STUDYING COGNITION VIA INFORMATION SEARCH IN TWO-PERSON GUESSING GAME EXPERIMENTS

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This paper is still in progress; for additional background please see:

Crawford, "Look-ups as the Windows of the Strategic Soul: Studying Cognition via Information Search in Game Experiments," manuscript, October 2006; and

Costa-Gomes and Crawford, "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study," *American Economic Review* (December 2006)

The paper builds on recent experimental papers that study subjects’ cognition in games by monitoring their searches for hidden but freely accessible information about payoffs.

The methods for monitoring search originated in MouseLab studies of decisions. (MouseLab is an automated way to track search as in eye-movement studies; see Payne, Bettman, and Johnson, *The Adaptive Decision Maker*, 1993. A modern analog, with more capabilities, is used in Wang, Spezio, and Camerer, "Pinocchio's Pupil: Using Eyetracking and Pupil Dilation To Understand Truth-telling and Deception in Games," 2006.)

These experiments all randomly and anonymously paired subjects to play series of different but related two-person games, with no feedback between games.

Suppressing learning from experience and repeated-game effects allows the designs to elicit subjects' initial responses, game by game.

This allows them to focus on strategic thinking—how players model others' decisions—uncontaminated by learning (which can make even amoebas converge to equilibrium).

("Eureka!" learning remains possible, but it can be tested for and seems to be rare.)

(The results yield insights into cognition that also help us think about how to model learning from experience, but that's another story.)

The previous experimental studies include:

Camerer, Johnson, Rymon, and Sen, "Cognition and Framing in Sequential Bargaining for Gains and Losses," in Kenneth Binmore, Alan Kirman, and Piero Tani (editors), *Frontiers of Game Theory*, 1993 (“CJ”); and

Johnson, Camerer, Sen, and Rymon, "Detecting Failures of Backward Induction: Monitoring Information Search in Sequential Bargaining," *JET* 2002 (“CJ”)

In these experiments subjects played series of two-person, three-round alternating-offers bargaining games, in which the "pie" varies across rounds to simulate discounting at a common rate; within a publicly announced extensive-form structure, each game was presented to subjects as a series of searchable pies. (Here and below, subjects were not allowed to write, and did not memorize the payoffs.)

These designs test backward-induction and social-preferences explanations of behavior in alternating-offers bargaining games.

Camerer and Johnson, "Thinking about Attention in Games: Backward and Forward Induction," in Isabel Brocas and Juan Carrillo (editors), *The Psychology of Economic Decisions, Volume Two: Reasons and Choices*, Oxford, 2004

In these experiments played simple extensive-form games with searchable payoffs, designed to test forward induction.

Costa-Gomes, Crawford, and Broseta, "Cognition and Behavior in Normal-Form Games: An Experimental Study," *Econometrica* 2001 ("CGCB")

In these experiments subjects played series of two-person matrix games with various kinds of iterated dominance or with unique pure-strategy equilibria without dominance; each game was presented to subjects as a set of independently searchable payoffs.

Costa-Gomes and Crawford, "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study," *American Economic Review* (December 2006) ("CGC")

In these experiments subjects played series of dominance-solvable two-person guessing games; each game was presented to subjects as a set of searchable payoff parameters.

These normal-form designs test equilibrium and iterated dominance against alternative decision rules such as the "level-*k*" types of Stahl and Wilson (1994 *JEBO*, 1995 *GEB*).

The experiments for the *AER* paper included two kinds of treatment:

A Baseline treatment, in which subjects played the games with other subjects

Several Robot/Trained Subjects ("R/TS") treatments, in which different subjects played the same series of games against a "robot" (framed as "the computer") and the computer played according to a pre-specified decision rule; those subjects were trained to identify the guesses the computer's rule yields and paid for their payoffs against the computer.

(There were six different R/TS treatments: one each for the decision rules or "types" *L1*, *L2*, *L3*, *D1*, *D2*, or *Equilibrium* as defined below.)

The *AER* paper focuses on reporting and analyzing Baseline subjects' guesses, which yield a clear view of the distribution of subjects' decision rules, but with some puzzles.

The current paper will report and analyze R/TS subjects' guesses and Baseline and R/TS subjects' information search data, with a focus on resolving the puzzles.

Outline

I begin by summarizing CGC's experimental design and results for guesses.

I then present representative search data and discuss how to model search behavior and use the data to draw inferences regarding cognition; combining search with a rudimentary analysis of cognitive process can better identify subjects' decision rules, sometimes even directly revealing the algorithms subjects use to choose their decisions.

Finally, I illustrate the possibilities for search analyses by showing how the search data (and R/TS results) can help to resolve the puzzles left open by the analysis of guesses.

CGC's experimental design

In CGC's guessing games, each player has his own lower and upper limit, both strictly positive; but players are not required to guess between their limits.

Guesses outside the limits are automatically adjusted up to the lower or down to the upper limit as necessary (a trick to enhance separation of rules via search).

Each player also has his own target, and his payoff increases with the closeness of his adjusted guess to his target times the other's adjusted guess.

The targets and limits vary independently across players and 16 games, with the targets either both less than one, both greater than one, or mixed.

(In the previous guessing experiments of Nagel (1995 *AER*) and Ho, Camerer, and Weigelt (1998 *AER*; "HCW"), targets and limits were always the same for both players, and varied either only across treatments or not at all.)

TABLE 3—STRATEGIC STRUCTURES OF THE GAMES

Game *ij*	Order played	Targets	Equilibrium	Rounds of dominance	Pattern of dominance	Dominance at both ends
1. $\alpha 2\beta 1$	6	Low	Low	4	A	No
2. $\beta 1\alpha 2$	15	Low	Low	3	A	No
3. $\beta 1\gamma 2$	14	Low	Low	3	A	Yes
4. $\gamma 2\beta 1$	10	Low	Low	2	A	No
5. $\gamma 4\delta 3$	9	High	High	2	S	No
6. $\delta 3\gamma 4$	2	High	High	3	S	Yes
7. $\delta 3\delta 3$	12	High	High	5	S	No
8. $\delta 3\delta 3$	3	High	High	5	S	No
9. $\beta 1\alpha 4$	16	Mixed	Low	9	S/A	No
10. $\alpha 4\beta 1$	11	Mixed	Low	10	S/A	No
11. $\delta 2\beta 3$	4	Mixed	Low	17	S/A	No
12. $\beta 3\delta 2$	13	Mixed	Low	18	S/A	No
13. $\gamma 2\beta 4$	8	Mixed	High	22	A	No
14. $\beta 4\gamma 2$	1	Mixed	High	23	A	Yes
15. $\alpha 2\alpha 4$	7	Mixed	High	52	S/A	No
16. $\alpha 4\alpha 2$	5	Mixed	High	51	S/A	No

Notes: Game identifiers: limits α for 100 and 500, β for 100 and 900, γ for 300 and 500, or δ for 300 and 900; targets 1 for 0.5, 2 for 0.7, 3 for 1.3, 4 for 1.5. Low targets are <1; high targets are >1; mixed targets are one <1, one >1. High equilibrium is determined by players' upper limits; low equilibrium is determined by players' lower limits. Rounds of dominance refers to the number player *i* needs to identify his equilibrium guess. Alternating dominance (A) occurs first for one player, then the other, then the first, etc.; simultaneous dominance (S) occurs for both players at once; and simultaneous then alternating dominance (S/A) is simultaneous in the first round and then alternating. Dominance at both ends refers to whether guesses are eliminated near both of a player's limits.

The 16 games subjects played are finitely dominance-solvable in 3-52 rounds, with essentially (because the only thing about a guess that matters is its adjusted guess) unique equilibria determined by the targets and limits in a simple way.

E.g. in game γ4δ3, player *i*'s limits and target are [300, 500] and 1.5 and player *j*'s are [300, 900] and 1.3. The product of targets 1.5 × 1.3 > 1, which implies that players' equilibrium adjusted guesses are determined (at least indirectly) by their upper limits. Player *i*'s equilibrium adjusted guess equals his upper limit of 500, but player *j*'s equilibrium adjusted guess is below his upper limit at 650.

The way in which equilibrium is determined here, by players' upper limits when the product of their targets is greater than 1, or by players' lower limits when the product of their targets is less than 1, is general in CGC's guessing games.

CGC's design exploits the discontinuity of the equilibrium correspondence when the product is 1 by including some games that differ mainly in whether the product is slightly greater, or slightly less, than 1; equilibrium responds very strongly to such differences, but empirically plausible non-equilibrium decision rules are largely unmoved by them.

The way in which equilibrium is jointly determined by both players' parameters also helps to separate the search implications of equilibrium and other rules.

Leading Strategic Decision Rules or *Types*

CGC's analysis of decisions, like Stahl and Wilson's (1995 *GEB*) and CGCB's, uses a structural non-equilibrium model of initial responses in which each subject's decisions are determined by one of several decision rules or *types*.

(Types play a central role in CGC's and CGCB's model of cognition, search, and decisions, which takes a procedural view of decision-making, in which a subject's type determines his search and his type and search determine his decision.)

I focus on CGC's normal-form types, assuming risk-neutrality with no social preferences:

L1, which best responds to a uniform random *L0* "anchoring type"

L2 (*L3*), which best responds to *L1* (*L2*)

Equilibrium, which makes its equilibrium decision

D1 (*D2*), which does one round (two rounds) of deletion of dominated decisions and then best responds to a uniform prior over the other's remaining decisions

Sophisticated, which best responds to the probabilities of other's decisions, as estimated from subjects' observed frequencies

Remarks:

L0 usually has 0 estimated frequency (or is confounded with the error structure).

Lk for *k* > 0 is rational, but deviates from equilibrium because it uses a simplified model of others' decisions. It is *k*-level rationalizable and so coincides with equilibrium in games that are *k*-dominance solvable. With plausible population type frequencies, this yields an inverse relationship between strategic complexity and equilibrium compliance as is often observed, e.g. CGCB, Table II.

Previous analyses have considered alternative definitions of *L2*, etc.: Stahl and Wilson's *L2* best responds to a noisy *L1*; and Camerer, Teck-Hua Ho, and Juin-Kuan Chong's ("A Cognitive Hierarchy Model of Games," 2004 *QJE*) *L2* best responds to an estimated mixture of *L1* and *L0*. CCG discuss the evidence.

By a quirk of our notation, *L2* (not *L1*) is *D1*'s cousin, and *L3* is *D2*'s. It is those pairs whose guesses are perfectly confounded in Nagel's games; and in two-person games, *Lk* guesses are *k*-rationalizable, just as *Dk-1*'s are.

CGC's Results for Guesses

The large strategy spaces and independent variation of targets and limits in CGC's design enhance separation of types' implications for decisions, to the point where many subjects' types can be precisely identified from guesses alone.

Of 88 subjects, 43 made guesses that complied *exactly* (within 0.5) with one type's guesses in 7-16 of the games (20 *L1*, 12 *L2*, 3 *L3*, and 8 *Equilibrium*).

E.g. CGC's Figure 2 shows the "fingerprints" of the 12 subjects whose apparent types were *L2*. Of their 192 guesses, 138 (72%) were exact. With games in Figure 2's unrandomized order, exact *L2* guesses track the pattern: 105, 175, 175, 300, 500, 650, 900, 900, 250, 225, 546, 455, 420, 525, 315, 315, more complex in randomized order.

Given how strongly CGC's design separates types' guesses (CGC's Figure 5), and that guesses could take 200-800 different rounded values in their games, these subjects' compliance is far higher than could occur by chance.

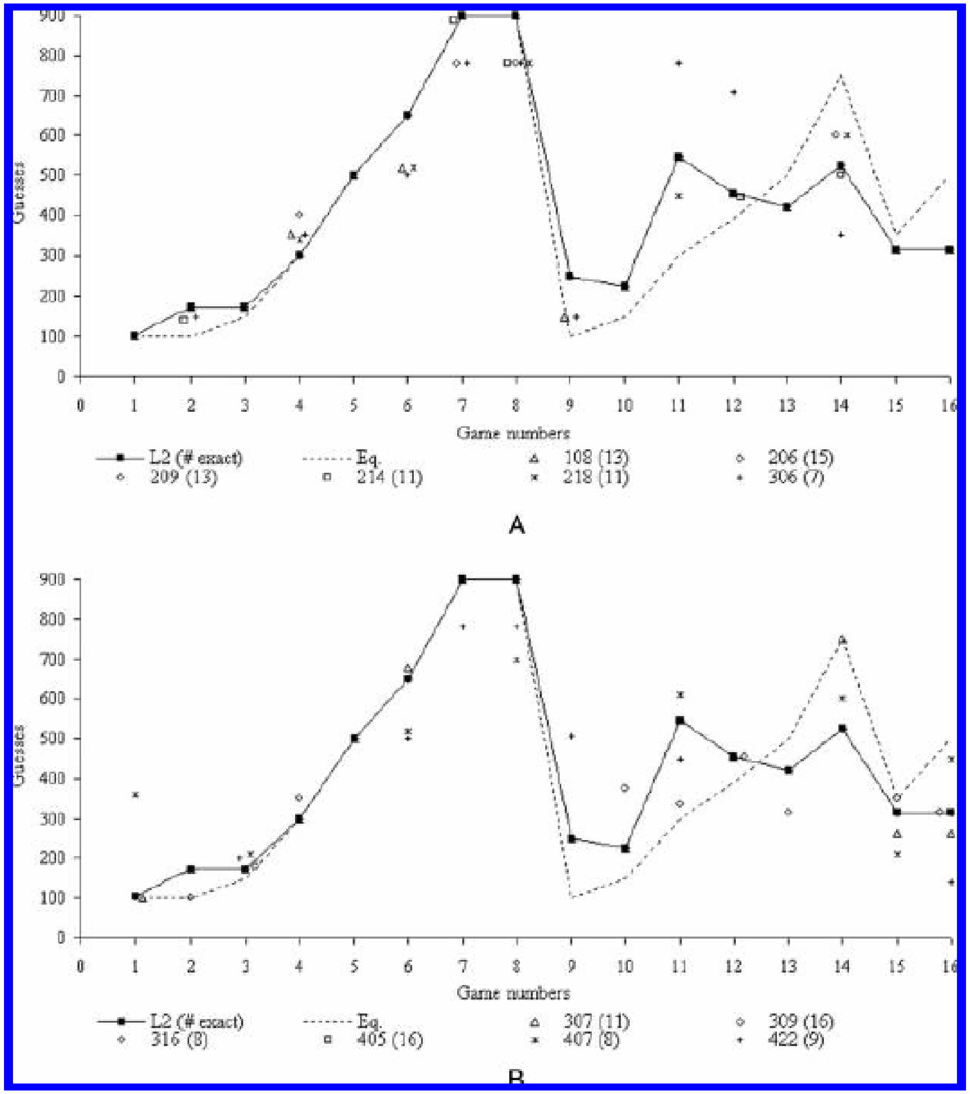


FIGURE 2. "FINGERPRINTS" OF 12 APPARENT *L2* SUBJECTS

Notes: Only deviations from *L2*'s guesses are shown. Of these subjects' 192 guesses, 138 (72 percent) were exact *L2* guesses.

Further, because the types specify precise, well-separated guess sequences in a very large space of possibilities, their high exact compliance rules out (intuitively or econometrically) alternative interpretations of their behavior.

In particular, because the types build in risk-neutral, self-interested rationality and perfect models of the game, the deviations from equilibrium of the 35 subjects whose apparent types are *L1*, *L2*, or *L3* can be attributed to non-equilibrium beliefs rather than irrationality, risk aversion, altruism, spite, or confusion.

(By contrast, in Stahl and Wilson's or CGCB's matrix-game designs, even a perfect fit does not distinguish a subject's best-fitting type from nearby omitted types; and in Nagel's and HCW's guessing-game designs, with large strategy spaces but with each subject playing only one game, the ambiguity is worse.)

CGC's other 45 subjects' types are less apparent from their guesses; but *L1*, *L2*, *L3*, and *Equilibrium* are still the only ones that show up in econometric estimates.

Unlike the common interpretation of Nagel's and HCW's results—that subjects are explicitly performing finitely iterated dominance—CGC's clear separation of *Lk* from *Dk-1* shows that *Dk* types don't exist in any significant numbers, at least in this setting.

Sophisticated, clearly separated from *Equilbrium*, also doesn't exist in significant numbers.

Two Puzzles Left Unresolved by CGC's Analysis of Guesses

A. What are Those Baseline "*Equilibrium*" Subjects Really Doing?

Consider the eight Baseline subjects with near-*Equilibrium* fingerprints (CGC's Figure 4).

Ordering the games by strategic structure, with CGC's 8 games with mixed targets on the right, shows that their deviations from equilibrium almost always occur with mixed targets.

Thus these subjects, whose compliance with *Equilibrium* guesses is "off the scale," are following a rule that only mimics *Equilibrium*, and only in games without mixed targets.

Yet all the ways we teach people to identify equilibria (best-response dynamics, equilibrium checking, iterated dominance) work just as well with mixed targets; thus whatever these subjects are doing, it's something we haven't thought of yet.

(*All* 44 of these subjects' deviations from *Equilibrium* (solid line) when it is separated from *L3* (dotted line) are in the direction of (and sometimes beyond) *L3* guesses. So they have a structure. But this could just reflect the fact that *L3* guesses are always less extreme.)

By contrast, CGC's *Equilibrium* R/TS subjects' compliance is equally high with and without mixed targets. (Those subjects were taught the usual ways to identify equilibria.)

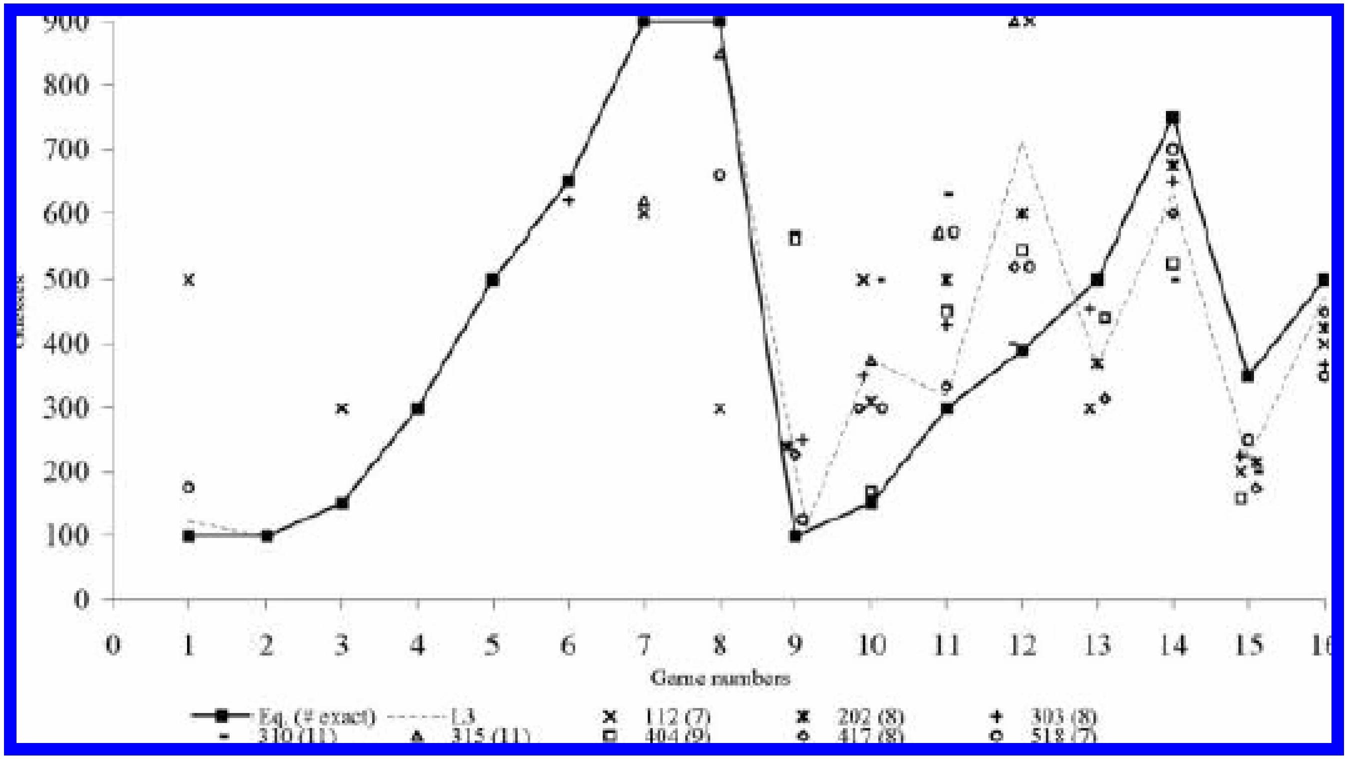
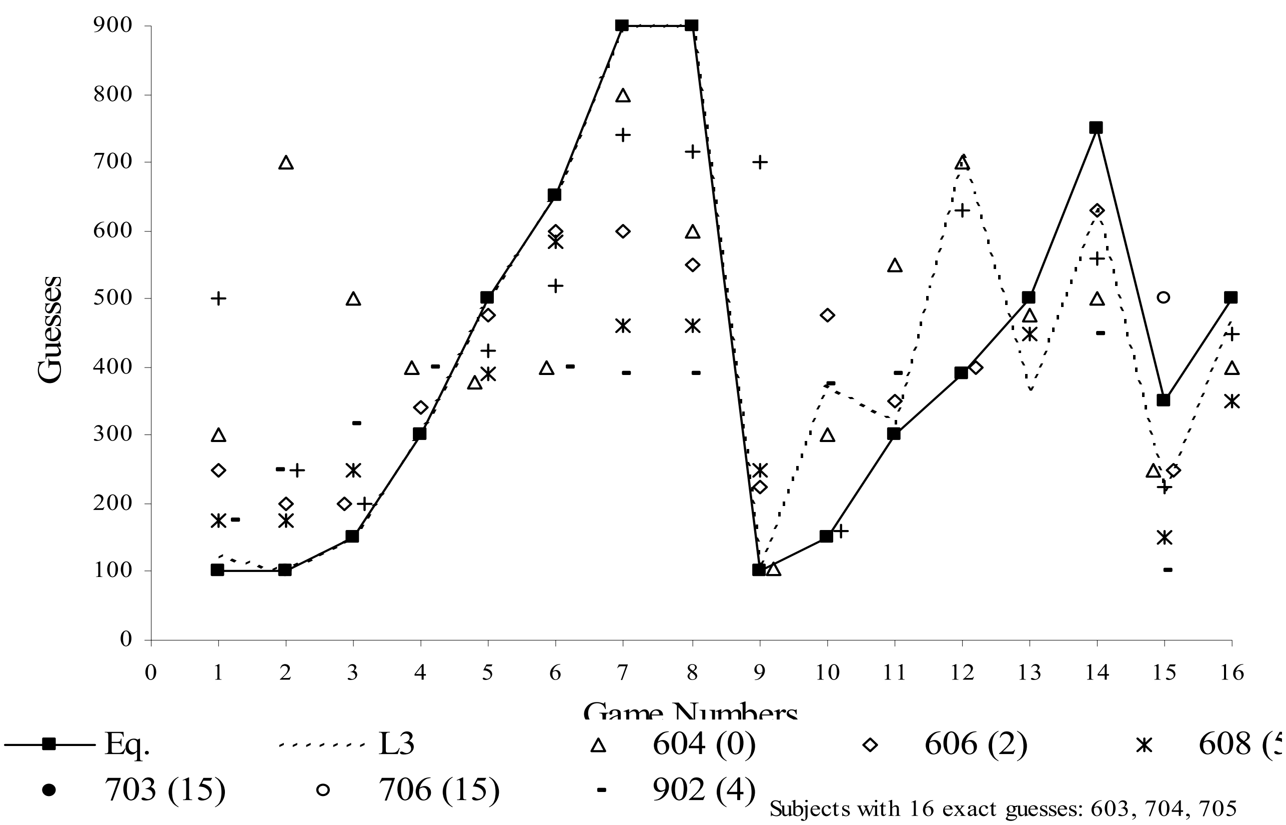


FIGURE 4. "FINGERPRINTS" OF EIGHT APPARENT *EQUILIBRIUM* SUBJECTS

Notes: Only deviations from *Equilibrium*'s guesses are shown. Of these subjects' 128 guesses, 69 (54 percent) were exact *Equilibrium* guesses.

Fingerprints of 10 UCSD Equilibrium R/TS Subjects
(only deviations from Eq.'s guesses are shown)



B. Why Are *Lk* the Only Types other than *Equilibrium* with Nonnegligible Frequencies?

CGC's analysis of decisions and search revealed significant numbers of subjects of types *L1*, *L2*, *Equilibrium*, or hybrids of *L3* and/or *Equilibrium*, and nothing else.

(More precisely, a careful analysis of the data, including CGC's specification test, reveals no other types that do better than a random model of guesses for more than one subject.)

Why do these rules predominate, out of the enormous number of possiblilities?

(Why, for instance, don't we get *Dk* rules, which are closer to what we teach?)

I suggest possible ways to resolve both puzzles after discussing the search analysis.

CGC’s Analysis of Information Search

In CGC’s design, within a publicly announced structure, each game was presented via MouseLab, which normally concealed the targets and limits but allowed subjects to look them up as often as desired, one at a time (click option, versus rollover option).

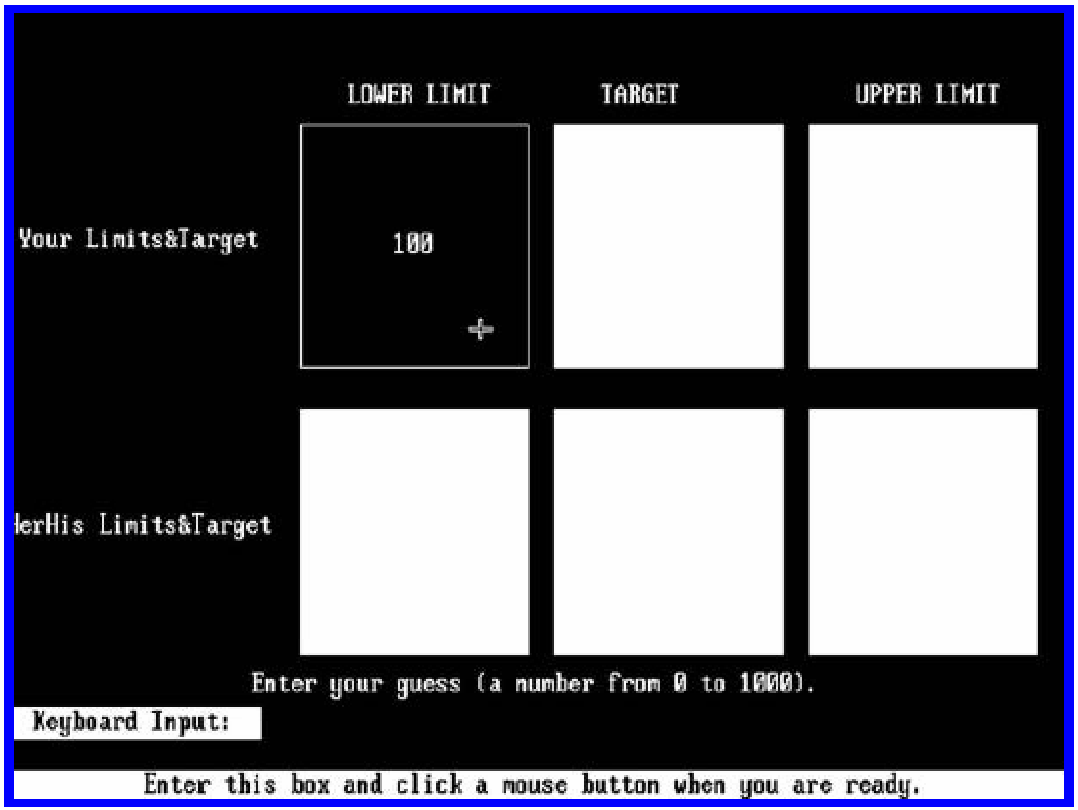


FIGURE 6. SCREEN SHOT OF THE MOUSELAB DISPLAY

With search costs as low as subjects' search patterns make them seem, free access made the entire structure effectively public knowledge, so the results can be used to test theories of behavior in complete-information versions of the games.

These designs also maintain tight control over subjects' motives for search by making information from previous plays completely irrelevant to current payoffs.

Design Desiderata for Studying Cognition via Search

CGC's design combines the strengths of Camerer and Johnson's and CGCB's designs.

Camerer and Johnson's design allows subjects to search for a small number of hidden payoff parameters (pies) within a simple, publicly announced structure, but makes their search patterns essentially one-dimensional, and so less informative than they could be.

CGCB's design makes search roughly three-dimensional (up-down in own payoffs, left-right in other's payoffs, transitions from own to other's payoffs) and independently separates the implications of leading types for search and decisions, but search is complex (8-16 payoffs in games with no common structure beyond being matrix games).

CGC's design has a simple parametric structure but makes search patterns high-dimensional, with leading types' search implications (almost) independent of the game.

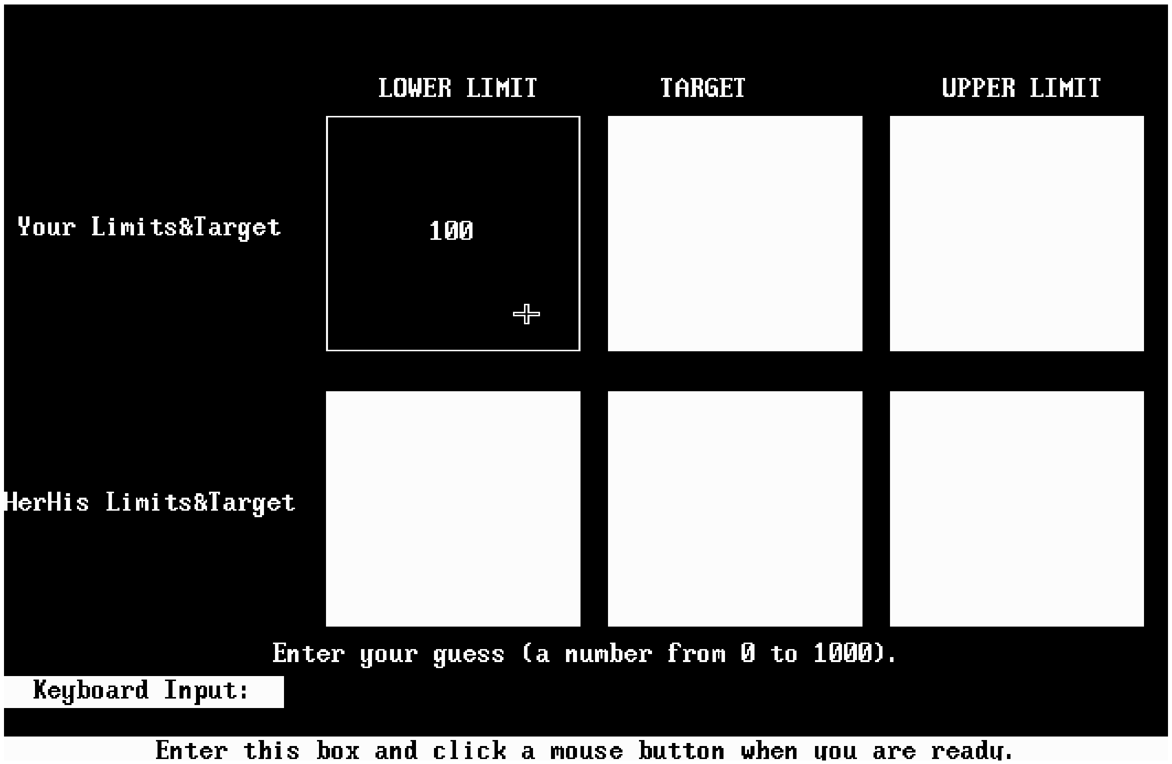
Search Data for Representative R/TS and Baseline Subjects

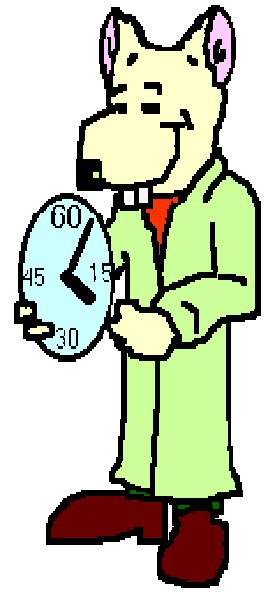
Let's start by comparing the search data for representative R/TS and Baseline subjects whose guesses conform closely to their assigned or estimated type, type by type, with the implications of CGC's theory of cognition and search (which is close to CGCB's theory, and was therefore specified almost completely before these data were generated).

Compliance with types' search implications as summarized in the tables will suggest that there some usable structure in the data, and then we can figure out how to model it.

But first….

SPEAK RODENT LIKE A NATIVE IN ONE EASY LESSON!





	a	*p*	*B*
You (*i*)	1	2	3
S/he (*j*)	4	5	6

MouseLab Box Numbers

Selected R/TS Subjects' Information Searches and Assigned Types' Search Implications

MouseLab box numbers

	a	*p*	*b*
You (*i*)	1	2	3
S/he (*j*)	4	5	6

Types' Search Implications

L1	{[4,6],2}
L2	{([1,3],5),4,6,2}
L3	{([4,6],2),1,3,5}
D1	{(4,[5,1], (6,[5,3]),2}
D2	{(1,[2,4]),(3,[2,6]),(4,[5,1],(6,[5,3]),5,2}
Eq	{[2,5],4} if pr. tar.<1,{[2,5],6} if > 1

Subject	**904**	**1716**	**1807**	**1607**	**1811**	**2008**	**1001**	**1412**	**805**	**1601**	**804**	**1110**	**1202**	**704**	**1205**	**1408**	**2002**
Type(#rt.)	L1 (16)	L1 (16)	L1 (16)	L2 (16)	L2 (16)	L2 (16)	L3 (16)	L3 (16)	D1 (16)	D1 (16)	D1 (3)	D2 (14)	D2 (15)	Eq (16)	Eq (16)	Eq (15)	Eq (16)
Alt.(#rt.)											L2 (16)						
Est. stvle	late	often	earlv	often	earlv				earlv								
Game																	
1	123456 4623	146462 134646 23	462513	135462 1313	134446 5213*4 6	111313 131313 5423	462135 21364* 246231 52	146231 564623 1	154356 423213 2642	254514 36231	154346 5213	135464 2646*1 313	246466 135464 641321 342462 422646 124625 5*1224 654646	123456 363256 565365 626365 652651 452262 6526	123456 424652 562525 6352*4 65	123123 456445 632132 11	142536 125365 253616 361454 613451 213452 63
2	123456 4231	462462 13	462132 25	135461 354621 3	134653 125642 313562 52	131313 566622 333	462135 642562 223146 2562*6 2	462462 546231 546231	514535 615364 23	514653 6213	515135 365462 3	135134 642163 451463 211136 414262 135362 *14654 6	123645 132462 426262 241356 462*13 524242 466135 6462	123456 525123 652625 635256 262365 456	123456 244565 565263 212554 146662 654251 44526* 31	123456 456123 643524 1	143625 361425 142523 625656 3

The subjects' frequencies of making their assigned types' (and when relevant, alternate types') exact guesses are in parentheses after the assigned type.
ct's look-up sequence means that the subject entered a guess there without immediately confirming it.

Selected Baseline Subjects' Information Searches and Estimated Types' Search Implications

MouseLab box numbers

	a	*p*	*b*
You (*i*)	1	2	3
S/he (*j*)	4	5	6

Types' Search Implications

L1	{[4,6],2}
L2	{([1,3],5),4,6,2}
L3	{([4,6],2),1,3,5}
D1	{(4,[5,1], (6,[5,3]),2}
D2	{(1,[2,4]),(3,[2,6]),(4,[5,1],(6,[5,3]),5,2}
Eq	{[2,5],4} if pr. tar.<1,{[2,5],6} if > 1

Subject	**101**	**118**	**413**	**108**	**206**	**309**	**405**	**210**	**302**	**318**	**417**	**404**	**202**	**310**	**315**
Type(#rt.)	L1 (15)	L1 (15)	L1 (14)	L2 (13)	L2 (15)	L2 (16)	L2 (16)	L3 (9)	L3 (7)	L1 (7)	Eq (8)	Eq (9)	Eq (8)	Eq (11)	Eq (11)
Alt.(#rt.)								Eq (9)	Eq (7)	D1 (5)	L3 (7)	L2 (6)	D2 (7)		
Alt.(#rt.)								D2 (8)			L2 (5)		L3 (7)		
Est. style	early/late	early	late	early	early	late	early	early	early	early	early	early	early	early/late	early
Game															
1	146246 213	246134 626241 32*135	123456 545612 3463*	135642	533146 213	1352	144652 313312 546232 12512	123456 123456 213456 254213 654	221135 465645 213213 45456* 541	132456 465252 13242* 1462	252531 464656 446531 641252 462121 3	462135 464655 645515 21354* 135462 426256 356234 131354 645	123456 254613 621342 *525	123126 544121 565421 254362 *21545 4*	213465 624163 564121 325466
2	46213	246262 2131	123564 62213*	135642 3	531462 31	135263 1526*2 *3	132456 253156 456545 463123 156562 62	123456 465562 231654 456*2	213546 566213 545463 21*266 54123	132465 132*46 2	255236 62*365 243563	462461 352524 261315 463562	123456 445613 255462 513565 23	123546 216326 231456 *62	134652 124653 656121 3
3	462*46	246242 466413 *426	264231	135642 53	535164 2231	135263	312456 5231*1 236545 5233** 513	123455 645612 3	265413 232145 563214 563214 523*65 4123	134652 1323*4	521363 641526 5263*6 52	462135 215634 *52	123456 123562 3	123655 463213	132465 544163 *3625

Modeling Cognition and Search

Different papers have taken different positions on how cognition shows up in search:

Camerer, Johnson, et al. gave roughly equal weight to look-up durations and to the numbers of look-ups of each pie ("acquisitions") and the transitions between pies.

Rubinstein, "Instinctive and Cognitive Reasoning: A Study of Response Times," *EJ* 2007, which considers some matrix games, considered only durations.

Camerer, Johnson, et al.'s and Rubinstein's analyses were also conducted at a very high level of aggregation, both across subjects and over time.

Gabaix, Laibson, Moloche, and Weinberg, "Costly Information Acquisition: Experimental Analysis of a Boundedly Rational Model," *American Economic Review*, 2006, focused on numbers of look-ups (not durations) and considered some aspects of their order too; they also conducted their analysis mostly at a high level of aggregation.

Studying Cognition via Numbers and Order of Subjects' Look-ups

CGCB and CGC argued that cognition is sufficiently heterogeneous and search sufficiently noisy that they are best studied at the individual level.

They also presumed that which look-ups subjects make, in which order, reveals at least as much information about cognition as durations or transition frequencies.

This should not be surprising, because simple theories of cognition more readily suggest roles for which look-ups subjects make, in which orders, than durations.

(No claim that durations are irrelevant was intended, just that they don't deserve the priority they have been given. CGCB (Table IV) do present some results on durations, under the heading of "gaze times.")

Types as Models of Cognition, Search, and Decisions

CGC's and CGCB's models of cognition, search, and decisions are based on a procedural view of decision-making, in which a subject's type determines his search, and his type and search then determine his decision. This is the key to linking them in the analysis.

(Because a type's search implications depend not only on what decisions it specifies, but why, something like a types-based model seems necessary here.)

Each type is naturally associated with algorithms that process payoff information into decisions; the analysis uses those algorithms as models of cognition, deriving a type's search implications under simple assumptions about how it determines search.

With their derived search implications, the types will provide a kind of basis for the enormous space of possible decision and search sequences, imposing enough structure to allow us to describe subjects' behavior in a comprehensible way, and to make it meaningful to ask how decisions and searches are related.

Cognition and Search

Without further assumptions, nothing precludes a subject's scanning and memorizing the information and going into his brain to figure out what to do, in which case search reveals nothing about cognition.

But subjects' actual searches appear to contain a lot of information about cognition.

We need to make enough additional assumptions to allow us to extract the signal from the noise in searches, but not so many that they distort the meaning of the signal.

CGC's (like CGCB's) additional assumptions are conservative in that they rest on types' minimal search implications, and they add as little structure to these as possible.

Types' minimal search implications in CGC's games can be derived from their *ideal guesses*, those they would make if they had no limits. (With automatic adjustment of guesses to fall within their limits, and quasiconcave payoffs, this is all they need to know.)

The left side of Table 4 lists formulas for types' ideal guesses in CGC's games.

The right side lists types' search implications, first in terms of our notation, then in terms of the box numbers in which MouseLab records the data.

TABLE 4—TYPES' IDEAL GUESSES AND RELEVANT LOOK-UPS

Type	Ideal guess	Relevant look-ups
L1	$p^i[a^j + b^j]/2$	$\{[a^j, b^j], p^i\} \equiv \{[4, 6], 2\}$
L2	$p^iR(a^j, b^j; p^j[a^i + b^i]/2)$	$\{([a^i, b^i], p^j), a^j, b^j, p^i\} \equiv \{([1, 3], 5), 4, 6, 2\}$
L3	$p^iR(a^j, b^j; p^jR(a^i, b^i; p^i[a^j + b^j]/2))$	$\{([a^j, b^j], p^i), a^i, b^i, p^j\} \equiv \{([4, 6], 2), 1, 3, 5\}$
D1	$p^i(\max\{a^j, p^ja^i\} + \min\{p^jb^i, b^j\})/2$	$\{(a^j, [p^j, a^i]), (b^j, [p^j, b^i]), p^i\} \equiv \{(4, [5, 1]), (6, [5, 3]), 2\}$
D2	$p^i[\max\{\max\{a^j, p^ja^i\}, p^j\max\{a^i, p^ia^j\}\}$ $+ \min\{p^j\min\{p^ib^j, b^i\}, \min\{p^jb^i, b^j\}\}]/2$	$\{(a^i, [p^i, a^j]), (b^i, [p^i, b^j]), (a^j, [p^j, a^i]), (b^j, [p^j, b^i]), p^j, p^i\}$ $\equiv \{(1, [2, 4]), (3, [2, 6]), (4, [5, 1]), (6, [5, 3]), 5, 2\}$
Eq.	$\{a^i$ if $p^ia^j \leq a^i$ or $\min\{p^ia^j, b^i\}$ if $p^ia^j > a^i\}$ if $p^ip^j < 1$ or $\{b^i$ if $p^ib^j \geq b^i$ or $\max\{a^i, p^ib^j\}$ if $p^ib^j < b^i\}$ if $p^ip^j > 1$	$\{[p^i, p^j], a^j\} \equiv \{[2, 5], 4\}$ if $p^ip^j < 1$ or $\{[p^i, p^j], b^j\}$ $\equiv \{[2, 5], 6\}$ if $p^ip^j > 1$
Soph.	[no closed-form expression; *Sophisticated*'s search implications are the same as *D2*'s]	$\{(a^i, [p^i, a^j]), (b^i, [p^i, b^j]), (a^j, [p^j, a^i]), (b^j, [p^j, b^i]), p^j, p^i\}$ $\equiv \{(1, [2, 4]), (3, [2, 6]), (4, [5, 1]), (6, [5, 3]), 5, 2\}$

Notes: The most basic operations are represented by the innermost look-ups, grouped within square brackets; these can appear in any order, but *may not* be separated by other look-ups. Other operations are represented by look-ups grouped within parentheses or curly brackets; these can appear in any order, and *may* be separated by other look-ups. *Equilibrium*'s minimal search implications are derived not directly from *Equilibrium*'s ideal guesses, but from p^ia^j when $p^ip^j < 1$ and p^ib^j when $p^ip^j > 1$ via Observation 1 (see on-line Appendix H).

Types' Search Implications

Evaluating a formula for a type's ideal guess requires a series of *operations*, some of which are *basic* in that they logically precede any other operation.

E.g. $[a^j+b^j]$ is the only basic operation for *L1*'s ideal guess, $p^i\,[a^j+b^j]/2$.

The search implications in Table 4 assume subjects perform basic operations one at a time via adjacent look-ups, remember their results, and otherwise rely on repeated look-ups rather than memory.

Basic operations will then be represented by adjacent look-up pairs that can appear in either order, but cannot be separated by other look-ups.

Such pairs are grouped within square brackets, as in $\{[a^j,b^j],p^i\}$ for *L1*.

Other operations can appear in any order and their look-ups can be separated.

They are represented by look-ups grouped within curly brackets or parentheses.

(Table 4 shows the look-ups associated with a type's operations in the order that seems most natural, if there is one; but this is not a requirement of the theory.)

Evidence on Cognition and Search
These assumptions are based on several sources of evidence:

(i) Camerer and Johnson's Robot/Trained Subjects' searches, which led them to characterize subgame-perfect equilibrium via backward induction search in terms of transitions between the second- and third-round pies

(ii) CGCB's Trained Subjects' searches, which suggest a similar view of *Equilibrium* search in matrix games

(iii) CGC's R/TS and Baseline subjects with high compliance with their types' guesses, whose searches suggest a similar view of *L1* and *L2* search

(CGC's specification analysis turned up only one clear violation of the proposed characterization of types' search implications: Baseline subject 415, whose apparent type was *L1* with 9 exact guesses, had 0 *L1* search compliance in 9 of the 16 games because s/he had no adjacent [a^j,b^j] pairs as we required for *L1*. Her/his look-up sequences were unusually rich in (a^j,p^i,b^j) and (b^j,p^i,a^j) triples, in those orders. Because the sequences were *not* rich in such triples with other superscripts, this is clear evidence that 415 was an *L1* who happened to be more comfortable with several numbers in working memory than our characterization of search assumes, or than our other subjects were. But because this violated our assumptions on search, this subject was "officially" estimated to be *D1*.)

Search Data for Representative R/TS and Baseline Subjects

The above search data suggest the following conclusions:

(i) There is little or no difference between the look-up sequences of R/TS and Baseline subjects of a given type (assigned for R/TS, apparent for Baseline); perhaps this is unsurprising, because R/TS subjects were not trained in search strategies

(ii) A subject’s type’s predicted sequence (Table 4) is unusually dense in his search sequences, at least for *L1* and *L2* (CGC's econometric analysis measures search compliance for a type as the density of its relevant sequences in the subject's sequences)

(iii) One can quickly learn to see the algorithms many subjects are using in the data. (And if we can learn to do it, the right kind of econometrics can do it too: many of CGC's subjects' types can be reliably identified from search alone (CGC's Table 7).)

(iv) For some subjects search is an important check on decisions; e.g. Baseline subject 309, with 16 exact *L2* guesses, misses some of *L2*'s relevant look-ups, avoiding deviations from *L2* only by luck (s/he later has a Eureka! moment between games 5 and 6, and from then on complies perfectly); reminiscent of CJ's finding that in their alternating-offers bargaining games, 10% of the subjects *never* looked at the last-round pie and 19% never looked at the second-round pie. Even if their decisions had conformed to subgame-perfect equilibrium, they could not have been making them for the reasons the theory assumes.)

CGC's Econometric Analysis of Guesses and Search

Most subjects' guesses-and-search type estimates reaffirm the guesses-only estimates.

For some the guesses-and-search type estimate resolves a tension between guesses-only and search-only estimates in favor of a type other than the guesses-only estimate.

In more extreme cases, a subject's guesses-only type estimate is excluded because it has 0 search compliance in 8 or more games, like subject 415.

Overall, search refines and sharpens the conclusions and confirms the absence of significant numbers of types other than *L1*, *L2*, *Equilibrium*, or hybrids of *L3* or *Equilibrium*.

TABLE 1—SUMMARY OF BASELINE AND OB SUBJECTS' ESTIMATED TYPE DISTRIBUTIONS

Type	Apparent from guesses	Econometric from guesses	Econometric from guesses, excluding random	Econometric from guesses, with specification test	Econometric from guesses and search, with specification test
L1	20	43	37	27	29
L2	12	20	20	17	14
L3	3	3	3	1	1
D1	0	5	3	1	0
D2	0	0	0	0	0
Eq.	8	14	13	11	10
Soph.	0	3	2	1	1
Unclassified	45	0	10	30	33

Note: The far-right-hand column includes 17 OB subjects classified by their econometric-from-guesses type estimates.

Possible Sources of Answers to the Puzzles

A. What are Those Baseline "*Equilibrium*" Subjects Really Doing?

(i) Can we tell how Baseline *Equilibrium* subjects find equilibrium in games without mixed targets: best-response dynamics, equilibrium checking, iterated dominance, or something else that doesn't "work" with mixed targets?

(The absence of Baseline *Dk* subjects suggests that they are not using iterated dominance. Best-response dynamics, perhaps truncated after 1-2 rounds, seems more likely. We plan to check by refining CGC's characterization of *Equilibrium* search and redoing the econometrics, separately with and without mixed targets.)

(ii) Is there any identifiable difference in Baseline *Equilibrium* subjects' search patterns in games with and without mixed targets? If so, how do the differences compare to those for *L1*, *L2*, or *L3* subjects?

(Our 20 Baseline *L1* subjects' compliance with *L1* guesses (CGC, Figure 1) is almost the same with and without mixed targets: unsurprisingly because the distinction is irrelevant to *L1*. But our 12 *L2* (CGC, Figure 2) and 3 *L3* subjects' compliance with apparent types' guesses is noticeably lower with mixed targets. This is curious, because for *L2* and *L3*, unlike for *Equilibrium*, games with mixed targets require no deeper understanding.)

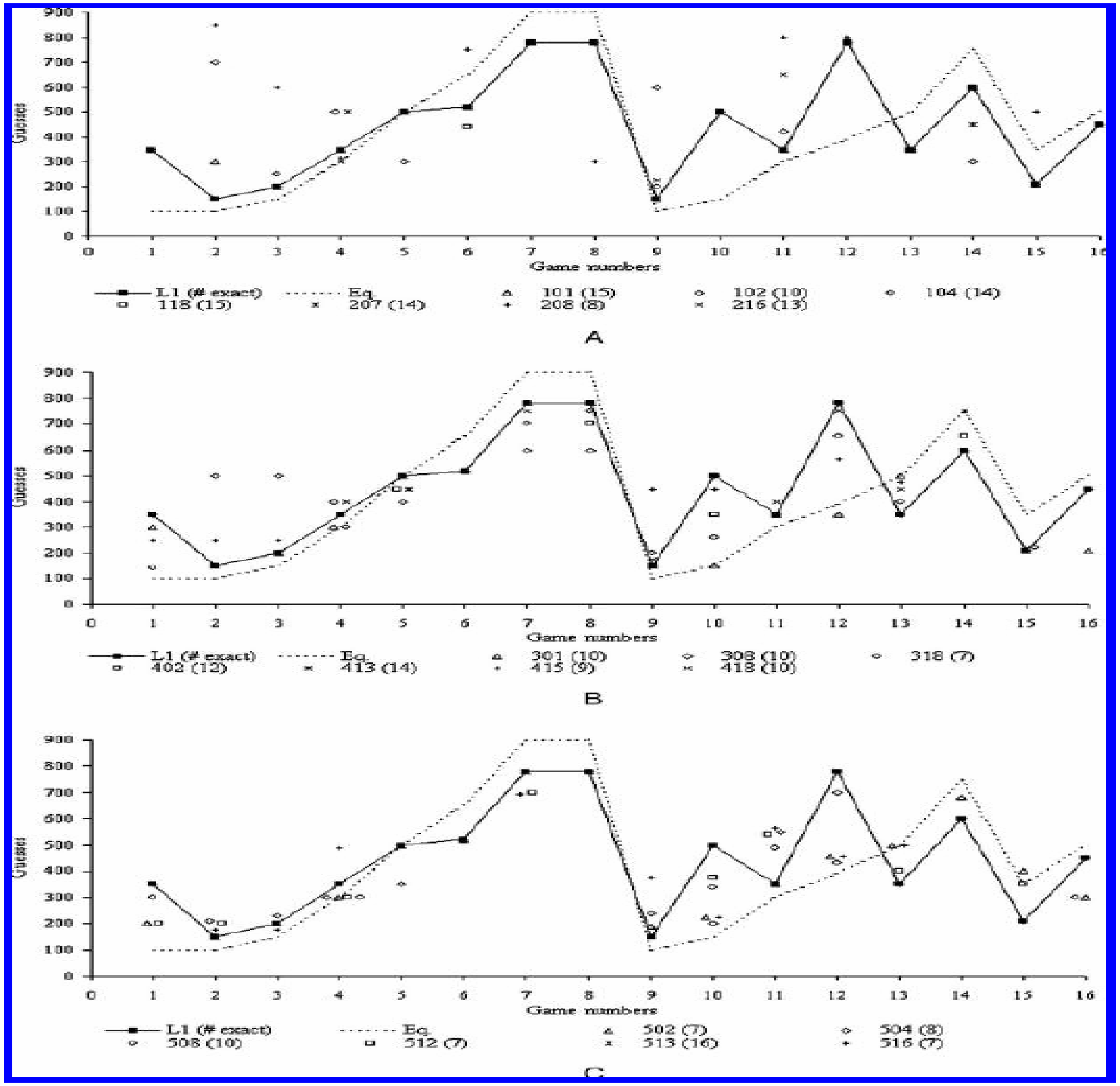


FIGURE 1. "FINGERPRINTS" OF 20 APPARENT *L1* SUBJECTS

Notes: Only deviations from *L1*'s guesses are shown. Of these subjects' 320 guesses, 216 (68 percent) were exact *L1* guesses.

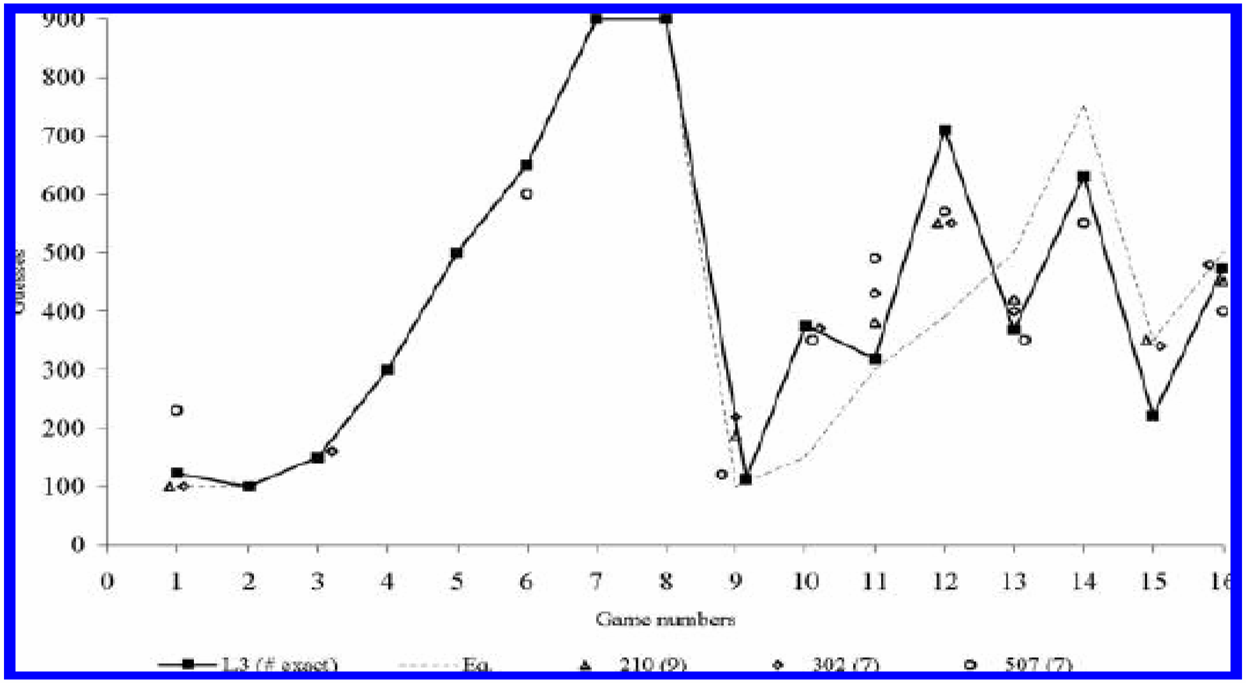


FIGURE 3. "FINGERPRINTS" OF THREE APPARENT *L3* SUBJECTS

Notes: Only deviations from *L3*'s guesses are shown. Of these subjects' 48 guesses, 23 (48 percent) were exact *L3* guesses.

(iii) Can we tell how R/TS *Equilibrium* subjects with high compliance manage to find their *Equilibrium* guesses even with mixed targets? How does their search in those games differ from Baseline *Equilibrium* subjects' search?

(CGC strove to make the R/TS *Equilibrium* training as neutral as possible, but something must come first. They were taught equilibrium-checking first, then best-response dynamics, then iterated dominance. To the extent that they used one of those methods, it explains why they have equal compliance with and without mixed targets. If they used something else, and it deviates from equilibrium in games with mixed targets, it might provide a clue to what the Baseline *Equilibrium* subjects did.)

(Note that CGC's Baseline subjects with high compliance for some type are, to the extent that we are confident in inferring their beliefs, like robot *un*trained subjects. These don't usually exist because you can't tell robot subjects how they will be paid without teaching them how the robot works, and so training them.

Thus CGC's design provides an unusual opportunity to separate the effects of training and strategic uncertainty, by comparing Baseline and R/TS subjects: Either *Equilibrium* is natural with mixed targets, but subjects don't see it without training; or *Equilibrium* is unnatural, and/or subjects don't believe that others, even with training, will make *Equilibrium* guesses with mixed targets.)

B. Why Are *Lk* the Only Types other than *Equilibrium* with Nonnegligible Frequencies?

(i) Most R/TS subjects could reliably identify their type's guesses, even *Equilibrium* or *D2*. (These average rates are for exact compliance, and so are quite high. Individual subjects' compliance was usually bimodal within type, on very high and very low.)

R/TS Subjects' Exact Compliance with Assigned Type's Guesses

	L1	***L2***	***L3***	***D1***	***D2***	***Eq.***
Number of subjects	25	27	18	30	19	29
% Compliance|Passed UT2	80.0	91.0	84.7	62.1	56.6	70.3
% Failed UT2	0.0	0.0	0.0	3.2	5.0	19.4

(ii) But there are noticeable signs of differences in difficulty across types:
(a) No one ever failed an *Lk* Understanding Test, while some failed the *Dk* and many failed the *Equilibrium* Understanding Test.

(b) For those who passed, compliance was highest for *Lk* types, then *Equilibrium*, then *Dk* types. This suggests that *Dk* is even harder than *Equilibrium*, but could just be an artifact of the more stringent screening of the *Equilibrium* Test.

(c) Among *Lk* and *Dk* types, compliance was higher for lower *k* as one would expect, except that *L1* compliance was lower than *L2* or *L3* compliance. (We suspect that this is because *L1* best responds to a random *L0* robot, which some subjects think they can outguess; while *L2* and *L3* best respond to a deterministic *L1* or *L2* robot.)

(d) Remarkably, 7 of our 19 R/TS *D1* subjects passed the *D1* Understanding Test, in which *L2* answers are wrong; and then "morphed" into *L2*s when making their guesses, significantly reducing their earnings. E.g. R/TS *D1* subject 804 made 16 exact *L2* (and so only 3 exact *D1*) guesses. (Recall that it is *L2* that is *D1*'s cousin.) This kind of morphing, in this direction, is the only kind that occurred. We view this as pretty compelling evidence that *Dk* types are unnatural.

(e) A comparison of *Lk*'s and *Dk-1*'s search and storage requirements may have something to add. (E.g. *Dk-1* requires more memory than *Lk*.)

Appendix 1: A "Theory" of Optimal Search for Hidden Payoff Information

I now sketch a simple model of optimal search with costs of storing numbers in working memory, which rationalizes the stylized facts of search behavior in these experiments.

The model views search for hidden payoff information as just another decision, and takes the formula that relates a type's desired decisions to the hidden parameters as given. Thus a subject's type determines both an optimal search pattern and an optimal decision.

Occurrence

The usual rationality assumption implies that a player will look up all costlessly available information that might affect his beliefs and best respond to his beliefs.

When, as here, observing a parameter will normally cause a nonnegligible change in beliefs and the optimal decision, this conclusion extends to all relevant information that is available at a sufficiently small but non-0 cost. (There is a lot of evidence that subjects perceive the cost of a look-up as close to negligible.)

Thus, if a type's decision depends on a hidden parameter, then that parameter must appear in the type's look-up sequence. But so far any order will do.

Adjacency

Assume that there is a cost of keeping numbers in working memory, which starts out small, but is much larger, even for one number, than the cost of a look-up; and that this cost increases with the number of stored numbers and is proportional to storage time. (Thus a player's "lifetime" total memory cost is the time integral of an increasing function of the number of stored numbers.) (There is some evidence for these assumptions too.)

Given these assumptions, a player minimizes his total memory plus look-up cost for evaluating an expression like *L1*'s ideal guess, $p^i [a^j+b^j]/2$, containing a basic operation like $[a^j+b^j]$, by processing $[a^j+b^j]$ separately, storing the result (in the meanwhile "forgetting" a^j and b^j), and combining the result with p^i.

(The alternative, processing $p^i a^j$ separately, storing the result, then processing $p^i b^j$ separately and combining it with $p^i a^j$, requires leaving more numbers in working memory longer: The sequence of numbers in memory for the method in the previous paragraph is 1, 2, 1, 2, 1; the sequence for the method in this paragraph is 1, 2, 1, 2, 3, 2, 1. The previous method also saves by eliminating the repeated look-up of p^i, but this is of second-order importance.)

I have only illustrated the cost savings from giving priority to basic operations, but I conjecture that the argument is general. If so, it justifies assuming that subjects perform basic operations one at a time via adjacent look-ups, remembering the results, and otherwise relying on repeated look-ups rather than memory.

The argument also seems likely to extend to CJ's extensive-form games, justifying their focus on transitions between pies from adjacent rounds.

The theory implies more than this, regarding both the order of operations (basic ones should come first) and how non-basic operations are executed. I defer such implications in favor of mentioning an issue regarding CGC's search data:

Many Baseline subjects usually look first at their apparent type's relevant sequence and then make irrelevant look-ups or stop (e.g. 108, 118, and 206, labeled "early" in the above look-up data). Others make irrelevant look-ups first, and look at the relevant sequence only near the end (e.g. 413, labeled "late"). Others repeat the relevant sequence many times (e.g. 101, labeled "early/late"). The theory is actually consistent with this kind of heterogeneity when look-up costs are negligible (but storage costs are not). Because MouseLab allows a subject to enter a tentative guess without confirming it (the *s in the look-up data), this kind of storage has zero cost in CGC's and CGCB's designs; and so subjects can satisfy their curiosity (early or late) without running up storage costs.

Appendix 2: Costa-Gomes, Crawford, and Broseta's Matrix-Game Experiments

CGCB adapted CJ's methods to study cognition via search for hidden payoffs in matrix games, eliciting initial responses to 18 games with various patterns of iterated dominance or unique pure-strategy equilibria without dominance (CGCB, Figure 2).

CGCB's design strongly separates leading types' implications for decisions.

Previous experiments (e.g. Stahl and Wilson, *GEB* 1995) found systematic deviations from the equilibrium decisions when players have pecuniary preferences (in games that probably disable social preferences).

CGCB's results for decisions replicated most patterns in previous experiments, with high equilibrium compliance with in games solvable by one or two rounds of iterated dominance but lower compliance in games solvable by three rounds or by the circular logic of equilibrium without dominance (CGCB, Table II).

CGCB's design replicated previous results in a way that allowed a more precise assessment of subjects' cognition, which confirms the view of subjects' behavior suggested by analyses of decisions alone, with some differences.

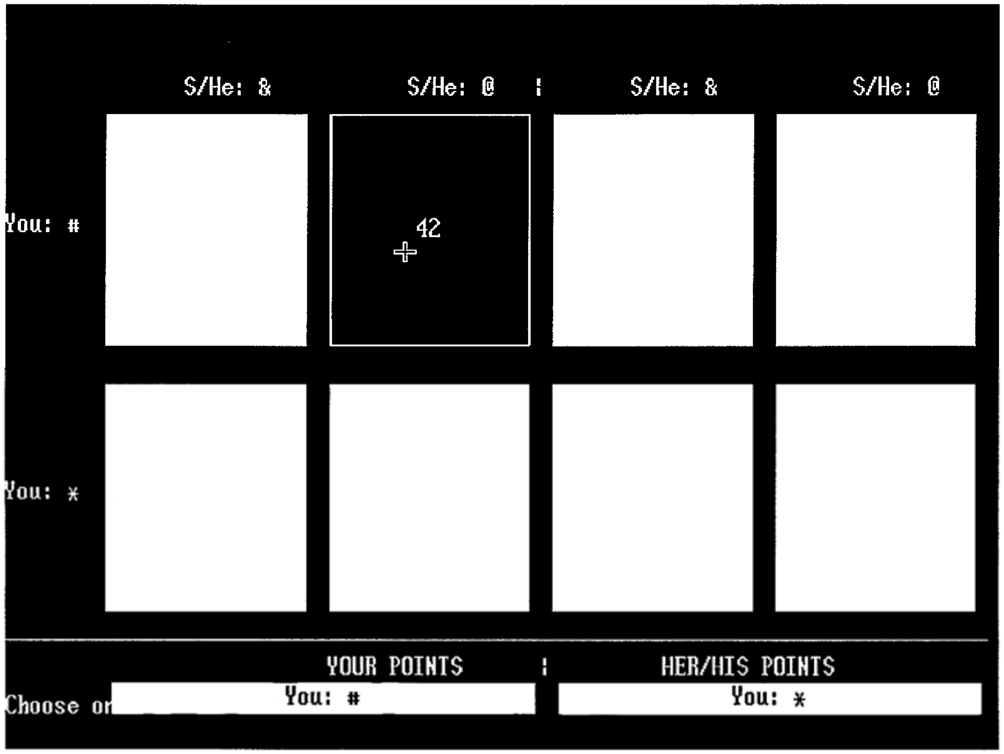
Monitoring Search via MouseLab in Matrix Games

Within a publicly announced structure, CGCB presented each game to subjects as a matrix via MouseLab, which normally concealed payoffs but allowed subjects to look up their own and their partner's payoffs for each decision combination as often as desired, one at a time (click option in MouseLab).

Row and Column players' payoffs were spatially separated to ease cognition and make search more informative.

(Subjects were always framed as Row players, although each played each of our games once as Row and once as Column player, in a sequence that disguised those relationships and randomized away effects of patterns in their structures.)

(Subjects were not allowed to write down the payoffs, and the frequencies with which they looked them up made clear that they did not memorize them.)



CGCB's Figure 1. MouseLab Screen Display (for a 2×2 game)

Separation of Types' Implications for Decisions in CGCB's Design: Figure 2

2A (1,2)	*A,P,N*	*D12,L2,E,S*
A	72,93	31,46
D	84,52	55,79

2B (1,2)	*A,P,N*	*D12,L2,E,S*
D	94,23	38,57
A	45,89	14,18

3A (2,1)	*D*	*A*
D12,L2,E,S	75,51	42,27
A,P,N	48,80	89,68

3B (2,1)	*D*	*A*
A,P,N	21,92	87,43
D12,L2,E,S	55,36	16,12

4A (2,1)		*D*	*A*
A,P,N	59,58	46,83	85,61
D12,L2,E,S	38,29	70,52	37,23

4B (2,1)	*A*	*D*
D12,L2,E,S	31,32	68,46
P	72,43	47,61
A,N	91,65	43,84

4C (1,2)	*D12,L2,E,S*	*A,P,N*
	28,37	57,58
A	22,36	60,84
D	51,69	82,45

4D (1,2)	*D12,L2,E,S*	*P*	*A,N*
D	42,64	57,43	80,39
A	28,27	39,68	61,87

5A (3,2)	*A,P,N*	*D12,L2,E,S*
A	53,86	24,19
P,N,D1,L2,S	79,57	42,73
D2,E	28,23	71,50

5B (3,2)	*A,P,N*	*D12,L2,E,S*
A	76,93	25,12
D2,E	43,40	74,62
P,N,D1,L2,S	94,16	59,37

6A (2,3)	*A*	*D2,E,S*	*P,N,D1,L2*
D12,L2,E,S	21,26	52,73	75,44
A,P,N	88,55	25,30	59,81

6B (2,3)	*D2,E*	*A*	*P,N,D1,L2,S*
A,P,N	42,45	95,78	18,96
D12,L2,E,S	64,76	14,27	39,61

7A (∞,∞)	*N,D12,L2,S*	*A,P*	*E*
L2,E,S	87,32	18,37	63,76
A,P,N,D12	65,89	96,63	24,30

7B (∞,∞)	*N,D12,L2,S*	*A,P*	*E*
A,P,N,D12	67,91	95,64	31,35
L2,E,S	89,49	23,53	56,78

8A (∞,∞)	*L2,E,S*	*A,P,N,D12*
E	72,59	26,20
A,P	33,14	59,92
N,D12,L2,S	28,83	85,61

8B (∞,∞)	*L2,E,S*	*A,P,N,D12*
A,P	46,16	57,88
E	71,49	28,24
N,D12,L2,S	42,82	84,60

9A (1,2)	*D12,L2,E,S*	*A,P,N*
	22,14	57,55
	30,42	28,37
A	15,60	61,88
D	45,66	82,31

9B (2,1)			*A*	*D*
A,P,N	56,58	38,29	89,62	32,86
D12,L2,E,S	15,23	43,31	61,16	67,46

(*A* = *Altruistic*, *P* = *Pessimistic* (minimax), *N* = *Naïve* (CGCB's name for *L1*) and *Optimistic* (maximax, decisions not separated from *Naïve*'s), *E* = *Equilibrium*, *S* = *Sophisticated*, *D12* = *D1* and *D2*, *D* = dominant decision = all types but *A*.)

Separation of Types' Implications for Search in CGCB's Design

CGCB's design also makes it possible to test and compare types via search.

They make two assumptions about how cognition affects search, *Occurrence* and *Adjacency* that are close to CGC's characterization of cognition and search.

In CGCB's display, a subject's searches can vary in three main dimensions:

(i) the extent to which his transitions are up-down in his own payoffs, which under *Occurrence* and *Adjacency* is (for a Row player) naturally associated with rationality in the decision-theoretic sense;

(ii) the extent to which his transitions are left-right in other's payoffs, which under *Occurrence* and *Adjacency* is associated with thinking about other's incentives;

(iii) the extent to which he makes transitions from own to other's payoffs and back for the same decision combination, which under *Occurrence* and *Adjacency* is associated with interpersonal fairness or competitiveness comparisons.

This variation allows strong separation of types' implications for search.

The independent separation of types' implications for decisions and search is an important strength of the design: Searches and decisions together, and their relationships, yield a much clearer view of a subject's type than decisions alone.

Some types' implications under Occurrence and Adjacency in game 3A (Column has dominant decision, "nonstrategic" Rows pick B and "strategic" Rows pick T)

	S/He: L	**S/He: R**	**S/He: L**	**S/He: R**
You: T	**75**	**42**	**51**	**27**
You: B	**48**	**89**	**80**	**68**
	Your	**Points**	**Her/His**	**Points**
	You: T		**You: B**	

Naïve (*L1*) compares expected payoffs of own decisions given a uniform prior over other's, via either up-down or left-right own payoff comparisons. Occurrence requires look-ups 75, 48, 42, and 89. Adjacency requires either the set of comparisons {(75,42), (48,89)} *or* the set of comparisons {(75,48), (42,89)}.

	S/He: L	S/He: R	S/He: L	S/He: R
You: T	**75**	**42**	**51**	**27**
You: B	**48**	**89**	**80**	**68**
	Your	**Points**	**Her/His**	**Points**
	You: T		**You: B**	

L2 needs to identify other's *Naïve* decision and *L2*'s best response to it; Occurrence requires all other's look-ups plus 75 and 48, the own look-ups for other's *Naïve* decision. Adjacency requires either the set of comparisons {(51,27), (80,68)} or the set of comparisons {(51,80), (27,68)} to identify other's Naïve decision, plus the comparison (75,48) to identify *L2*'s best response.

If *Equilibrium* has a dominant decision it needs only to identify it. If not, it can use iterated dominance or equilibrium-checking, decision combination by combination or via "best-response dynamics." Occurrence requires look-ups 51, 27, 80, 68, 75, and 48. Adjacency requires comparisons (51,27), (80,68), and (75,48).

CGCB's Results

The most frequent estimated types are *Naïve* (*L1*) and L2, each nearly half of the population.

Incorporating search compliance into the econometric analysis shifts the estimated type distribution toward *Naïve*, at the expense of *Optimistic* and *D1*.

Part of this shift occurs because *Naïve*'s search implications explain more of the variation in subjects' searches and decisions than *Optimistic*'s, which are too unrestrictive to be useful in the sample; another part occurs because *Naïve*'s search implications explain more of the variation in subjects' searches and decisions than *D1*'s, which are more restrictive, but too weakly correlated with subjects' decisions.

D1 also loses some frequency to *L2*, even though their decisions are weakly separated in CGCB's design, because *L2*'s search implications explain much more of the variation in subjects' searches and decisions.

Overall, CGCB's analysis of decisions and search yields a significantly different interpretation of behavior than their analysis of decisions alone. The analysis suggests a strikingly simple view of behavior, with *Naïve* and *L2* 65-90% of the population and *D1* 0-20%, depending on confidence in their model of search.