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Economic growth with subsistence consumption

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Abstract

Four stylized facts of economic growth in DCs are set up initially. Despite its obvious simplicity the linear growth model with subsistence consumption is able to reproduce two of them: a rise in the saving rate along with per capita income as well as β-divergence. The rate of convergence shows extraordinarily low values at early stages of economic development. Hence, the big diversity in growth rates can partly be explained to represent transitional phenomena. An extension of the basic model additionally allows an explanation of the hump-shaped pattern of growth. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The development process is one of transition. Gersovitz (1988, p. 383)

If one takes Gersovitz (1988) literally and is interested in an explanation of economic growth and development in terms of growth theory, one is led to ask what class of growth models is consistent with this view. In addition, it is reasonable to ask what class of growth models is able to reproduce the main

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stylized facts of (aggregate) economic growth primarily applying to the lower range of per capita income:[1]

1. A big diversity in the growth rates of per capita income including zero and even negative growth;[2]
2. a positive correlation between the saving rate and per capita income;[3] and
3. a positive correlation between the growth rate and the level of per capita income, i.e. β-divergence.[4]
4. More generally, many authors report β-divergence for the lower range of per capita income and β-convergence for the upper range of per capita income, i.e. a hump-shaped pattern of growth.[5]

Beside an increase in total factor productivity, the accumulation of reproducible inputs (physical and human capital) represents one of the major forces of growth. Especially in the long run, the accumulation of physical and human capital needs to be financed by internal saving.[6] Within the development literature it is stressed that saving in the case of developing countries (DCs) is determined by the willingness as well as the ability to save (e.g., Hemmer, 1988, pp. 150–159). The usual constant-intertemporal-elasticity-of-substitution (CIES) formulation of preferences abstracts from the requirement of a minimum consumption level in order to sustain life. However, the requirement of subsistence consumption undoubtedly restricts the possibilities to substitute consumption intertemporally and hence the ability to save at least for the lower range of income. Several questions arise which are of fundamental importance: Does the requirement of subsistence consumption influence the process of growth beyond this threshold? If so, how long does it take for the influence of subsistence consumption on growth to vanish? How does the requirement of subsistence consumption interact with other essential mechanisms of growth? The paper in hand seeks to answer these questions systematically within the context of simple endogenous growth models with Stone–Geary preferences. In addition, it will be shown that these models provide a potential

[1] The general stylized facts of economic growth were formulated by Kaldor (1961). Romer (1989) enlarges Kaldor's list by five other prominent features of the data.

[2] This corresponds to Kaldor's (1961) sixth fact. For the case of low-income countries this empirical regularity is particularly marked; see Romer (1989) for a discussion.

[3] The empirical evidence in favor of this empirical regularity is overwhelming (e.g., Aghevli et al., 1990; Ogaki et al., 1996).

[4] For empirical evidence, see Baumol (1986), Zind (1991), and Pritchett (1997).

[5] For empirical evidence, see Dollar (1992), Baumol et al. (1989), and Easterly (1994).

[6] As far as specific consumption activities (nutrition, health efforts, and education) are productive, the accumulation of human capital does not necessarily require the renunciation of consumption. This aspect is ignored within this paper; for this see Steger (2000).

explanation of the stylized facts listed above. It is assumed that the economy under study is symmetric to the rest of the world with respect to preferences and technology. As Rebelo (1992) stresses, the aim of this methodological assumption is to rule out explanations of differences in growth experiences that are based solely on the existence of cross-country differences in preferences and technology.

In order to address very specific theoretical as well as empirical questions, Stone–Geary preferences (henceforth SGP) have been widely applied within recent growth literature. According to Rebelo (1992), a broad class of endogenous growth models is inconsistent with cross-country diversity in growth rates in the face of international capital markets. As a solution to this theoretical problem, he suggests a linear growth model with SGP. Easterly (1994) uses a constant-elasticity-of-substitution production function with the Jones–Manuelli property and SGP to discuss the threshold effects of different policy measures on the long-run growth rates. Ben-David (1994) applies a neoclassical growth framework extended by subsistence consumption to demonstrate the possibility of multiple balanced-growth equilibria.

The paper is organized as follows: Section 2 concisely discusses the concept of subsistence and its empirical importance. In Section 3 a linear growth model with SGP is analyzed systematically. The quantitative convergence implications are investigated in addition to the qualitative convergence implication. In Section 4 the basic model is extended by diminishing marginal returns to capital as well as by the general meaning of policy-induced distortions. Section 5 summarizes and concludes with some final considerations.

2. The subsistence level of consumption

Subsistence is a widely used concept with varying meanings and definitions. Sharif (1986) comprehensively surveys the concept, its importance in the context of different theories, and its measurement. Subsistence as a mode of production can be distinguished from subsistence as a mode of consumption. The first is usually defined as (mostly agricultural) production for home-consumption, while the latter denotes a standard of living that allows for the satisfaction of the minimum (physical and mental) basic needs of life.

The interpretation of subsistence as a mode of consumption corresponds to the concept of the poverty line which is used to identify that part of the population which is regarded as absolutely poor. The World Bank (1990) employs two such poverty lines. The lower poverty line amounts to \$275, while the upper poverty line amounts to \$370 (in 1985 PPP prices). In addition, Ben-David (1994) reports calculations from Stigler (1945) who estimates the subsistence level of consumption defined as the least-cost requirement for sustaining an individual's dietary needs as approximately \$300 a year (in 1980 US \$).

The relevance of the obvious requirement of subsistence consumption for growth is straightforward. The requirement of subsistence consumption restricts the ability to save. Consequently, within the framework of all growth models which explain growth to result from the accumulation of reproducible factors, the restriction of the ability to save crucially influences the process of growth. Intuitively, this influence is greater the nearer the economy is located at subsistence. How appropriate or relevant is this concept? There are clearly other heavy burdens that can inhibit growth in these countries: Poor and deteriorating infrastructure, a bad institutional set-up, as well as macroeconomic instability. However, if a large part of the population is concerned with nothing else but staying alive, these other issues are of minor importance for the poorest economies (Ben-David, 1994).

In order to obtain an impression of the empirical relevance of subsistence consumption, Table 1 displays the proportional difference between income and the subsistence level of consumption. The latter is first identified with the lower (proportional difference I) and second with the upper poverty line (proportional difference II) as used by the World Bank (1990). The third column shows that income exceeds the lower poverty line for the group of low-income economies only marginally. Moreover, income even falls short of the upper poverty line as shown in the fourth column. Subsistence consumption considerations seem to be of minor importance for middle-income economies. In the case of high-income economies the requirement of subsistence consumption is nearly irrelevant.

The meaning of the requirement of subsistence consumption for intertemporal consumption decisions can be easily formalized by means of an intertemporal Stone–Geary utility function. The subsistence level of consumption, $\bar{c} = 0$, is interpreted as that amount of consumption which is a necessary prerequisite for

Table 1
Per capita income in excess of subsistence
Source: World Bank (1990, pp. 178–180).

Group of countries[a] (number of countries)	GNP per capita[b]	Proportional difference I[c]	Proportional difference II[d]
Low-income economies (36)	320	0.14	−0.16
China and India (2)	340	0.19	−0.32
Other low-income (34)	280	0.02	−0.09
Lower-middle-income economies (34)	1380	0.80	0.73
Upper-middle-income economies (14)	3240	0.92	0.89
High-income economies (25)	17,080	0.98	0.98

[a] Country grouping according to World Bank (1990, p. x).
[b] In 1988 dollars.
[c] Proportional difference between GNP and the lower poverty line ($275).
[d] Proportional difference between GNP and the upper poverty line ($370).

sustaining life. Solely consumption that exceeds the subsistence level creates well being, i.e. the instantaneous utility function is defined on the range $c \geq \bar{c}$:

$$U[\{c(t)\}] = \int_0^\infty \frac{[c(t) - \bar{c}]^{1-\theta} - 1}{1-\theta} e^{-(\rho - n)t} dt. \tag{1}$$

The instantaneous utility function in Eq. (1) is twice continuously differentiable and strictly concave. The intertemporal utility function assumes that consumption is additively separable and that the value of the utility functional converges to a finite value which requires $\rho > n$. The usual CIES function can be regarded as a special case of Eq. (1) with $\bar{c} = 0$. This might approximately be justified in the case of developed countries. However, for DCs subsistence consumption considerations are of major importance and should be taken into account within theoretical analyses. The instantaneous Stone–Geary utility function implies a variable intertemporal elasticity of substitution (IES) for two immediate points in time:

$$\sigma(c) \equiv -\frac{u'(c)}{u''(c)c} = \frac{c - \bar{c}}{\theta c}.$$

Several properties of the IES are noteworthy: (i) the IES is zero for consumption equal to the subsistence level of consumption; (ii) the IES increases with the level of consumption; and (iii) the IES asymptotically converges to θ^{-1} as consumption grows without bound. These properties make good economic sense. If income equals the subsistence level of consumption, the individual is simply unable to substitute consumption intertemporally and the IES equals zero. For an increasing income level the individual must first achieve subsistence consumption letting intertemporal considerations guide its decisions only for that portion of the budget which exceeds subsistence.

3. A linear growth model with subsistence consumption

3.1. The model

The fundamental importance of the requirement of subsistence consumption for the process of growth mainly relevant to low-income countries is analyzed. For this a simple linear one-sector growth model with SGP is employed.[7] The economy considered is closed and the representative consumer–producer house-

[7] The linear growth model can be interpreted to represent the basic version of a broad class of endogenous growth models (Romer, 1989; Rebelo, 1992).

hold is assumed to maximize its dynastic lifetime utility subject to several constraints.

$$\max_{\{c(t)\}} \int_0^\infty \frac{[c(t)-\bar{c}]^{1-\theta}-1}{1-\theta} e^{-(\rho-n)t} dt$$
$$\text{s.t.} \quad \dot{k}(t) = (A-\delta-n)k(t) - c(t)$$
$$k(0)=k_0, \quad k(t) \geq 0$$
$$\bar{c} \leq c(t) \leq Ak(t). \tag{2}$$

All variables are expressed in per capita terms and t represents the time index. c denotes consumption, $\bar{c}$ the subsistence level of consumption, θ a constant preference parameter, ρ the individual time preference rate, and n the constant rate of population growth, respectively. Gross output is a linear function of capital, $y = Ak$, where k denotes the stock of physical and human capital and A a constant technology parameter.

The first-order conditions of the dynamic problem (Eq. (2)) consist of two linear differential equations in c and k together with two boundary conditions including the transversality condition. This differential equation system can be easily solved analytically yielding the solution:

$$c(t) = \bar{c} + [c(0) - \bar{c}] e^{\theta-1(A-\delta-\rho)t} \tag{3}$$

$$k(t) = \bar{k} + [k(0) - \bar{k}] e^{\theta-1(A-\delta-\rho)t} \tag{4}$$

$$\text{with} \quad \bar{k} \equiv \frac{\bar{c}}{A-\delta-n} \quad \text{and} \quad A-\delta-n>0.$$

3.2. Implications

3.2.1. Dynamic equilibria

The solutions (3) and (4) immediately show that there are, in principle, three possible dynamic equilibria: Two steady states with consumption and capital being constant and one asymptotic balanced-growth equilibrium with consumption and capital asymptotically growing with constant growth rates.

The first steady state is the *subsistence steady state* with consumption equal to the level of subsistence consumption and capital equal to the level of subsistence capital, $\bar{k}$. It represents a low-level-development equilibrium that applies whenever the net marginal product is low relative to the time preference rate.[8] In this case the individuals are simply not willing to postpone consumption and the subsistence steady state is the long-run solution of the dynamic problem (Eq. (2)).

[8] In contrast, a poverty trap requires that, for a given set of parameters, the economy possesses multiple dynamic equilibria. The lowest of these which is stable is called a poverty trap (Azariadis, 1996).

The second steady state might be considered to represent merely a theoretical possibility which is not very probable. If the net marginal product of capital just equals the time preference rate, the economy displays zero growth with consumption and capital probably above subsistence. An external resource transfer would increase the level of consumption and capital but cannot stimulate sustained growth. This type of equilibrium with zero growth and consumption above subsistence will be worked out in a more plausible way in Section 4.2.1.

The third possibility is represented by an asymptotic balanced-growth equilibrium. Whenever the net marginal product of capital exceeds the time preference rate, the economy pursues unbounded growth. The growth rates of consumption and capital converge towards their common asymptotic balanced-growth-equilibrium value (time index omitted):

$$\lim_{t\to\infty}\frac{\dot{c}}{c}=\lim_{t\to\infty}\frac{\dot{k}}{k}=\theta^{-1}(A-\delta-\rho). \tag{5}$$

The solutions (3) and (4) allow the explicit formulation of the stable arm of the saddle-path leading to the subsistence steady state or — depending on the constellation of parameters — towards the asymptotic balanced-growth equilibrium:

$$c=z(k-\bar{k})+\bar{c} \quad \text{with} \quad z\equiv A-\delta-n-\theta^{-1}(A-\delta-\rho)>0. \tag{6}$$

Given an initial condition for capital, the policy function (6) immediately shows the corresponding value of consumption so that all necessary first-order conditions (including the transversality condition) are satisfied.

3.2.2. Transitional dynamics

Using the policy function (6), the net saving rate defined as the relation between net investment and net output reads as follows:

$$s=\frac{A-\delta-\rho}{\theta(A-\delta-n)}\,\frac{k-\bar{k}}{k}. \tag{7}$$

It should be noted explicitly that a value of the net saving rate equal to zero implies a constant stock of capital. That is, the amount of gross investment just suffices to replace depreciation, δk, and to enlarge the stock of capital in accordance with population growth, nk. As far as the initial capital stock exceeds its subsistence level, Eq. (7) demonstrates that the saving rate is positive whenever $A-\delta-\rho>0$ and negative whenever $A-\delta-\rho<0$. In both cases, the saving rate monotonically converges to its dynamic-equilibrium value which reads $s^*=(A-\delta-\rho)/\theta(A-\delta-n)$ for $A-\delta-\rho>0$ and $s^*=0$ for $A-\delta-\rho<0$.

Growth theory (especially endogenous growth theory) as well as traditional development theory assign a dominant role to internal saving and investment for economic growth and development. Within the latter, the saving rate is considered

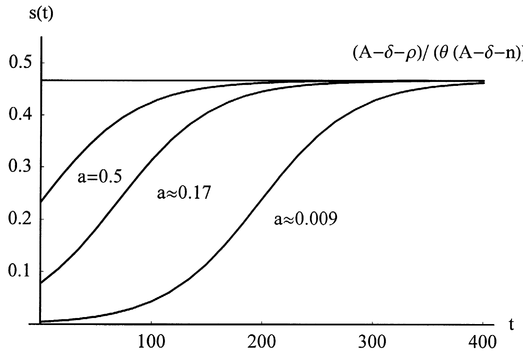

Fig. 1.

to be determined by the *ability to save* and by the *willingness to save* (e.g., Hemmer, 1988, pp. 150–159). These two concepts can be assigned directly to the determinants of the saving rate as expressed in Eq. (7). The ability to save (sometimes called 'economic or investible surplus') at any moment in time is defined as the proportional difference between the current and the subsistence level of income. Within the framework of the present model, the ability to save is represented by the term $(k-\bar{k})/k$. It is near zero for k near $\bar{k}$ and converges to one as k approaches infinity, i.e. approximately the entire output is disposable for saving as capital grows without bound. The willingness to save is determined by the preference and technology parameters and is represented by the term $(A-\delta-\rho)/\theta(A-\delta-n)$. Fig. 1 shows the time paths of the saving rate for different starting values of the stock of capital in relation to its subsistence level, i.e. for different initial abilities to save $a \equiv (k(0)-\bar{k})/k(0)$.[9]

In the case of developed economies, a is sufficiently close to one, so that the analysis of the requirement of subsistence consumption is nearly irrelevant for this group of countries. However, in the case of low-income countries the consideration of subsistence consumption might significantly influence the process of economic development.

The rise in the saving and the investment rate causes the growth rate of capital to increase as well. The time path of the growth rate of capital, which equals the growth rate of income, monotonically increases and converges towards its asymptotic balanced-growth-equilibrium value as illustrated by Fig. 2.

The smaller the a, the longer it takes for the transition towards the asymptotic balanced-growth equilibrium. For $a \cong 0.009$, the growth rate of capital requires

[9] The following set of parameters is employed: $A=0.1$, $\theta=3$, $\delta=0.02$, $\rho=0.01$, $n=0.03$, $\bar{c}=2$. These parameter values are consistent with usual calibrations. In the case of the linear growth model, the constant marginal product of capital must be determined by the choice of A. Kaldor (1961, p. 178) reports an average rate of profit on capital for the UK (1870–1914) of 10.5%.

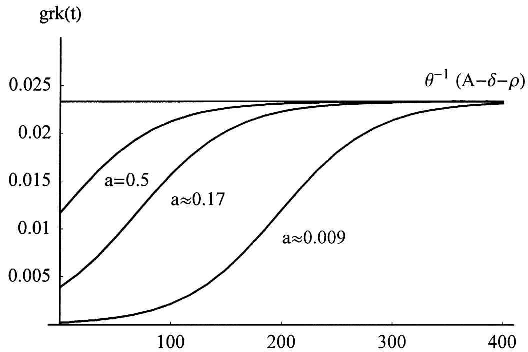

Fig. 2.

nearly 200 years to eliminate half the distance between its initial and its balanced-growth-equilibrium value.

Despite its obvious simplicity, the linear growth model with SGP is extraordinarily useful because it is able to reproduce two of the stylized facts enumerated in the Introduction, i.e. the positive correlation between the saving rate and the level of per capita income [stylized fact (2)] as well as β-divergence [stylized fact (3)]. In addition, the model displays important properties which are in line with theoretical and empirical research: (i) As Rebelo (1992) has pointed out, the model is able to explain internationally divergent growth dynamics even in the presence of international capital markets. (ii) The model avoids the counterfactual implication of extraordinarily high interest rates at early stages of economic development that characterizes most versions of the neoclassical model (King and Rebelo, 1993). (iii) The model predicts that the IES increases with per capita income which is in line with empirical research on the IES (e.g., Ogaki et al., 1996). However, the model clearly fails to reproduce the hump-shaped pattern of growth [stylized fact (4)]. In addition, provided that symmetry with respect to preference and technology parameters is supposed the model has difficulty in explaining the big diversity in growth rates for the group of low-income countries [stylized fact (1)]. More specifically, as far as divergent growth experiences are viewed to represent transitional phenomena, the ability to explain these different growth rates essentially depends on the implied rate of convergence.

3.3. Convergence considerations

Within the context of convergence analyses, qualitative and quantitative issues arise which are both of fundamental importance for the development process. The qualitative aspect concerns the question whether the model implies (conditional) β-convergence or (conditional) β-divergence. The quantitative aspect is concerned with the rate of convergence towards the dynamic equilibrium which provides important information about whether the emphasis of analysis should be placed on

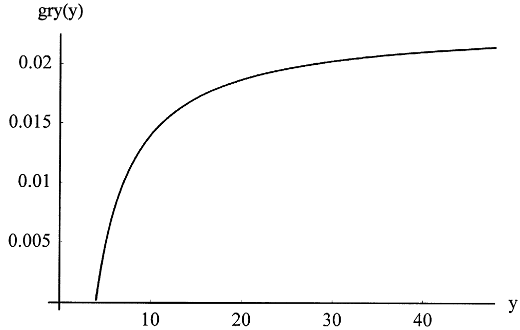

Fig. 3.

transitional or balanced-growth dynamics. The analysis of the rate of convergence is meaningful irrespective of the qualitative convergence implication.[10]

The linear growth model with subsistence consumption implies conditional *β-divergence*. The growth rate of income increases with the level of income conditioned on a (unique) balanced-growth equilibrium. Fig. 3 illustrates the relation between the growth rate, gry, and the level of income, y.[11]

The reason for this divergence mechanism, which might in analogy to the neoclassical convergence mechanism be labeled as 'subsistence-divergence mechanism', is simply that in the case of growth the ability to save rises continuously. Consequently, the saving and investment rates increase as well. Unlike the neoclassical model, the linear growth model displays a constant marginal and average product of capital. As a result, the productive contribution of the entire stock of the reproducible factors remains constant and an increase in the investment rate fully translates into an increase in the growth rate of output.

In the next step, consider the quantitative implications of the transition process. For this, the current state of the economy is expressed in terms of a variable, $x(t)$, such that this variable monotonically converges to a stationary value denoted by x^*, which represents the dynamic equilibrium of the economy. The instantaneous speed or rate of convergence can be reasonably defined as (the negative of) that share of the distance between the current state and the dynamic equilibrium which is eliminated during the current period:

$$\lambda(t) \equiv -\frac{\dot{x}(t)}{x(t) - x^*}. \tag{8}$$

The instantaneous rate of convergence (Eq. (8)) is only constant if the underlying differential equation, $\dot{x}(t) = F[x(t)]$, is linear. For non-linear differential

[10] A model might imply β-divergence during the transition to the balanced-growth equilibrium. At the same time it is meaningful to ask how fast it converges to its balanced-growth equilibrium.

[11] Based on the same set of parameters which were employed previously with $a \approx 0.009$.

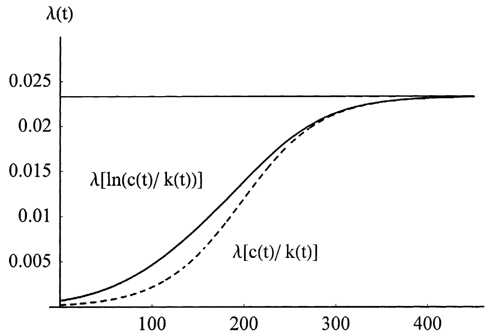

Fig. 4.

equations, the rate of convergence varies with $x(t) - x^*$.[12] Because the economy in question exhibits unbounded growth $x(t) - x^*$ cannot be expressed in terms of the original variables. This distance must rather be expressed in terms of variables which are functions of the original variables and converge to stationary values.

The instantaneous rate of convergence is expressed on the basis of the consumption–capital ratio. Both the c/k-ratio and the logarithm of the c/k-ratio are employed. The rate of convergence calculated on the basis of the c/k-ratio shows the instantaneous as well as true rate of convergence. This rate of convergence is true in the sense that it describes the actual rate of convergence of the economy expressed in original variables. However, empirical estimates on the rate of convergence are usually based on logarithmic variables. Therefore, the logarithm of the c/k-ratio is additionally employed in order to obtain a perfect comparability. Fig. 4 plots the time paths of the instantaneous rates of convergence. As one would expect, the rate of convergence based on the logarithm of the c/k-ratio, $\lambda[\ln(c(t)/k(t))]$, is higher compared to the rate of convergence based on the original c/k-ratio, $\lambda[(c(t)/k(t))]$. However, the qualitative behavior of both rates of convergence is identical and the quantitative difference does not seem to be substantial.

The asymptotic value of the rate of convergence amounts to 2.3%. However, Fig. 4 demonstrates that the rate of convergence locally around the asymptotic balanced-growth equilibrium is not a good estimate for the global converging

[12] Because the differential equations which describe the dynamics of the economy are mostly non-linear, the rate of convergence is usually calculated on the basis of a first-order Taylor series approximation around a stationary value and is, therefore, only valid within a 'small neighborhood' around the stationary value.

behavior.[13] At early stages of economic development, the rate of convergence is extraordinarily low (below 0.1% per year at the initial point in time) and increases only slightly. As a result, the time span required for the transition towards the asymptotic balanced-growth equilibrium is extremely long. As demonstrated above, the saving rate and the growth rate of capital require a very long time span for the transition towards their balanced-growth-equilibrium values with half-life times of about 200 years. Therefore, the big diversity in growth rates [stylized fact (1)] can partly be explained as representing a transition phenomenon. However, if international symmetry with respect to preferences and technology is supposed, the range of possible growth rate differences is restricted.

4. Extensions of the basic linear growth model

4.1. The model

The basic linear growth model with SGP is extended in two directions. First, the presence of fixed factors such as land or (raw) labor is important at low incomes but becomes negligible as income grows without bound. As a result, the production function is allowed to exhibit diminishing returns to the reproducible factor, which eventually converge to a lower bound (Jones and Manuelli, 1990). Second, in order to capture the influence of detrimental government policy, a general index reflecting several distortions is incorporated into the production function. The aggregate production function of the private sector for gross output 'net of distortions' is assumed to read as follows:

$$y(t) = (1-\tau)\left[Ak(t) + Bk(t)^{\alpha} \right], \tag{9}$$

where A, B, and α (with $0<\alpha<1$) denote constant technology parameters and y denotes gross output. The distortions summarized in τ (with $0 \leq \tau 1$) reduce the marginal product of the reproducible factors and can be considered to result mainly from bad government policies which are frequently found in DCs. The extent of distortions is considered to vary internationally. Examples include income or investment taxes (e.g., Barro, 1990; Jones and Manuelli, 1990), inflation taxes (e.g., Easterly et al., 1992; Easterly, 1994), bad institutional set-ups (e.g., Rebelo, 1992), sectoral distortions due to unequal taxation of different sectors, dual exchange rate systems, and sectorally discriminating tariffs and

[13] In contrast, Ortigueira and Santos (1997) demonstrate that the local rate of convergence around the dynamic equilibrium is a good estimate for the global convergence behavior within a wide range of capital in the case of the neoclassical and the Uzawa–Lucas model.

import quotas (e.g., Rebelo, 1992; Easterly, 1994).[14] The production function (9) exhibits the Jones–Manuelli property, i.e. the marginal product of the reproducible factors decreases, however, converging to a positive constant as capital grows without bound:

$$\lim_{k \to \infty} \frac{\partial y}{\partial k} = (1-\tau) A > 0.$$

The model with subsistence consumption and a Jones–Manuelli technology including a general index of distortions reads as follows:

$$\begin{aligned}
&\max_{\{c(t)\}} \int_0^{\infty} \frac{[c(t)-\bar{c}]^{1-\theta}-1}{1-\theta} e^{-(\rho-n)t} \mathrm{d}t \\
&\text{s.t.} \quad \dot{k}(t) = (1-\tau)\left[Ak(t)+Bk(t)^{\alpha}\right] - (\delta+n)k(t) - c(t) \\
&k(0) = k_0, \quad k(t) \geq 0 \\
&\bar{c} \leq c(t) \leq (1-\tau)\left[Ak(t)+Bk(t)^{\alpha}\right].
\end{aligned} \tag{10}$$

The necessary first-order conditions of the preceding dynamic problem in the case of interior solutions are:[15]

$$\dot{c}(t) = (c(t)-\bar{c})\theta^{-1}\left[(1-\tau)\left(A+\alpha Bk(t)^{\alpha-1}\right)-\delta-\rho\right] \tag{11}$$

$$\dot{k}(t) = (1-\tau)\left[Ak(t)+Bk(t)^{\alpha}\right] - (\delta+n)k(t) - c(t). \tag{12}$$

4.2. Implications

4.2.1. Dynamic equilibria

In order to discuss the implications of the model, the canonical system (11) and (12) that governs the dynamics of the economy is investigated. The model displays two steady states with stationary values of consumption and capital and one (asymptotic) balanced-growth equilibrium with unbounded growth. Eq. (11) illustrates that there are three possibilities for the long-run evolution of consumption: (i) zero growth with consumption at subsistence [$\dot{c}=0,\ c^{*}=\bar{c}$]; (ii) zero growth with consumption above subsistence [$\dot{c}=0,\ c^{*}>\bar{c}$]; and (iii) unbounded growth [$c \to \infty$].

[14] The incorporation of an index of distortions into the production function enables a meaningful explanation of different long-run growth rates while assuming symmetry with respect to preferences and technology between the economy under study and the rest of the world (Easterly et al., 1992; Rebelo, 1992; Easterly, 1994).

[15] Because the Hamiltonian of the problem (10) is concave in the control and the state, the necessary conditions are also sufficient for a maximum. In addition, an optimal trajectory must satisfy the transversality condition: $\lim_{t\to\infty} e^{-(\rho-n)t}\lambda(t)k(t) = 0$.

The subsistence equilibrium with zero growth and consumption at subsistence results whenever the private net marginal product of capital evaluated at the subsistence capital stock falls short of the time preference rate, i.e. $(1-\tau)(A+\alpha B\bar{k}^{\alpha-1})-\delta-\rho<0$. As a result of the diminishing marginal returns to capital, the validity of the preceding inequality evaluated at the subsistence level of capital implies that this inequality holds for each initial stock of capital above subsistence. The steady-state value of capital is implicitly defined by $(1-\tau)(A\bar{k}+B\bar{k}^{\alpha})-(\delta+n)\bar{k}-\bar{c}=0$.[16] This unique steady state is (locally) saddle-point stable.[17]

The second steady state with consumption above subsistence applies whenever the private net marginal product of capital exceeds the time preference rate at the initial point in time, i.e. $(1-\tau)(A+\alpha Bk(0)^{\alpha-1})-\delta-\rho>0$ with $k(0)>\bar{k}$. In this case, the individuals are able and willing to substitute consumption intertemporally and consumption and capital initially increase. However, as the economy grows the private marginal product of capital decreases. The asymptotic marginal product of capital might be too low in order to guarantee unbounded growth: $(1-\tau)A-\delta-\rho\leq 0$. Eventually the golden rule will apply, i.e. the private net marginal product of capital equals the time preference rate and the economy converges to a unique steady state with $k^*=[(\delta+\rho-(1-\tau)A)/((1-\tau)\alpha B))]^{1/\alpha-1}$ and $c^*=[(1-\tau)A-\delta+n)]k^*+B(k^*)^{\alpha}$. Again, this steady state is (locally) saddle-point stable.

The asymptotic balanced growth equilibrium applies whenever the growth condition is even valid asymptotically, i.e. $(1-\tau)A-\delta-\rho>0$. The economy pursues unbounded growth. From Eq. (11) it is clear that the asymptotic growth rate of consumption is $\lim_{t\to\infty}\frac{\dot{c}}{c}=\theta^{-1}[(1-\tau)A-\delta-\rho]$. In addition, Eq. (12) indicates that consumption and capital must expand asymptotically at the same rate if the growth rate of capital should be constant asymptotically (time index omitted):

$$\lim_{t\to\infty}\frac{\dot{c}}{c}=\lim_{t\to\infty}\frac{\dot{k}}{k}=\theta^{-1}[(1-\tau)A-\delta-\rho]. \tag{13}$$

As has been shown in the literature, the private rate of return to the reproducible inputs is primarily affected by government policies, which were summarized by the index of overall distortions. Consequently, international variations in detrimental government policies that reduce the marginal product of the reproducible factors represent a potential explanation of internationally diverging growth experiences.

[16] This equation might have two solutions for $\bar{k}$; one less than the golden-rule value for k and one greater. In this case, the lower value is the relevant one (Easterly, 1994).

[17] An appendix available from the author upon request proves the saddle-point stability.

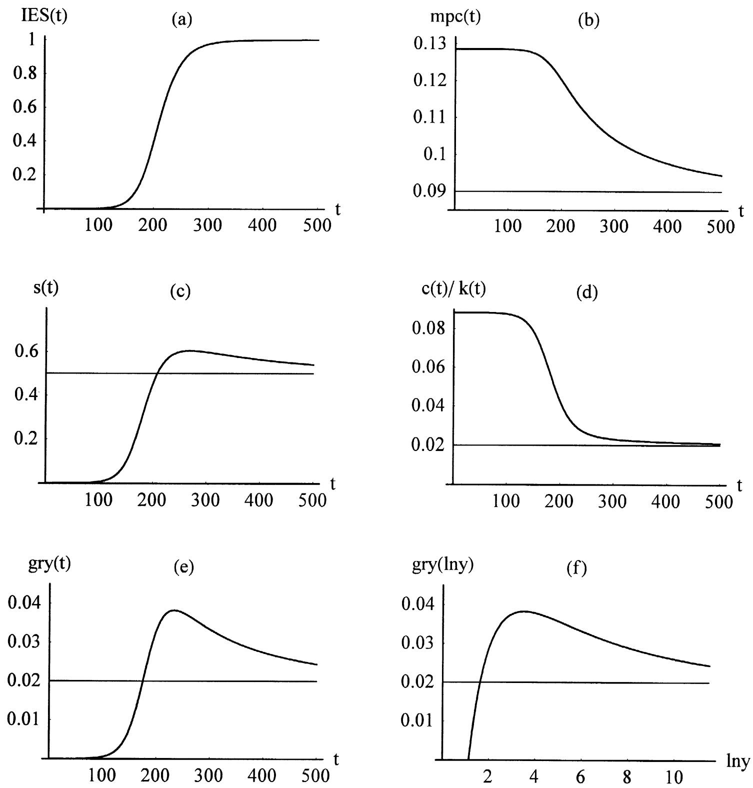

Fig. 5.

4.2.2. Transitional dynamics: simulation results

In order to illustrate the dynamics of the model under study, the transition process in the case of unbounded growth is simulated. This signifies that the system of differential equations (11) and (12) is approximated numerically, supposing that the growth condition holds.[18]

Fig. 5 shows the time paths of the intertemporal elasticity of substitution, IES(t), the marginal product of capital, mpc(t), the saving rate, $s(t)$, the consumption–capital ratio, $c(t)/k(t)$, the growth rate of income, gry(t), and the relation between the growth rate of income and the logarithm of income, gry$(\ln y)$, respectively. Several points should be emphasized.

The growth rate of income displays non-monotonic dynamics, specifically a hump-shaped relation is observed. Growth initially accelerates, reaching a maxi-

[18] The following set of parameters is employed: $\tau = 0.1$, $A = 0.1$, $B = 0.1$, $\alpha = 0.8$, $\delta = 0.02$, $n = 0.03$, $\theta = 1$, $\rho = 0.05$, and $\bar{c} = 2$.

mum growth rate and subsequently decelerates (Fig. 5e and f). In other words, the model implies conditional β-divergence for the lower range of income and conditional β-convergence for the higher range of income. Eventually, the growth rate converges towards its asymptotic balanced-growth-equilibrium value. Therefore, the Jones–Manuelli model with subsistence consumption is able to reproduce an important stylized fact of economic growth [stylized fact (4)].[19]

This pattern of evolution is the result of two opposing forces, as can be illustrated on the basis of the simulation results: First, at early stages of economic development the IES increases (Fig. 5a) reflecting an increase in the ability to save. As a result, the saving rate rises initially (Fig. 5c), causing the growth rate of income to rise as well, the subsistence-divergence mechanism. Second, the marginal product of capital falls with an increasing level of capital (Fig. 5b). Economically, this bears two implications: (i) A fall in the rate of return to the factors that can be accumulated reduces the incentives to save.[20] (ii) Along with the marginal product the average product of capital decreases as well. Consequently, the productive contribution of the whole stock of capital to output growth is reduced and the growth rate of output falls. Both mechanisms together [(i) and (ii)] constitute the neoclassical convergence mechanism. In the present case, the subsistence-divergence mechanism dominates the neoclassical convergence mechanism at early stages of economic development, while the reverse is true for later stages of development.

The c/k-ratio falls monotonically and converges towards its asymptotic balanced-growth-equilibrium value (Fig. 5d). Therefore, the c/k-ratio is used to give a rough estimate of the average rate of convergence. Analogous to the definition of the instantaneous rate of convergence used in the previous section, the average rate of convergence valid for any time interval $[t, t+v]$ can be defined as: $\lambda_a(t, t+v) \equiv -([x(t-v)-x(t)]/v)/(x(t)-x^*)$. The average rate of convergence for $t=0$ and $v=200$ amounts to $\lambda_a(0,200) \cong 0.0034$. That is, during the first 200 years on average 0.34% of the distance between the initial state and the balanced-growth-equilibrium gets eliminated per year. As before, this value is extraordinarily low and the implied time span required for the transition to the balanced-growth equilibrium is extremely long.

At last, Fig. 5 shows that the saving rate (Fig. 5c) and the growth rate of income (Fig. 5e) seem to be nearly constant during the first 100 years; in fact they increase very slowly. Empirically, this pattern of dynamics could be interpreted as representing a balanced-growth equilibrium with low saving and low growth.

[19] It should be noted explicitly that this result essentially hinges on the value of the parameter α. If this technology parameter is reduced sufficiently, the marginal product of capital decreases more rapidly. As a result, a monotonic rise in the growth rate of income occurs.

[20] The interaction of this mechanism together with the increase of the saving rate as a result of a rise in the ability to save explains the slight overshooting of the saving rate (Fig. 5c).

However, within the frame of the present model it represents a transitional phenomenon. Following this early stage of economic development, saving and growth increase significantly. This pattern of dynamics is consistent with several theoretical approaches and empirical observations on the stages of economic development. Reynolds (1983) comprehensively describes the growth experiences of the 'third world' over the time period from 1850 to 1980. He stresses the distinction between extensive and intensive growth.[21] According to Reynolds, most countries experienced a turning point, i.e. extensive turned into intensive growth.[22] Within the present model, this turning point occurs after about 100 years as Fig. 5(e) indicates. Therefore, the growth model under study is able to explain this transition from extensive to intensive growth endogenously. However, the implication of a smoothly increasing growth rate can be questioned on empirical grounds. Pritchett (1998), for example, reveals a high instability in growth rates that is especially marked for DCs. This finding does not necessarily contradict the models presented in this paper. The models are compatible with single or infrequently occurring large permanent shocks. In this case the time path of the growth rate of per capita income would show a discontinuity. Provided that the shocks are uncorrelated across countries, the average pattern of growth appears nonetheless consistent with the transitional dynamics described above.

5. Summary and conclusion

The requirement of subsistence consumption unambiguously affects the process of economic growth. It clearly restricts the ability to save not only for levels of per capita income at or slightly above subsistence. The intertemporal elasticity of substitution, which reflects both the ability and the willingness to save, increases with the level of per capita consumption and asymptotically converges to a constant. As a result, the requirement of subsistence consumption causes the growth rate of income to increase. It therefore represents an important mechanism of β-divergence, which might be labeled as subsistence-divergence mechanism. For realistic and widely employed parameter values, the rate of convergence is exceptionally low at early stages of economic development and the time span required for the transition towards the asymptotic balanced-growth equilibrium is correspondingly long.

Despite its obvious simplicity, the linear growth model with subsistence consumption is able to reproduce two of the stylized facts enumerated in the

[21] Extensive growth is defined as population growth with per capita income being constant, while intensive growth is defined as population growth with a rise in per capita income (Reynolds, 1983, p. 943).

[22] Reynolds' concept of 'turning point' resembles the take-off period originated by Rostow (1956).

Introduction. The model implies a rise in the saving rate along with the level of per capita income [stylized fact (2)] as well as β-divergence [stylized fact (3)]. On account of the extraordinarily low values of the rate of convergence for low incomes, different growth rates can partly be explained to represent transitional phenomena [stylized fact (1)]. However, if international symmetry with respect to preferences and technology is supposed, the possible range of growth rates is restricted. Hence, the linear growth model with subsistence consumption has some difficulty in explaining the big diversity in growth experiences observable for the group of DCs. In addition, the model clearly fails to reproduce the hump-shaped pattern of growth [stylized fact (4)].

An extension of the basic model by a general index of policy-induced distortions, which is sensibly allowed to vary internationally, permits a more satisfactory explanation of stylized fact (1). The possible range of long-run growth rates enhances despite maintaining the symmetry assumption. In the case of unbounded growth one observes policy continuity, i.e. the growth rate falls steadily as the extent of distortions increases (Jones and Manuelli, 1990). The model is further extended by diminishing marginal returns to the factors that can be accumulated in order to reproduce the remaining stylized fact (4). The interaction between the subsistence-divergence mechanism and the neoclassical convergence mechanism produces an acceleration subsequently followed by a deceleration of growth, i.e. a hump-shaped pattern of growth [stylized fact (4)].

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