\phi}}\right] \tag{7}$$

$$h_t^r = h_t^o = h_t \tag{8}$$

(9) $$f_t^1 = \frac{(\eta_w - 1)}{\eta_w}\tilde{w}_t\left[\frac{(1-\gamma)}{c_{ot}} + \frac{\gamma}{c_{rt}}\right]h_t^d\left(\frac{w_t}{\tilde{w}_t}\right)^{\eta_w} + \theta^w\beta E_t\left(\frac{\pi_{t+1}}{\pi_t^\chi}\right)^{\eta_w-1}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\eta_w-1}f_{t+1}^1$$

(10) $$f_t^2 = \nu h_t^\phi h_t^d\left(\frac{w_t}{\tilde{w}_t}\right)^{\eta_w} + \theta^w\beta E_t\left(\frac{\pi_{t+1}}{\pi_t^\chi}\right)^{\eta_w}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\eta_w}f_{t+1}^2$$

(11) $$f_t^1 = f_t^2$$

(12) $$h_t = s_t^w h_t^d$$

(13) $$s_t^w = (1-\theta^w)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\eta_w} + \theta^w\left(\frac{w_{t-1}}{w_t}\right)^{-\eta_w}\left(\frac{\pi_t}{\pi_{t-1}^\chi}\right)^{\eta_w}s_{t-1}^w$$

(14) $$w_t^{1-\eta} = (1-\theta^w)\tilde{w}_t^{1-\eta_w} + \theta^w\left(\frac{w_{t-1}\pi_{t-1}^\chi}{\pi_t}\right)^{(1-\eta_w)}$$

(15) $$\frac{u_t k_t}{h_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

(16) $$mc_t = k_{gt}^{-\alpha_g}r_{k,t}^\alpha w_t^{1-\alpha}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$$

(17) $$x_t^1 = \mu p_t^* y_t mc_t + \theta\beta E_t\frac{\lambda_{t+1}}{\lambda_t}x_{t+1}^1$$

(18) $$x_t^2 = y_t p_t^{*1-\epsilon} + \theta\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{p_t^*}{p_{t+1}^*}\pi_{t+1}^{-1}x_{t+1}^2$$

(19) $$x_t^2 = x_t^1$$

(20) $$1 = (1-\theta)p_t^{*1-\epsilon} + \theta\pi_t^{\epsilon-1}$$

(21) $$c_t = \gamma c_{rt} + (1-\gamma)c_{ot}$$

(22) $$h_t = \gamma h_{rt} + (1-\gamma)h_{ot} = h_{rt} = h_{ot}$$

(23) $$i_t = (1-\gamma)i_{ot}$$

(24) $$k_t = (1-\gamma)k_{ot}$$

(25) $$\tau_t = \gamma\tau_{rt} + (1-\gamma)\tau_{ot}$$

(26) $$y_t = c_t + i_t + g_t$$

(27) $$k_{gt}^{\alpha_g} h_t^{d\,1-\alpha}(u_t k_t)^{\alpha} = s_t y_t$$

(28) $$s_t = (1-\theta)p_t^{*-\epsilon} + \theta\pi_t^{\epsilon} s_{t-1}$$

(29) $$\lambda_t = R_t \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}}$$

(30) $$R_t - R = \phi_\pi(\pi_t - \pi)$$

(31) $$\frac{b_t}{R_t} - \frac{b_{t-1}}{\pi_t} + \tau_t = g_t$$

(32) $$\tau_t - \tau = \phi_b(b_{t-1} - b) + \phi_g(g_t - g)$$

(33) $$\tau_{ot} - \tau_o = \tau_{rt} - \tau_r$$

(34) $$g_t = c_{gt} + i_{gt}$$

(35) $$k_{gt+1} = (1 - \delta^g)k_{gt} + i_{gt}$$

and processes for c_{gt} and i_{gt}.

Set of variables:
k_{ot}, k_t, c_{ot}, c_{rt}, c_t, λ_t, q_t, i_{ot}, i_t, u_t,
r_t^k, h_t, h_t^d, h_t^r, h_t^o, s_t, s_t^w, f_t^1, f_t^2, x_t^1,
x_t^2, w_t, $\tilde{w}_t$, mc_t, g_t, p_t^*, μ_t, R_t, b_t, τ_t,
τ_{ot}, τ_{rt}, k_{gt}, π_t, c_{gt}, i_{gt}

With the functional form for the investment adjustment cost given as:

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

and the capacity utilization cost given as:

$$a(u_t) = \delta + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$$

we can rewrite Equation (4),

$$\lambda_t = \lambda_t q_t \left(1 - S\left(\frac{i_{ot}}{i_{o,t-1}}\right) - i_{ot} S'\left(\frac{i_{ot}}{i_{o,t-1}}\right)\right) - \beta E_t \lambda_{t+1} q_{t+1} S'\left(\frac{i_{o,t+1}}{i_{o,t}}\right) i_{o,t+1}$$

as

$$\lambda_t = \lambda_t q_t \left(1 - S\left(\frac{i_{ot}}{i_{o,t-1}}\right) - \frac{i_{ot}}{i_{o,t-1}} \kappa \left(\frac{i_{ot}}{i_{o,t-1}} - 1\right)\right) + \beta E_t \lambda_{t+1} q_{t+1} \kappa \left(\frac{i_{o,t+1}}{i_{o,t}} - 1\right)\left(\frac{i_{o,t+1}}{i_{o,t}}\right)^2$$

and Equation (5),

$$r_t^k = q_t a'(u_t)$$

as

$$r_t^k = q_t(\delta_1 + \delta_2(u_t - 1))$$

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