Lying for Strategic Advantage: Rational and Boundedly Rational Misrepresentation of Intentions

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Starting from an example of the Allies' decision to feint at Calais and attack Normandy on D-Day, this paper models misrepresentation of intentions to competitors or enemies. Allowing for the possibility of bounded strategic rationality and rational players' responses to it yields a sensible account of lying via costless, noiseless messages. In some leading cases, the model has generically unique pure-strategy sequential equilibria, in which rational players exploit boundedly rational players, but are not themselves fooled. In others, the model has generically essentially unique mixed-strategy sequential equilibria, in which rational players' strategies protect all players from exploitation. (*JEL* C72, D72, D80)

> Lord, what fools these mortals be!
> —Puck, *A Midsummer Night's Dream*, Act 3

> You may fool all the people some of the time; you can even fool some of the people all the time; but you can't fool all of the people all the time.
> —Abraham Lincoln

> Now give Barnum his due.
> —John Conlisk (2001)

Lying for strategic advantage about planned actions, or *intentions*, is a common feature of economic and political as well as military life. Such lying frequently takes the extreme form of active misrepresentation, as opposed to less than full, honest disclosure. Examples range from the University of California's three consecutive "last chance" voluntary early retirement incentive programs in the early 1990's; to ex-President George Bush's regrettably memorable 1988 campaign promise, "Read my lips: no new taxes"; the nearly universal practice of lying about planned currency devaluations; Nathan Rothschild's pretense of having received early news of a British defeat at Waterloo; Hitler's 1939 nonaggression pact with Stalin; and the U.S. Department of Defense's short-lived (but, remarkably, publicly acknowledged) Office of Strategic Influence (James Dao and Eric Schmitt, 2002).[1] In other cases, the effects of active misrepresentation are duplicated by tacit exploitation of widespread misperceptions, as in accelerationist monetary policy; periodic but unpredictable investment tax credits, or regularizations of the status of illegal immigrants; the failure to disclose known product safety hazards; and deceptive accounting practices in the private or the public sector.[2]

These examples share two common features.

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[1] Roland Benabou and Guy Laroque (1992) give several examples concerning lying to manipulate financial markets, including the possibly apocryphal story of Rothschild's pretense, which allegedly allowed him to make large clandestine purchases of British government securities at depressed prices. Examples in international politics are easy to find, and it is probably no accident that there is a board game called Diplomacy, in which success depends on forming unenforceable agreements with other players and then being the first to break them.

[2] See, for example, Paul Krugman's (2001) discussion of the current Bush administration's use of "creative" accounting to make the 2001 tax cut appear feasible without dipping into the Social Security surplus. Krugman's *New York Times* columns provide many other examples of lying by public or corporate officials.

All involve misrepresentation via agreements, statements, or nonstatements that in themselves have little or no direct costs. And all involve situations in which the parties have predominantly conflicting interests, so that successful deception benefits the deceiver only at the expense of the deceived. Nonetheless, the misrepresentation often succeeds. In fact, in many of the examples the public has so completely internalized the logic of misrepresentation that criticism of the gullibility of those deceived is as common as criticism of the misrepresentation itself.[3]

Theory lags behind the public's intuition. The examples' common features suggest that, to a first approximation, they can be modeled as communication via costless messages ("cheap talk") in a zero-sum two-person game. In such a model, however, costless messages must be ignored in equilibrium: If one player could benefit by responding to the other's message, his response would hurt the other player, who would therefore do better to make his message uninformative. Thus, in equilibrium no information is conveyed by costless messages, but neither is anyone fooled by them.[4]

This result is appealing in its simplicity, but it leaves us with no systematic way to think about a ubiquitous phenomenon with important consequences. This paper proposes a tractable characterization of bounded strategic rationality in a leading class of zero-sum two-person communication games. The characterization allows a complete analysis of the interaction between possibly rational or boundedly rational senders and receivers, which gives a sensible account of misrepresentation of intentions to competitors or enemies.

My analysis is inspired in part by Kenneth Hendricks and R. Preston McAfee's (2002; henceforth "HM") analysis of misrepresentation of intentions via what they call "feints" or "inverted signaling." HM's primary interest, like mine, is in economic and political applications; but they motivate their analysis by Operation Fortitude, the Allies' successful attempt to mislead the Germans about their intention to land in Normandy rather than the Pas de Calais (the obvious choice *ex ante*) on D-Day, June 6, 1944, and I will follow them in this.[5] Their model is a zero-sum two-person game. First the attacker chooses (possibly randomly) between two possible locations and allocates a fixed budget of force between them. Next, the defender privately observes a binary signal whose probability distribution depends on the attacker's allocation, and allocates (possibly randomly) his own budget of force between the two locations. The attack location and players' force allocations then determine their payoffs. The attacker's allocation is like a noisy message to the defender; but as in other models of costly signaling, its large direct payoff implications sometimes allow equilibria in which it is not ignored.

HM assume that the payoff function and the conditional probability distribution of the signal are both symmetric across locations. They show, under plausible additional assumptions, that equilibrium in their game must involve some attempt by the attacker to misrepresent his intentions (allocating force to both locations with positive probability) and that his attempt succeeds (inducing the defender to allocate force to both locations with positive probability). For, if the defender ignored his observation of the signal, the attacker would assign all of his force to his intended attack location; but if the defender anticipated this, the attacker would prefer to allocate some force to the other location.

HM identify equilibria in their model in two

[3] Mike Royko's (1988) prescient view of Bush's "Read my lips" promise is an entertaining example. An official who does not lie about his country's plan to devalue its currency risks being removed from office, if not institutionalized. Other examples are explicitly covered by proverbs such as "All's fair in love and war."

[4] See Crawford and Joel Sobel's (1982) and Joseph Farrell's (1993) analyses of strategic communication of private information and intentions. Crawford and Sobel's equilibria have no active misrepresentation, only intentional vagueness, taking the extreme form of no transmission if the Sender's and Receiver's preferences differ enough to make the game effectively zero-sum; Farrell coined the term "babbling" for such equilibria. Farrell and Matthew Rabin (1996) and Crawford (1998) survey the theory.

[5] The deception was so successful that the Germans kept 19 divisions in Calais for several critical days after D-Day. HM summarize the history; see also Gordon Harrison (1951, especially Appendix A) and Anthony Kemp (1994). HM also give several examples of economic and political misrepresentation, in which firms distort their exploration or bidding strategies to mislead competitors, political candidates campaign in areas they believe are unimportant to divert their opponents' efforts from areas they consider crucial, and so on.

cases, distinguished by the signal's informativeness. When the signal is not very informative, they identify "full-defense" equilibria, in which the attacker deterministically allocates most of his force to one attack location but randomizes the location itself, and the defender allocates all of his own force deterministically, to the location the signal suggests is more likely to be attacked. When the signal is more informative, they identify "split-defense" equilibria, in which the attacker randomizes his allocation and attack location in such a way that the defender can draw no inference from the signal, and the defender also randomizes his allocation. In these equilibria, with positive probability the attacker allocates more than half his force to the location he does *not* attack. HM also obtain intriguing comparative statics results, showing that when the signal is not very informative, a reduction in noise hurts the attacker; but that when it is more informative, a reduction in noise benefits the attacker.

HM stress that their explanation of misrepresentation depends on the noisiness of the signal: "With perfect observability, feints differ from the standard analysis in inconsequential ways. In particular, were the Germans to observe the actual allocation of allied forces, it would not have been possible for the Allies to fool the Germans. Thus, imperfect observation is a critical element for modeling feints."

HM's analysis makes significant progress in understanding the phenomenon of misrepresentation, but it has three troubling aspects. I shall describe them from the point of view of Operation Fortitude, although they are equally troubling in other applications.

First, the cost to the Allies of faking the preparations for an invasion of Calais was small compared to that of the preparations for the actual invasion of Normandy, hence more like cheap talk than HM's identification of feints with sizeable fractions (sometimes more than half) of the attacker's force would suggest. The Germans knew as well as the Allies that it was feasible to fake, or conceal, invasion preparations at no great cost. In a standard equilibrium analysis, they would then rationally ignore both the faked evidence that the attack would be at Calais and the lack of evidence that it would be at Normandy. But they did not—and Allied planners did not expect them to, with anything like certainty.

Second, HM's analysis does not reflect the asymmetry between Normandy and Calais that is arguably the most salient feature of Operation Fortitude.[6] Why not feint at Normandy and attack at Calais instead, particularly if the deception has a fair chance of success? Allied planners rejected Calais in favor of Normandy early in their planning, mainly (but not entirely) because the proximity to England that made it the obvious attack location was also obvious to the Germans, who were expected to defend it so heavily that on balance, Normandy would be preferable (Harrison, 1951). Neither Allied planners' choice of Normandy nor the fact that they did not explicitly randomize it is inconsistent with HM's equilibria per se, because they assign positive probabilities to both attack locations and in a mixed-strategy equilibrium in beliefs, a player need not bear any uncertainty about his own decision (Robert Aumann and Adam Brandenburger, 1995). But Allied planners were not indifferent between the locations, and an explanation that treats their choice as an accidental feature of the history may miss something important.

Finally, an analysis of equilibrium in a game without precedent—of which Operation Fortitude and D-Day are perhaps the quintessential example—implicitly rests on the assumption that players' rationality and beliefs are at least mutual knowledge (Aumann and Brandenburger, 1995, Theorem A). These assumptions are more than usually strained in HM's model, whose equilibria involve a delicate balance of wholly or partly mixed strategies that depend on the details of the signal distribution.

This paper shows that allowing for the *possibility* of bounded strategic rationality yields a sensible account of misrepresentation of intentions in a simpler game, and with costless and noiseless messages. The model and analysis fully reflect the low message costs, the importance of payoff asymmetry across actions, and the difficulty of justifying a delicate equilibrium analysis of a game without precedent just noted.

The model is based on the class of zero-sum two-person perturbed Matching Pennies games in Figure 1. Two players, a Sender (analogous

[6] HM's only reference to the asymmetry is to note that when their signal is not very informative, if the attacker's payoffs make one location easier to attack, that location is *more* likely to be attacked.

		Receiver	
		Left	Right
Sender	Up	a (Sender), $-a$ (Receiver)	0, 0
	Down	0, 0	1, -1

FIGURE 1. THE UNDERLYING GAME

to the Allies) and a Receiver, choose simultaneously between two pure actions, U for Up (analogous to attacking at Calais) or D for Down for the Sender and L for Left (analogous to defending Normandy) and R for Right for the Receiver. I assume throughout that $a > 1$, which corresponds to the lesser difficulty of an unanticipated invasion of Calais. Before playing this *underlying game*, the Sender sends the Receiver a costless, nonbinding, noiseless message, u or d, about his intended action, with u (d) representing action U (D) in a commonly understood language (Farrell, 1993). Players then choose their actions simultaneously. The structure of the game is common knowledge.[7]

In a standard equilibrium analysis of this game, in any equilibrium (subgame perfect or not) the Sender's message must be uninformative, in that the probability that he plays U conditional on his message is independent of the message; and the Receiver must ignore it, in the sense that the probability that he plays L is independent of the Sender's message.[8] The underlying game must therefore be played according to its unique mixed-strategy equilibrium, in which the Sender plays U with probability $1/(1 + a)$ and the Receiver plays L with probability $1/(1 + a)$, with respective expected payoffs $a/(1 + a)$ and $-a/(1 + a)$.[9] Thus, communication is ineffective and misrepresentation is unsuccessful.

The closest precedents for a nonequilibrium analysis of such games are Farrell (1988) and Rabin (1994), who study preplay communication about intentions via cheap talk, mainly in games whose players have substantial common interests, using extended notions of rationalizability.[10] My model is similar in spirit, but it relaxes the equilibrium assumption in a way that imposes more structure on behavior. I assume that the Sender's and the Receiver's roles in the above game are filled by players chosen randomly from separate distributions, each assigning positive prior probability to certain boundedly rational or *Mortal* decision rules, or *types*, as well as to a *Sophisticated* type. The players do not observe each other's types; but the structure of the game, including the type distributions, is common knowledge.[11] The analysis is otherwise completely standard.

[7] These games differ from HM's in having costless and noiseless messages, separate from the attacker's force allocation; simultaneous, zero-one allocations of force to locations; and a payoff asymmetry across actions. Because each side's forces were actually somewhat dispersed, the discrete force allocations in the present model should be thought of as representing *principal* attack or defense locations.

[8] The Sender can make his message uninformative either by always sending the same message or by randomizing his message independently of his action.

[9] This equilibrium illustrates a strategic principle noted by John Von Neumann and Oskar Morgenstern (1953 [first edition 1944 (!)], pp. 175–76): Counterintuitively, the Sender's probability of playing U is (like the Receiver's probability of playing L) a decreasing function of a. Thus, in this equilibrium, the Allies are more likely to attack Normandy and the Germans are more likely to defend Calais. But for plausible values of a (say, $a \leq 1.5$), the probability that, *ex post*, the Allies attack Normandy *and* the Germans defend Calais, is $a^2/(1 + a)^2 \leq 0.36$, so this explanation of the history is unlikely as well as accidental. Crawford and Dennis E. Smallwood (1984) analyze the comparative statics of payoff changes in general two-person zero-sum games with mixed-strategy equilibria, identifying the general principle that underlies this result.

[10] See also Miguel Costa-Gomes (2002), who extends Rabin's analysis to interpret experimental data.

[11] Thus the model adopts a view of human nature close to Lincoln's, which is more nuanced and arguably more realistic than Puck's. The idea of behavioral types has a long history in game theory, going back to David Kreps and Robert Wilson's (1982) and Paul Milgrom and John Roberts' (1982) analyses of Selten's Chain Store Paradox. Dilip Abreu and Rajiv Sethi (2001) provide an overview and an interesting application to bargaining. However, this and most subsequent work on behavioral types has focused on the limit of sequential equilibria as the prior probability of behavioral types approaches zero, which often differ discontinuously from sequential equilibria when the prior probability is zero. In my analysis, by contrast, sequential equilibrium normally varies continuously with the prior probabilities of *Mortal* types, and the interest of the analysis depends on those probabilities being nonnegligible. My analysis also differs in deriving *Mortals'* behavior directly from the structure of the communication game, in a way similar in spirit to Philip Reny's (1992) notion of explicable equilibrium, which models "trembles" via "complete theories."

Sophisticated players satisfy the usual mutual knowledge of beliefs and rationality assumptions with respect to each other, and can use their knowledge of the structure to predict the probability distributions of *Mortal* players' strategies. Thus, the *Sophisticated* type represents the ideal of a fully strategically rational player in this setting.

Mortal players can be thought of as maximizing expected payoffs, if desired, but the beliefs about other players' strategies that rationalize their behavior generally differ from equilibrium beliefs. Instead they use step-by-step procedures of a kind that are empirically plausible in communication games, which generically determine unique, pure strategies but avoid simultaneous determination of the kind used to define equilibrium. In the words of Reinhard Selten (1998, p. 433):

> Basic concepts in game theory are often circular in the sense that they are based on definitions by implicit properties ... . Boundedly, [*sic*] rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found. Each step of the procedure is simple, even if many case distinctions by simple criteria may have to be made.

Because *Mortals'* strategies are determined independently of each other's and *Sophisticated* players' strategies, they can be treated as exogenous, and the analysis can focus on a *reduced game* between the possible *Sophisticated* players in each role.[12]

In the reduced game, *Sophisticated* players' strategy choices weigh the responses of *Sophisticated* opponents against those of *Mortal* opponents. The possibility of *Mortal* opponents fundamentally alters the game from *Sophisticated* players' point of view. The reduced game is no longer zero-sum, because a *Mortal* opponent's payoff may differ from a *Sophisticated* opponent's. Its messages are no longer cheap talk, in that a *Sophisticated* Sender's message directly influences his expected payoff via *Mortal* Receivers' responses. Finally, the reduced game has incomplete information; and although a Sender's message is nominally about his intentions, it conveys information to a *Sophisticated* Receiver about his type, and only indirectly about his intentions. These differences allow the analysis to yield more insight into misrepresentation than the standard analysis.

Sequential equilibrium in the reduced game is determined by the prior probability distribution of *Mortal* players' behavior. The key aspects of this distribution are the probabilities that the Sender is *Mortal* and lies, or is *Mortal* and tells the truth; that the Receiver is *Mortal* and believes, or inverts, the Sender's message; and the actions chosen by each kind of *Mortal* player in the underlying game. I lump together *Mortal* Senders who lie, or tell the truth, as the types *Liars* and *Truthtellers*, and *Mortal* Receivers who believe, or invert, the Sender's message as the types *Believers* and *Inverters.*

As explained in Section I, many plausible boundedly rational Sender decision rules effectively assume that their attempts to misrepresent will succeed. They therefore respond to the payoff advantage ($a > 1$) of U against L over D against R by sending a message meant to induce the Receiver to play L (d for *Liars;* u for *Truthtellers*) and then playing U on the equilibrium path. I assume that all *Mortal* Senders behave this way, which greatly simplifies the analysis without significantly distorting its conclusions.

Given this assumption, the model allows a simple characterization of sequential equilibrium. It is shown in Section II that when the probabilities of *Sophisticated* Sender and Receiver types are relatively high, there is a generically unique or essentially unique sequential equilibrium in mixed strategies, similar to the babbling message followed by mixed-strategy equilibrium in the underlying game of the standard analysis. In such an equilibrium, *Sophisticated* Senders' and Receivers' mixed strategies fully offset each other's gains from exploiting *Mortal* players, *Sophisticated* players in each role have the same expected payoffs as their *Mortal* counterparts, and all players' expected

[12] An equilibrium analysis of the reduced game generalizes standard equilibrium analysis of the original game, in that common knowledge that all players are *Sophisticated* would make their beliefs common knowledge and therefore the same, and so in equilibrium (Aumann and Brandenburger, 1995). Similar techniques are used by Colin Camerer et al. (2002) to analyze a reduced game with sophisticated and adaptive learners, elucidating a phenomenon they call "strategic teaching."

payoffs are exactly as in the standard analysis. Thus, not only does sequential equilibrium vary continuously with the type probabilities, most of the standard analysis's conclusions are unaffected by increases in the probabilities of *Mortal* types from zero to moderate levels.

By contrast, when the probabilities of *Sophisticated* Sender and Receiver types are relatively low, there is a generically unique sequential equilibrium in pure strategies.[13] In such an equilibrium, a *Sophisticated* Receiver can perfectly predict a *Sophisticated* Sender's action, and vice versa. Thus, their communication is "disciplined," as in Farrell and Robert Gibbons (1989); but here the discipline comes from the implicit presence of *Sophisticated* players' *Mortal* alter egos rather than real other players.

In this case, interest centers on when a *Sophisticated* Sender "fools" a *Sophisticated* Receiver in equilibrium, either playing U while the Receiver plays L or playing D while the Receiver plays R. A *Sophisticated* Sender has the *ability* to fool a *Sophisticated* Receiver whenever the prior probability of one *Mortal* Sender type is high enough, by pooling with that type's message. When the probability of a *Sophisticated* Sender is low enough and the probability of a *Believer* is in an intermediate range, there is a "Fortitude" sequential equilibrium in which a *Sophisticated* Sender sends u but plays D—feinting at Calais and attacking Normandy—and both a *Sophisticated* Receiver and a *Believer* play R—defending Calais.[14] However, assuming that *Mortal* Senders play U on the equilibrium path, in any pure-strategy sequential equilibrium a *Sophisticated* Receiver plays R following either message. Thus, there are no "reverse-Fortitude" equilibria in which a *Sophisticated* Receiver defends Normandy but a *Sophisticated* Sender attacks Calais.[15]

This explanation of Operation Fortitude works if both Allies and Germans are *Sophisticated*, and so does not require postulating an unexplained difference in behavior. It is also less subtle, and perhaps therefore more credible, than HM's explanation: *Sophisticated* Allied planners (or *Mortal* planners who make a point of lying to Germans) conceal their preparations for invading Normandy and fake preparations for invading Calais, knowing that the cost of faking is low; that the Germans may be the type of *Mortal* who can be fooled this way; and that even *Sophisticated* Germans prefer to defend Calais. *Mortal* Germans who believe the Allies' messages are fooled because they are too literal-minded (or perhaps too clever) to see through the deception.[16] *Sophisticated* Germans see the possibility of deception, but still prefer to defend Calais because they think the Allies are probably *Mortal*, and they prefer *ex ante* to be "fooled" at Normandy by *Sophisticated* Allies over being "fooled" at Calais by both types of *Mortal* Allies.

Sophisticated players' sequential equilibrium strategies, pure or mixed, depend only on the payoffs and parameters that reflect simple, portable facts about behavior that could be learned in many games, even if imperfectly analogous to the present one. This is important for applications, particularly those as unprecedented as Operation Fortitude. And in the model's pure-strategy sequential equilibria, *Sophisticated* players' strategies are their unique extensive-form rationalizable strategies, identifiable by at most three steps of iterated conditional dominance (Makoto Shimoji and Joel Watson, 1998).

With regard to welfare, *Sophisticated* players in either role do at least as well in equilibrium as their *Mortal* counterparts, by definition. In the

[13] In signaling games, sequential equilibria are usually both essentially nonunique and, in cheap-talk games, inessentially nonunique due to the ambiguity of costless messages (Crawford and Sobel, 1982). The present analysis avoids this ambiguity by assuming that *Mortal* Receivers react to the literal meanings of messages, and other players know this. It avoids essential nonuniqueness because *Mortal* Senders' behavior ensures that both messages have positive probability, and Senders' and Receivers' interests are opposed.

[14] The probability of a *Believer* must be in an intermediate range because if the probability of *Inverters* is too high, a *Sophisticated* Sender will prefer to fool them rather than both *Believers* and *Sophisticated* Receivers. If, instead, the probability of an *Inverter* is in an intermediate range, there is an equivalent sequential equilibrium in which the roles of the messages are interchanged but everything else is the same.

[15] For other parameter values, there are mixed-strategy equilibria in which this has positive probability.

[16] As will be clear in Section I, *Mortal* Germans are fooled if they believe they are an even number of steps ahead of the Sender, in the hierarchy of iterated-best-response types. Other types of *Mortal* German are not fooled because they believe they are an odd number of steps ahead; but *Sophisticated* Allied planners know that (in this case) such types are less likely than *Mortal* Germans who will be fooled by feinting at Calais.

mixed-strategy sequential equilibria that arise when the probabilities of a *Sophisticated* Sender and Receiver are both relatively high, in each role *Sophisticated* and *Mortal* players have identical expected payoffs: Perhaps surprisingly, the prevalence of *Sophisticated* players fully protects *Mortal* players from exploitation. By contrast, in pure-strategy sequential equilibria (and in the other mixed-strategy sequential equilibria), *Sophisticated* players in either role have strictly higher payoffs than their *Mortal* counterparts. Their advantage in such equilibria comes from their ability to avoid being fooled (except by choice, when it is the lesser of two evils) and their ability to choose which type(s) of opponent to fool.

These results suggest that an adaptive analysis of the dynamics of the type distribution, in the style of Conlisk (2001), would show that *Sophisticated* and *Mortal* players can coexist in long-run equilibrium whether or not *Sophisticated* players have higher costs, justifying the assumptions about the type probabilities maintained here.

The rest of the paper is organized as follows. Section I discusses the behavior of *Mortal* players and the details of constructing the reduced game. Section II characterizes the model's sequential equilibria, showing how they depend on the payoffs and the type distribution. Section III compares *Mortal* and *Sophisticated* Sender and Receiver types' equilibrium welfares and briefly discusses an adaptive model of the evolution of the type distribution. Section IV discusses related work, and Section V is the conclusion.

I. The Model

In this section, I motivate the assumptions about the behavior of *Mortal* players and discuss the details of constructing the reduced game between *Sophisticated* players.

A Sender's feasible pure strategies are (message, action|sent u, action|sent d) = (u, U, U), (u, U, D), (u, D, U), (u, D, D), (d, U, U), (d, U, D), (d, D, U), or (d, D, D). A Receiver's pure strategies are (action|received u, action|received d) = (L, L), (L, R), (R, L), or (R, R). Table 1 lists some plausible boundedly rational decision rules for *Mortal* Senders and Receivers.[17]

TABLE 1—PLAUSIBLE BOUNDEDLY RATIONAL DECISION RULES FOR SENDERS AND RECEIVERS

Sender rule	Behavior (b.r. ≡ best response)	message, action|sent u, action|sent d
Credible ≡ *W0*	tells the truth	u, U, D
W1 (*Wily*)	lies (b.r. to *S0*)	d, D, U
W2	tells truth (b.r. to *S1*)	u, U, D
W3	lies (b.r. to *S2*)	d, D, U

Receiver rule	Behavior	action|received u, action|received d
Credulous ≡ *S0*	believes (b.r. to *W0*)	R, L
S1 (*Skeptical*)	inverts (b.r. to *W1*)	L, R
S2	believes (b.r. to *W2*)	R, L
S3	inverts (b.r. to *W3*)	L, R

Such rules find strong empirical support in experiments with communication games (Andreas Blume et al., 2001), and closely resemble rules that find strong support in other game experiments (Dale Stahl and Paul Wilson, 1995; Costa-Gomes et al., 2001). They also play a prominent role in the classical literature on deception, from Sun Tzu, who advocates *W1* in *The Art of War* (c. 400–320 B.C.; quoted in Donald Daniel and Katherine Herbig, 1981), continuing through Erving Goffman (1969), who uses *W0* and *S0* as the foundation of a richer taxonomy of decision rules.[18]

Note that, with the exception of *S0*, the Receiver decision rules can all be viewed as outguessing simple models of the Sender, and so effectively assume that the Sender's attempts to

[17] I assume for convenience that *Credible* Senders play u, U, D rather than d, U, D, even though both strategies are truthful and both yield the Sender the same payoff, 0, if his message is always believed. *Credible* Senders could be given a strict preference for u, U, D by slightly perturbing the underlying game's payoffs.

[18] Lowell Thomas (1924) recounts an intriguing example of a feint by General Edmund Allenby and Colonel T. E. Lawrence in which types depend on positions in social networks: "... Lawrence also started the rumour through the Arab army that Emir Feisal's host intended to launch its main attack against Deraa railway junction between Amman and Damascus. 'As a matter of fact,' Lawrence remarked, 'we had every intention of attacking Deraa, but we spread the rumour so far and wide that the Turks refused to believe it. Then in deadly secrecy we confided to a chosen few in the inner circle that we really were going to concentrate all of our forces against Amman. But we were not.' This 'secret,' of course, leaked out and was betrayed to the Turks, who immediately shifted the greater part of their forces to the vicinity of Amman, exactly as Allenby and Lawrence had planned."

misrepresent will fail. Similarly, except for *W0*, the Sender decision rules can all be viewed as outguessing simple models of the Receiver, and so effectively assume that their attempts to misrepresent will succeed. As a result, the Sender rules all respond to the payoff advantage ($a > 1$) of U against L over D against R by sending a message meant to induce the Receiver to play L, and then playing U on the equilibrium path. Both of these features are typical of simple step-by-step procedures, which tend to be too simple to reflect the danger of playing U in the perturbed Matching Pennies game, in which equilibrium inherently involves "implicit properties" (Selten, 1998, p. 433). This is not inevitable, but it is a challenge to find simple, uncontrived rules that avoid the trap.

In the analysis, I stylize this tendency by assuming that all *Mortal* Senders expect their attempts to misrepresent to succeed, and therefore send a message meant to induce the Receiver to play L, and then play U on the equilibrium path.[19] *Liars* therefore send message d and play U on the equilibrium path, but would play D if they sent message u. They include all of Table 1's *Wily* Senders, Wj, with j odd, and any other *Mortal* Senders who always lie. *Truthtellers* send message u and play U on the equilibrium path, but would play D if they sent message d. They include all *Wily* Senders, Wj, with j even (including *W0, Credible*, as an honorary *Wily* type), and other *Mortal* Senders who tell the truth. Similarly, *Inverters* play L (R) following message u (d) and *Believers* play R (L) following message u (d). *Inverters* include all *Skeptical* Receivers, Sk, with k odd, and other *Mortal* Receivers who invert the Sender's message; and *Believers* include all *Skeptical* Receivers, Sk, with k even (including *S0, Credulous*, as an honorary *Skeptical* type) and other *Mortal* Receivers who believe the Sender's message.[20]

The behavior of a Sender population can be summarized by $s_l \equiv \Pr\{\text{Sender is a } Liar\}$, $s_t \equiv \Pr\{\text{Sender is a } Truthteller\}$, and $s_s \equiv \Pr\{\text{Sender is } Sophisticated\}$, where $s_l + s_t + s_s = 1$; and that of a Receiver population by $r_i \equiv \Pr\{\text{Receiver is an } Inverter\}$, $r_b \equiv \Pr\{\text{Receiver is a } Believer\}$, and $r_s \equiv \Pr\{\text{Receiver is } Sophisticated\}$, where $r_i + r_b + r_s = 1$. To avoid trivialities, I assume that these type probabilities are all strictly positive in both populations. I also ignore nongeneric parameter configurations, and all if and only if (henceforth, iff) statements should be interpreted as generic statements.

Because *Liars* always send message d and *Truthtellers* always send message u on the equilibrium path, both messages have positive prior probability. Further, because *Inverters* and *Believers* always choose different actions for a given message, a *Sophisticated* Sender is always pooled with exactly one *Mortal* Sender type.

After receiving a message for which a *Sophisticated* Sender's strategy specifies playing U with probability 1, a *Sophisticated* Receiver's best response is R (because all *Mortal* Senders also play U). Otherwise, his best response may depend on his posterior probability or *belief*, z, that the Sender is *Sophisticated.* If x is the message and y is a *Sophisticated* Sender's probability of sending message u, a *Sophisticated* Receiver's belief is $z \equiv f(x, y)$, where $f(u, y) \equiv ys_s/(s_t + ys_s)$ and $f(d, y) \equiv (1 - y)s_s/[(1 - y)s_s + s_l]$, by Bayes' Rule.

Figure 2 gives the payoff matrix of the reduced game between a *Sophisticated* Sender and Receiver, using these observations to derive *Sophisticated* players' expected payoffs. (Greek capital letters identify strategy combinations that are pure-strategy equilibria of the reduced game, sequential or not, for some parameter configurations.) If, for example, a *Sophisticated* Sender's strategy is u, U, D and a *Sophisticated* Receiver's strategy is R, L, the former plays U and the latter plays R when he receives message u. Thus, all Sender types play U, *Inverters* play L, *Believers* and *Sophisticated* Receivers play R, a *Sophisticated* Sender's expected payoff is ar_i, and a *Sophisticated* Receiver's is 0. If, instead, a *Sophisticated* Sender's strategy is u, D, U and a *Sophisticated* Receiver's strategy is L, R, the former plays D and the latter plays L when he receives message u. All other Sender types play U, *Inverters* play L, and *Believers*

[19] The continuity of equilibria implies that the analysis is robust to small deviations from this assumption.

[20] That such types can be lumped together in this way illustrates a kind of paradox of bounded strategic rationality, in that, with a finite number of possibilities for guessing and outguessing, it is as bad to be too much wilier, or more skeptical, than one's opponent as to be too much less wily, or skeptical. By contrast, in Conlisk's (2001) model *Tricksters* always find a way to outwit *Suckers*, just as Puck does with mortals.

Sender \ Receiver	L,L	L,R	R,L	R,R
u,U,U	$a(r_i+r_s), -a$	$a(r_i+r_s), -a$	$ar_i, 0$ **A**	$ar_i, 0$ **B**
u,U,D	$a(r_i+r_s), -a$	$a(r_i+r_s), -a$	$ar_i, 0$ **A′**	$ar_i, 0$ **B′**
u,D,U	$r_b, -as_t/(s_s+s_t)$	$r_b, -as_t/(s_s+s_t)$	$(r_b+r_s), -s_s/(s_s+s_t)$	$(r_b+r_s), -s_s/(s_s+s_t)$ **Γ**
u,D,D	$r_b, -as_t/(s_s+s_t)$	$r_b, -as_t/(s_s+s_t)$	$(r_b+r_s), -s_s/(s_s+s_t)$	$(r_b+r_s), -s_s/(s_s+s_t)$ **Γ**
d,U,U	$a(r_b+r_s), -a$	$ar_b, 0$ **Δ**	$a(r_b+r_s), -a$	$ar_b, 0$ **E**
d,U,D	$r_i, -as_l/(s_s+s_l)$	$(r_i+r_s), -s_s/(s_s+s_l)$	$r_i, -as_l/(s_s+s_l)$	$(r_i+r_s), -s_s/(s_s+s_l)$ **Z**
d,D,U	$a(r_b+r_s), -a$	$ar_b, 0$ **Δ′**	$a(r_b+r_s), -a$	$ar_b, 0$ **E′**
d,D,D	$r_i, -as_l/(s_s+s_l)$	$(r_i+r_s), -s_s/(s_s+s_l)$	$r_i, -as_l/(s_s+s_l)$	$(r_i+r_s), -s_s/(s_s+s_l)$ **Z′**

FIGURE 2. PAYOFF MATRIX OF THE REDUCED GAME BETWEEN A *SOPHISTICATED* SENDER AND RECEIVER

Sender \ Receiver	L	R
U	$a(r_i+r_s)$; $-a$	ar_i ; 0
D	r_b ; $-a(1-z)$	(r_b+r_s) ; $-z$

FIGURE 3A. "u" GAME FOLLOWING MESSAGE u

Sender \ Receiver	L	R
U	$a(r_b+r_s)$; $-a$	ar_b ; 0
D	r_i ; $-a(1-z)$	(r_i+r_s) ; $-z$

FIGURE 3B. "d" GAME FOLLOWING MESSAGE d

play R. A *Sophisticated* Sender's expected payoff is r_b; and a *Sophisticated* Receiver's, whose posterior belief that the Sender is *Mortal* is $1 - s_s/(s_t + s_s) \equiv s_t/(s_t + s_s)$, is $-as_t/(s_t + s_s)$.

Figure 3 (Panels A and B) gives the payoff matrices of the reduced "u" and "d" games following messages u and d, as determined by a *Sophisticated* Receiver's belief, $z \equiv f(x, y)$.

Because messages have no direct costs, the only difference between type populations in which the frequencies of *Mortal* Senders and Receivers are interchanged is in which message fools which type. Figure 2 reflects this symmetry, in that simultaneous permutations of the probabilities of *Liars* and *Truthtellers,* and of *Believers* and *Inverters,* yield an equivalent game. Figure 3's u and d games are identical except for interchanged roles of r_i and r_b, because they differ only in whether *Inverters* or *Believers* are fooled.

II. Analysis

In this section I characterize the sequential equilibria of the reduced game, as functions of the payoff a and the type probabilities.[21] Table 2 and Figure 4 give the sequential equilibria for the various possible parameter configurations.

Proposition 1, proved in the Appendix, is the basic characterization result:

PROPOSITION 1: *Unless either* $r_b > r_i$, $ar_b + r_i < 1$, *and* $s_s > as_t$, *or* $r_i > r_b$, $ar_i + r_b < 1$, *and* $s_s > as_l$, *the reduced game has a generically unique sequential equilibrium in pure strategies, in which a Sophisticated Sender's and Receiver's strategies are as given in Table* 2 *and Figure* 4. *In these sequential*

[21] Sequential equilibrium combines the standard notion of sequential rationality with consistency restrictions on players' beliefs. Because both messages always have positive probability, zero-probability updating is not an issue, and any notion that captures the idea of sequential rationality would yield the same results.

TABLE 2—SEQUENTIAL EQUILIBRIA OF THE REDUCED GAME BETWEEN A *SOPHISTICATED* SENDER AND RECEIVER

(E) d, U, U; R, R iff $r_b > r_i$, $ar_b + r_i > 1$, and $r_i > 1/(1 + a)$ [true iff $r_b > r_i > 1/(1 + a)$]
(E′) d, D, U; R, R iff $r_b > r_i$, $ar_b + r_i > 1$, and $r_i < 1/(1 + a)$
(Γ) u, D, U; R, R iff $r_b > r_i$, $ar_b + r_i < 1$, $r_b > 1/(1 + a)$, and $s_s < as_t$
($\Gamma_{\rm m}$) m, D, U; R, R iff $r_b > r_i$, $ar_b + r_i < 1$, $r_b > 1/(1 + a)$, and $s_s > as_t$
(Γ′) u, D, D; R, R iff $r_b > r_i$, $ar_b + r_i < 1$, $r_b < 1/(1 + a)$, and $s_s < as_t$ [true iff $r_i < r_b < 1/(1 + a)$]
($\Gamma'_{\rm m}$) m, M_u, M_d; M_u, M_d iff $r_b > r_i$, $ar_b + r_i < 1$, $r_b < 1/(1 + a)$, and $s_s > as_t$
(B) u, U, U; R, R iff $r_i > r_b$, $ar_i + r_b > 1$, and $r_b > 1/(1 + a)$ [true iff $r_i > r_b > 1/(1 + a)$]
(B′) u, U, D; R, R iff $r_i > r_b$, $ar_i + r_b > 1$, and $r_b < 1/(1 + a)$
(Z) d, U, D; R, R iff $r_i > r_b$, $ar_i + r_b < 1$, $r_i > 1/(1 + a)$, and $s_s < as_l$
($Z_{\rm m}$) m, U, D; R, R iff $r_i > r_b$, $ar_i + r_b < 1$, $r_i > 1/(1 + a)$, and $s_s > as_l$
(Z′) d, D, D; R, R iff $r_i > r_b$, $ar_i + r_b < 1$, $r_i < 1/(1 + a)$, and $s_s < as_l$ [true iff $r_b < r_i < 1/(1 + a)$]
($Z'_{\rm m}$) m, M_u, M_d; M_u, M_d iff $r_i > r_b$, $ar_i + r_b < 1$, $r_i < 1/(1 + a)$, and $s_s > as_l$

Note: In the table m refers to a probability mixture over messages u and d, and M_u (M_d) refers to the player's part of the relevant mixed-strategy equilibrium in the u (d) game; both are described precisely in Proposition 1.

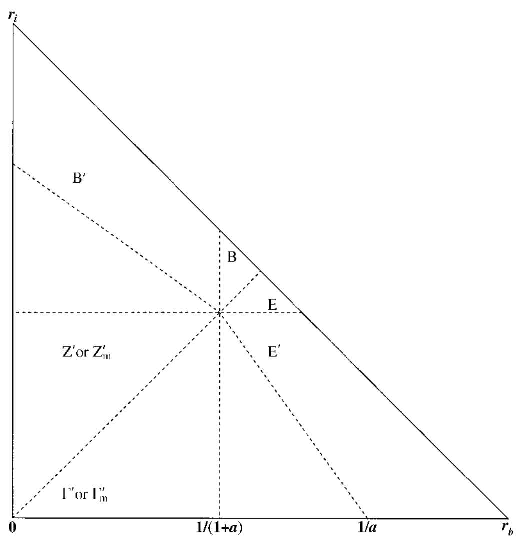

FIGURE 4. SEQUENTIAL EQUILIBRIA WHEN $a = 1.4$

Note: Subscript m denotes sequential equilibria when $s_s > as_t$ (as_l) in Γ or Γ′ (Z or Z′)

equilibria, a Sophisticated Receiver's strategy is R, R; *and a Sophisticated Sender plays* U (D) *on the equilibrium path iff* $a \max\{r_b, r_i\} + \min\{r_b, r_i\} > (<) 1$ *and sends message* d (u) *iff* $r_b > (<) r_i$. *Sophisticated players' sequential equilibrium strategies are their unique extensive-form rationalizable strategies, identifiable by at most three steps of iterated conditional dominance.*

If, instead, either (*i*) $r_b > r_i$, $ar_b + r_i < 1$, *and* $s_s > as_t$; *or* (*ii*) $r_i > r_b$, $ar_i + r_b < 1$, *and* $s_s > as_l$, *the reduced game has a generically unique or essentially unique mixed-strategy sequential equilibrium, in which a Sophisticated Sender's and Receiver's strategies are as given in Table* 2 *and Figure* 4. *In case* (*i*), *if* $r_b < 1/(1 + a)$, *there are multiple mixed-strategy sequential equilibria, in each of which a Sophisticated Sender sends message* u *with probability* y, *where* $as_t/s_s < y < 1 - as_l/s_s$. *Each of these* y *values leads to* u *and* d *games with different, unique mixed-strategy equilibria. In these equilibria a Sophisticated Sender plays* U *with probability* $1 - a/(1 + a)[ys_s/(s_t + ys_s)] = [1 - as_t/ys_s]/(1 + a)$ *in the* u *game and* $1 - a/(1 + a)[(1 - y)s_s/\{s_l + (1 - y)s_s\}] = [1 - as_l/(1 - y)s_s]/(1 + a)$ *in the* d *game; a Sophisticated Receiver plays* L *with probability* $[1 - (1 + a)r_i]/(1 + a)r_s$ *in the* u *game and* $[1 - (1 + a)r_b]/(1 + a)r_s$ *in the* d *game; a Sophisticated Sender's equilibrium expected payoff is* $a/(1 + a)$; *and a Sophisticated Receiver's equilibrium expected payoff is* $-a/(1 + a)$.[22]

In case (*i*), *if* $r_b > 1/(1 + a)$, *there is a unique mixed-strategy sequential equilibrium, in which a Sophisticated Sender sends message* u *with probability* $y = s_t/as_s$ *and plays* D *in the* u *game and* U *in the* d *game; a Sophisticated Receiver plays* R *in the* u *game and* R *in the* d *game; a Sophisticated Sender's expected payoff is* $(s_t/as_s)(r_b + r_s) + (1 - s_t/as_s)ar_b$; *and a Sophisticated Receiver's expected payoff is* $-s_t/[a(1 + a)s_s]$.

[22] Thus, there are multiple sequential equilibria, with different distributions of payoffs; but sequential equilibrium is generically essentially unique in that all sequential equilibria have the same expected payoffs, in each role, for *Sophisticated* and *Mortal* players.

In case (ii), where $r_i > r_b$, $ar_i + r_b < 1$, and $s_s > as_l$, the conclusions are the same as in case (i), but with the roles of r_i and r_b, and of s_l and s_t, reversed.

The asymmetry across actions of Proposition 1's conclusion that a *Sophisticated* Receiver's strategy is R, R in *all* pure-strategy sequential equilibria is an important part of the model's explanation of Operation Fortitude. The conclusion is trivial if the probability of a *Mortal* Sender is high enough to make R a dominant strategy in the underlying game; but, somewhat surprisingly, it remains valid even if the probability is not high enough, as long as the game has a pure-strategy sequential equilibrium. It superficially resembles the babbling equilibrium of the standard analysis, but it actually stems from the assumption that *Mortal* Senders play U.[23] In a pure-strategy sequential equilibrium, a *Sophisticated* Sender's deviation from his equilibrium message "proves" to a *Sophisticated* Receiver that the Sender is *Mortal*, making R the Receiver's best response off the equilibrium path. If a *Sophisticated* Sender plays U on the equilibrium path, the conclusion is immediate. If, instead, a *Sophisticated* Sender plays D on the equilibrium path while a *Sophisticated* Receiver plays L, the *Sophisticated* Sender's message fools only the most frequent type of *Mortal* Receiver, at a payoff gain of 1 per unit of probability. But such a *Sophisticated* Sender could reverse his message and action, again fooling the most frequent type of *Mortal* Receiver, but now at a payoff gain of $a > 1$ per unit, a contradiction.

Given that a *Sophisticated* Receiver plays R, R, the rest of Proposition 1's conclusions concerning pure-strategy equilibria are straightforward. Because a *Sophisticated* Sender cannot truly fool a *Sophisticated* Receiver in equilibrium, whichever action he chooses in the underlying game, it is always best to send the message that fools whichever type of *Mortal* Receiver, *Believer* or *Inverter*, is more likely. The only remaining choice is whether to play U or D, when, with the optimal message, the former action fools $\max\{r_b, r_i\}$ *Mortal* Receivers at a gain of a per unit and the latter fools them at a gain of 1 per unit, but also "fools" r_s *Sophisticated* Receivers. Simple algebra reduces this question to whether $a \max\{r_b, r_i\} + \min\{r_b, r_i\} > 1$ or < 1.

It is clear from Figure 4 that the model's pure-strategy sequential equilibria avoid the perverse comparative statics of equilibrium mixed strategies with respect to a in the standard analysis, noted in footnote 9. Within the region that supports a given pure-strategy equilibrium, a does not affect *Mortal* or *Sophisticated* players' strategies at all. However, as intuition suggests, increasing a always enlarges the set of type frequencies that support equilibria in which a *Sophisticated* Sender's equilibrium action is U (B, B′, E, or E′).

Proposition 1's conclusions concerning mixed-strategy equilibria in case (i) if $r_b < 1/(1 + a)$ (Γ'_m), or in case (ii) if $r_i < 1/(1 + a)$ (Z'_m), are straightforward extensions of the standard analysis to parameter configurations in which the probabilities of a *Sophisticated* Sender and Receiver are both high. But in case (i) if $r_b > 1/(1 + a)$ (Γ_m), or case (ii) if $r_i > 1/(1 + a)$ (Z_m), the model has unique mixed-strategy sequential equilibria with a different character, in which randomization is confined to a *Sophisticated* Sender's message, and serves to "punish" a *Sophisticated* Receiver for deviating from R, R in a way that relaxes the $s_s \leq as_t$ or $s_s \leq as_l$ constraint whose violation prevents a *Sophisticated* Sender from realizing the higher expected payoff of equilibrium Γ or Z. These equilibria are otherwise similar to the pure-strategy equilibria Γ or Z for adjoining parameter configurations, and converge to them as the relevant population parameters converge.

In both kinds of mixed-strategy equilibrium, players' strategies are determined by simple, portable behavioral parameters as for pure-strategy equilibria; but both share some of the delicacy of HM's equilibria, and of mixed-strategy equilibria more generally.

To assess more fully the model's ability to explain Operation Fortitude, reconsider the sequential equilibria Γ or Γ′ in which a *Sophisticated* Sender feints at Calais and attacks at Normandy, while a *Sophisticated* Receiver defends Calais. In general, the conditions for those equilibria are $r_b > r_i$, $ar_b + r_i < 1$, and $s_s <$

[23] Even the mixed-strategy equilibria identified in Proposition 1 differ from the babbling equilibria of the standard analysis. Here, the Sender's message is informative, and a *Sophisticated* Receiver's mixed strategy depends on the message, although the effect of this dependence is neutralized by his opponent's strategy.

TABLE 3—EXPECTED PAYOFFS OF *MORTAL* AND *SOPHISTICATED* SENDER AND RECEIVER TYPES ($r_b > r_i$)

Sender type	E or E′ equilibrium message, action, and payoff	Γ or Γ′ equilibrium message, action, and payoff	Γ_m equilibrium message, action(s), and payoff	Γ'_m equilibrium message, action(s), and payoff
Liar	d, U, ar_b	d, U, ar_b	d, U, ar_b	d, U, $a/(1 + a)$
Truthteller	u, U, ar_i	u, U, ar_i	u, U, ar_i	u, U, $a/(1 + a)$
Sophisticated	d, U, ar_b	u, D, $r_b + r_s$	m, D|u, U|d, $(s_t/as_s)(r_b + r_s) + (1 - s_t/as_s)ar_b$	m, M_u|u, M_d|d, $a/(1 + a)$

Receiver type	E or E′ equilibrium action|u, action|d, and payoff	Γ or Γ′ equilibrium action|u, action|d, and payoff	Γ_m equilibrium action|u, action|d, and payoff	Γ'_m equilibrium action|u, action|d, and payoff
Believer	R, L, $-a(s_l + s_s)$	R, L, $-as_l - s_s$	R, L, $-as_l - s_s[(s_t/as_s) + (1 - s_t/as_s)a] = -a(s_l + s_s) - s_t/a + s_t$	R, L, $-a/(1 + a)$
Inverter	L, R, $-as_t$	L, R, $-as_t$	L, R, $-as_t$	L, R, $-a/(1 + a)$
Sophisticated	R, R, 0	R, R, $-s_s$	R, R, $-s_s(s_t/as_s) = -s_t/a$	M_u, M_d, $-a/(1 + a)$

as_t. $r_b > r_i$ reflects a preponderance of *Believers* over *Inverters* that is plausible, so I assume it.[24] Given this, suppose $r_b = cr_i$ and $s_l = cs_t$ for some constant c. Then, Γ or Γ′ is sequential iff $r_b < c/(ac + 1)$ and $s_s < a/(1 + a + c)$. When $a = 1.4$, as in Figure 4, and $c = 3$, which seem plausible values, these conditions reduce to $r_b < 0.58$ and $s_s < 0.26$, reasonable parameter ranges.[25]

III. Welfare Analysis

This section conducts a welfare analysis of the model's sequential equilibria, comparing the expected payoffs of *Mortal* and *Sophisticated* types. The comparisons use actual rather than anticipated expected payoffs for *Mortal* types, whose beliefs may be incorrect. I focus on cases in which $r_b > r_i$; transposition yields the results when $r_i > r_b$.

Table 3 lists all types' messages, actions, and expected payoffs on the possible sequential equilibrium paths, extending Figure 2's payoff calculations and Proposition 1's characterization of equilibrium behavior from *Sophisticated* to *Mortal* players.[26] The table shows that *Sophisticated* players in either role have expected payoffs at least as high as their *Mortal* counterparts'. This much is true by definition, because *Sophisticated* players can always mimic *Mortal* players; but in pure-strategy equilibria, *Sophisticated* players have strictly higher payoffs. *Sophisticated* Senders' advantage over *Mortal* Senders in these equilibria stems from their ability to avoid being fooled and to choose which type(s) to fool. *Sophisticated* Receivers' advantage comes from their ability to avoid being fooled, or to choose the least costly way to be "fooled."

Sophisticated players enjoy a smaller advantage in the mixed-strategy sequential equilibria Γ_m or Z_m, but for similar reasons. By contrast, in the mixed-strategy sequential equilibria Γ'_m

[24] If $r_i > r_b$ the sequential equilibria Z or Z′ would duplicate the outcomes of Γ or Γ′, with inverted messages.

[25] Higher values of a make the first condition more stringent and the second less stringent. The plausibility of the Γ or Γ′ equilibria may be further enhanced by the human tendency to overrate one's own strategic sophistication relative to others'. Further, Γ_m is behaviorally similar to Γ, and so one might relax Γ's restriction on s_s, at the cost of a random prediction of a *Sophisticated* Sender's message. The closest the model can come to a reverse-Fortitude sequential equilibrium, in which a *Sophisticated* Sender attacks at Calais while a *Sophisticated* Receiver defends Normandy, is in the sequential equilibria E or E′, in which a *Sophisticated* Sender feints at Normandy and attacks Calais, fooling *Believers* but not *Sophisticated* Receivers. The conditions for those equilibria are $r_b > r_i$ and $ar_b + r_i > 1$. Again assuming $r_b > r_i$ and $r_b = cr_i$, E or E′ is sequential iff $r_b > c/(ac + 1)$. When $a = 1.4$ and $c = 3$, this condition reduces to $r_b > 0.58$, which seems less realistic than the conjunction of $r_b < 0.58$ and $s_s < 0.26$.

[26] Table 3 sometimes combines the equilibrium-path outcomes of more than one equilibrium, to save space. The listed actions may therefore differ from *Sophisticated* players' sequential equilibrium strategies.

or Z'_m, *Sophisticated* players' equilibrium mixed strategies completely offset each other's gains from fooling *Mortal* Receivers, and in each role, *Sophisticated* and *Mortal* players have the same expected payoffs.[27] Thus, in this case, the prevalence of *Sophisticated* players protects *Mortal* players from exploitation.

IV. Related Work

This section briefly discusses related work.

Sobel (1985) was the first to propose an equilibrium explanation of lying, studying an "enemy" Sender's incentives in repeated interaction to build and eventually exploit a reputation for being a "friend" of the Receiver's. His analysis focused on communication of private information in settings with asymmetric information about the Sender's preferences, as opposed to the asymmetric information about the Sender's and the Receiver's strategic thinking analyzed here. Benabou and Laroque (1992) extended Sobel's analysis to allow the Sender to have noisy private information, and used it to analyze the use of inside information to manipulate financial markets.[28]

Farrell and Gibbons' (1989) analysis of costless communication to multiple audiences has already been mentioned. Glenn Loury (1994) provides a different perspective on the issues that arise with multiple audiences.

Finally, Conlisk (2001) studied the adaptive dynamics of selection in favor of types with higher payoffs among a different set of types, *Trickster, Avoider,* and *Sucker,* taking the "fooling technology" as given (footnote 20). He showed that if those types have successively lower costs they can coexist in long-run equilibrium, proving (in a special case) P. T. Barnum's dictum, "There's a sucker born every minute, and two to take him."

Section III's welfare analysis could be used to conduct an analysis of adaptive dynamics like Conlisk's. I conjecture that unless *Sophisticated* players have higher costs, their payoff advantage in pure-strategy (and some mixed-strategy) equilibria will lead their relative frequencies to grow until the population frequencies enter the region of mixed-strategy equilibria in which all types' expected payoffs are equal (region Γ'–Z' in Figure 4). (Because *Liars* do better than *Truthtellers* when there are more *Believers* than *Inverters,* and *Believers* do better than *Inverters* when there are more *Truthtellers* than *Liars,* there is also a tendency for the dynamics to approach the diagonal in Figure 4.) The population can then be expected to drift among a continuum of neutral steady states in the Γ'–Z' region.[29] If *Sophisticated* players have slightly higher costs, the population frequencies should approach and remain near the boundary of the Γ'–Z' region, without entering it.[30] This would also allow *Sophisticated* and *Mortal* players to coexist in long-run equilibrium, justifying the assumptions about the type frequencies maintained here.

V. Conclusion

In this paper, I have proposed a way to model active misrepresentation of intentions to competitors or enemies. The model focuses on the strategic interaction between rational and boundedly rational types of players in a game of conflicting interests, with one-sided preplay communication via costless messages.

Allowing for the possibility of bounded rationality yields a sensible account of lying via costless, noiseless messages, and simplifies many aspects of the analysis of games with communication. For many parameter configurations, in contrast to a standard analysis of communication with conflicting interests, the model has generically unique pure-strategy sequential equilibria, which can be identified by iterated elimination of conditionally dominated strategies. In these equilibria rational players exploit boundedly rational players, but are not

[27] Here, *Truthtellers'* and *Liars'* strategies are both in the support of a *Sophisticated* Sender's mixed strategy.

[28] See also John Morgan and Philip Stocken's (2003) analysis of financial analysts' incentives to reveal private information to investors, and some of the references cited there.

[29] I am grateful to Kang-Oh Yi for this observation.

[30] This conclusion is not immediate because the present model has two player populations and a more complex pattern of payoff advantages than Conlisk's model. There, taking cost differences into account, in pairwise interactions *Tricksters* do better than *Suckers, Suckers* better than *Avoiders,* and *Avoiders* better than *Tricksters.* Here, with equal costs for *Mortals* and higher costs for *Sophisticated* players, *Sophisticated* players may do better or worse than their *Mortal* counterparts, depending on the parameters and the costs.

themselves fooled. This part of the analysis suggests an explanation of the Allies' decision to feint at Calais and attack at Normandy on D-Day, and of why the Allies did not instead feint at Normandy and attack at Calais.

For other parameter configurations, the model has generically unique or essentially unique mixed-strategy sequential equilibria, in which rational players' equilibrium strategies offset each other's gains from fooling boundedly rational players, completely protecting them from exploitation. Those equilibria share some of the delicacy of mixed-strategy equilibria in other games.

In all of the model's equilibria, players' strategies are determined by simple, portable behavioral parameters. Thus the analysis reduce strategic questions like those that underlie Operation Fortitude to empirical questions about behavioral parameters, which can be discussed using evidence from imperfectly analogous settings.[31] I hope that the methods for modeling bounded strategic rationality presented here can elucidate strategic communication when players' interests are not in conflict, and can also be used to create behaviorally realistic models of strategic behavior in other applications.

Appendix

PROOF OF PROPOSITION 1:

I begin by characterizing the equilibria of the u and d games (Figure 3), as determined by a *Sophisticated* Receiver's belief, z, that the Sender is *Sophisticated.*

LEMMA 1: *The* u *game has a generically unique equilibrium as follows:*

(*i*) U, R *is a pure-strategy equilibrium iff* $r_i > 1/(1 + a)$;
(*ii*) D, L *is a pure-strategy equilibrium iff* $r_b > a/(1 + a)$ *and* $z > a/(1 + a)$;
(*iii*) D, R *is a pure-strategy equilibrium iff* $r_i < 1/(1 + a)$ *and* $z < a/(1 + a)$; *and*
(*iv*) *there is a mixed-strategy equilibrium, with* Pr{*Sophisticated Sender plays* U} $= 1 - a/(1 + a)z$, Pr{*Sophisticated Receiver plays* L} $= [1 - (1 + a)r_i]/(1 + a)r_s$, *Sophisticated Sender's expected payoff* $a/(1 + a)$, *and Sophisticated Receiver's expected payoff* $-a/(1 + a)$, *iff* $r_i < 1/(1 + a)$, $r_b < a/(1 + a)$ *and* $z > a/(1 + a)$.

The d *game has a generically unique equilibrium as follows:*[32]

(*i*) U, R *is a pure-strategy equilibrium iff* $r_b > 1/(1 + a)$;
(*ii*) D, L *is a pure-strategy equilibrium iff* $r_i > a/(1 + a)$ *and* $z > a/(1 + a)$;
(*iii*) D, R *is a pure-strategy equilibrium iff* $r_b < 1/(1 + a)$ *and* $z < a/(1 + a)$; *and*
(*iv*) *there is a mixed-strategy equilibrium, with* Pr{*Sophisticated Sender plays* U} $= 1 - a/(1 + a)z$, Pr{*Sophisticated Receiver plays* L} $= [1 - (1 + a)r_b]/(1 + a)r_s$, *Sophisticated Sender's expected payoff* $a/(1 + a)$, *and Sophisticated Receiver's expected payoff* $-a/(1 + a)$, *iff* $r_b < 1/(1 + a)$, $r_i < a/(1 + a)$ *and* $z > a/(1 + a)$.

PROOF:

Straightforward calculations, noting that (U, L) is never an equilibrium, and, because $r_i > 1/(1 + a)$ and $r_b > a/(1 + a)$ or vice versa are inconsistent, the conditions for (i)–(iv) are mutually exclusive and (with nongeneric exceptions) collectively exhaustive.

Lemmas 2–3, which correspond to the pure- and mixed-strategy cases considered in Proposition 1, characterize the sequential equilibria of the reduced game.

[31] Even field experiments are possible. Before the battle of Midway on June 3–6, 1942, the Americans had broken the Japanese Naval code, and had taken care that the Japanese did not learn this. But the Japanese had coded symbols for locations whose meanings were not yet all known. The Americans thought "AF" was Midway Island, but they weren't sure. So they sent out a radio signal, in clear, that Midway was short of water. Sure enough, the next day the Japanese were telling each other, in code, that AF was short of water, and the Americans then felt sure they knew what AF meant (Gordon Prange, 1982). One can of course imagine Japanese sophisticated enough to use this opportunity to fool the Americans about the meaning of AF, which would have helped them far more than knowing which island was short of water. But the Americans seemed sure that the Japanese were *Mortal*, and they were right—or at least, the Japanese were sure their code could not be broken, and this error was enough to make them effectively *Mortal.*

[32] The characterization here is identical to that for the u game, with the roles of r_b and r_i interchanged.

LEMMA 2: *Unless either* $r_b > r_i$, $ar_b + r_i < 1$, *and* $s_s > as_t$, *or* $r_i > r_b$, $ar_i + r_b < 1$, *and* $s_s > as_l$, *the reduced game has a generically unique sequential equilibrium in pure strategies, in which a Sophisticated Sender's and Receiver's strategies are as given in Table* 2 *and Figure* 4. *In these sequential equilibria, a Sophisticated Receiver's strategy is* R, R; *and a Sophisticated Sender plays* U (D) *on the equilibrium path iff a* $\max\{r_b, r_i\} + \min\{r_b, r_i\} > (<) 1$ *and sends message* d (u) *iff* $r_b > (<) r_i$. *Sophisticated players' sequential equilibrium strategies are their unique extensive-form rationalizable strategies, identifiable by at most three steps of iterated conditional dominance.*

PROOF:

Because all types have positive prior probability and *Liars* and *Truthtellers* send different messages, all messages have positive probability in equilibrium. Further, in any pure-strategy sequential equilibrium, a *Sophisticated* Sender's message is pooled with either *Liars'* or *Truthtellers'* message, so a deviation to the other message makes $z = 0$. In the u or d game that follows such a deviation, R is a conditionally dominant strategy for a *Sophisticated* Receiver; and a *Sophisticated* Sender's unique best response is U (D) iff $r_i > (<) 1/(1 + a)$ in the u game and U (D) iff $r_b > (<) 1/(1 + a)$ in the d game by Lemma 1.

All that remains is to identify the strategy combinations in Figure 2 that are equilibria for some parameter configurations, use these conditions to check which configurations make them sequential, and check the other conclusions of the lemma.

Identifying the configurations by the Greek capital letters in Figure 2, Δ and Δ' are equilibria iff $r_b > ½$. For Δ to be sequential, U, L must be an equilibrium in the u game when $z = 0$, which is never true. For Δ' to be sequential, D, L must be an equilibrium in the u game when $z = 0$, which is never true. Thus neither Δ nor Δ' is ever sequential. Similarly, A and A′ are equilibria iff $r_i > ½$, but neither A nor A′ is ever sequential.

E and E′ are equilibria iff $r_b > r_i$ and $ar_b > r_b + r_s$, which reduces to $ar_b + r_i > 1$. For E to be sequential, U, R must be an equilibrium in the u game when $z = 0$, which is true iff $r_i > 1/(1 + a)$. Thus E is sequential iff $r_b > r_i$, $ar_b + r_i > 1$, and $r_i > 1/(1 + a)$, where the second condition is implied by the first and third. For E′ to be sequential, D, R must be an equilibrium in the u game when $z = 0$, which is true iff $r_i < 1/(1 + a)$. Thus E′ is sequential iff $r_b > r_i$, $ar_b + r_i > 1$, and $r_i < 1/(1 + a)$. Similarly, B and B′ are equilibria iff $r_i > r_b$ and $ar_i + r_b > 1$; B is sequential iff $r_i > r_b$, $ar_i + r_b > 1$, and $r_b > 1/(1 + a)$, where the second condition is implied by the first and third; and B′ is sequential iff $r_i > r_b$, $ar_i + r_b > 1$, and $r_b < 1/(1 + a)$.

Γ and Γ' are equilibria iff $s_s < as_t$, $r_b > r_i$, and $r_b + r_s > ar_b$, which reduces to $ar_b + r_i < 1$. For Γ to be sequential, U, R must be an equilibrium in the d game when $z = 0$, which is true iff $r_b > 1/(1 + a)$. Thus Γ is sequential iff $s_s < as_t$, $r_b > r_i$, $ar_b + r_i < 1$, and $r_b > 1/(1 + a)$. For Γ' to be sequential, D, R must be an equilibrium in the d game when $z = 0$, which is true iff $r_b < 1/(1 + a)$. Thus Γ' is sequential iff $s_s < as_t$, $r_b > r_i$, $ar_b + r_i < 1$, and $r_b < 1/(1 + a)$, where the second condition is implied by the first and third. Similarly, Z and Z′ are equilibria iff $s_s < as_l$, $r_i > r_b$, and $ar_i + r_b < 1$; Z is sequential iff $s_s < as_l$, $r_i > r_b$, $ar_i + r_b < 1$, where the second condition is implied by the first and third; and $r_i > 1/(1 + a)$. and Z′ is sequential iff $s_s < as_l$, $r_i > r_b$, $ar_i + r_b < 1$, and $r_i < 1/(1 + a)$.

In each case, the generic uniqueness of *Sophisticated* players' best responses can be verified by iterated conditional dominance, starting with the pure-strategy equilibria in the 2×2 u and d games. The remaining conclusions are easily verified by inspection.

LEMMA 3: *If either* (*i*) $r_b > r_i$, $ar_b + r_i < 1$, *and* $s_s > as_t$; *or* (*ii*) $r_i > r_b$, $ar_i + r_b < 1$, *and* $s_s > as_l$, *the reduced game has a generically unique or essentially unique mixed-strategy sequential equilibrium, in which a Sophisticated Sender's and Receiver's strategies are as given in Table* 2 *and Figure* 4. *In case* (*i*), *if* $r_b < 1/(1 + a)$, *there are multiple mixed-strategy sequential equilibria, in each of which a Sophisticated Sender sends message* u *with probability y, where* $as_t/s_s < y < 1 - as_l/s_s$. *Each of these y values leads to* u *and* d *games with different, unique mixed-strategy equilibria. In these equilibria a Sophisticated Sender plays* U *with probability* $1 - a/(1 + a)[ys_s/(s_t + ys_s)] = [1 - as_t/ys_s]/(1 + a)$ *in the* u *game and* $1 - a/(1 + a)[(1 - y)s_s/$

$\{s_l + (1 - y)s_s\}] = [1 - as_l/(1 - y)s_s]/(1 + a)$ *in the* d *game; a Sophisticated Receiver plays* L *with probability* $[1 - (1 + a)r_i]/(1 + a)r_s$ *in the* u *game and* $[1 - (1 + a)r_b]/(1 + a)r_s$ *in the* d *game; a Sophisticated Sender's equilibrium expected payoff is* $a/(1 + a)$; *and a Sophisticated Receiver's equilibrium expected payoff is* $-a/(1 + a)$.

In case (*i*), *if* $r_b > 1/(1 + a)$, *there is a unique mixed-strategy sequential equilibrium, in which a Sophisticated Sender sends message* u *with probability* $y = s_t/as_s$ *and plays* D *in the* u *game and* U *in the* d *game; a Sophisticated Receiver plays* R *in the* u *game and the* d *game; a Sophisticated Sender's expected payoff is* $(s_t/as_s)(r_b + r_s) + (1 - s_t/as_s)ar_b$, *and a Sophisticated Receiver's expected payoff is* $-s_t/[a(1 + a)s_s]$.

In case (*ii*), *where* $r_i > r_b$, $ar_i + r_b < 1$, *and* $s_s > as_l$, *the conclusions are the same as in case* (*i*), *but with the roles of* r_i *and* r_b, *and of* s_l *and* s_t, *reversed.*

PROOF:

In case (i), if $r_b < 1/(1 + a)$, and if $z < a/(1 + a)$ in either the u or the d game, D, R would be its unique equilibrium. But then, in this case, a *Sophisticated* Sender would prefer to send the message that led to that game with probability 1, and with $z = 0$, players would also have pure best responses in the other game by Lemma 1. But the proof of Lemma 2 shows that there are no pure-strategy sequential equilibria in this case when $s_s > as_t$. If, instead, $z > a/(1 + a)$ in each game, in this case the u and d games have unique mixed-strategy equilibria as characterized in Lemma 1. $ys_s/(s_t + ys_s) > a/(1 + a)$ and $(1 - y)s_s/[s_l + (1 - y)s_s] > a/(1 + a)$ provided that $as_t/s_s < y < 1 - as_l/s_s$, which is always feasible when $s_s > as_t$. Because a *Sophisticated* Sender's expected payoff is $a/(1 + a)$ in either the u or the d game, he is willing to randomize with any such y. The rest of the proof in this case is a straightforward translation of the conclusions of Lemma 1.

In case (i), if $r_b > 1/(1 + a)$, the d game always has a unique pure-strategy equilibrium U, R, with expected payoff ar_b for a *Sophisticated* Sender. If $y = 0$, the u game would be off the equilibrium path, so message u would make $z = 0$, and the u game would have a unique pure-strategy equilibrium D, R, with payoff $r_b + r_s > ar_b$ for a *Sophisticated* Sender. Thus there cannot be an equilibrium in this case with $y = 0$. Similarly, if $y = 1$, iff $r_b < a/(1 + a)$ the u game has a unique mixed-strategy equilibrium, with $z = s_s/(s_t + s_s) > a/(1 + a)$ and expected payoff $a/(1 + a) < ar_b$ for a *Sophisticated* Sender. If $y = 1$ and $r_b > a/(1 + a)$, the u game has a unique pure-strategy equilibrium, D, L, and expected payoff $r_b < ar_b$ for a *Sophisticated* Sender. Thus there cannot be an equilibrium with $y = 1$. Because $r_b + r_s > ar_b > a/(1 + a)$ in this case, a *Sophisticated* Sender's optimal choice of y maximizes $y(r_b + r_s) + (1 - y)ar_b$ subject to the constraint that D, R is an equilibrium in the u game, which is true in this case iff $z = ys_s/(s_t + ys_s) \leq a/(1 + a)$, or equivalently $y \leq s_t/as_s$. Thus, a *Sophisticated* Sender's optimal message strategy is $y = s_t/as_s$.[33] The rest of the proof follows directly from Lemma 1, noting that a *Sophisticated* Receiver's expected payoff is $-(s_t/as_s)[(s_t/as_s)s_s]/[s_t + (s_t/as_s)s_s] = -s_t/[a(1 + a)s_s]$.

Lemma 3 completes the proof of Proposition 1.

REFERENCES

Abreu, Dilip and Sethi, Rajiv. "Evolutionary Stability in a Reputational Model of Bargaining." Unpublished manuscript, Princeton University, 2001.

Aumann, Robert and Brandenburger, Adam. "Epistemic Conditions for Nash Equilibrium." *Econometrica*, September 1995, *63*(5), pp. 1161–80.

Benabou, Roland and Laroque, Guy. "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility." *Quarterly Journal of Economics*, August 1992, *107*(3), pp. 921–58.

Blume, Andreas; DeJong, Douglas; Kim, Yong-Gwan and Sprinkle, Geoffrey. "Evolution of Communication with Partial Common Interest." *Games and Economic Behavior*, October 2001, *37*(1), pp. 79–120.

Camerer, Colin; Ho, Teck-Hua and Chong, Juin-Kuan. "Sophisticated Experience-Weighted Attraction Learning and Strategic Teaching

[33] A *Sophisticated* Sender's best response is not $y = 1$ in this case, because in the reduced game *Mortal* players' responses are treated as part of the payoff function, which effectively constrain his choice of y.

in Repeated Games." *Journal of Economic Theory*, May 2002, *104*(1), pp. 137–88.

Conlisk, John. "Costly Predation and the Distribution of Competence." *American Economic Review*, June 2001, *91*(3), pp. 475–84.

Costa-Gomes, Miguel. "A Suggested Interpretation of Some Experimental Results on Preplay Communication." *Journal of Economic Theory*, May 2002, *104*(1), pp. 104–36.

Costa-Gomes, Miguel; Crawford, Vincent P. and Broseta, Bruno. "Cognition and Behavior in Normal-Form Games: An Experimental Study." *Econometrica*, September 2001, *69*(5), pp. 1193–235.

Crawford, Vincent P. "A Survey of Experiments on Communication via Cheap Talk." *Journal of Economic Theory*, February 1998, *78*(2), pp. 286–98.

Crawford, Vincent P. and Smallwood, Dennis E. "Comparative Statics of Mixed-Strategy Equilibria in Non-Cooperative Two-Person Games." *Theory and Decision*, May 1984, *61*(3), pp. 225–32.

Crawford, Vincent P. and Sobel, Joel. "Strategic Information Transmission." *Econometrica*, November 1982, *50*(6), pp. 1431–51.

Daniel, Donald and Herbig, Katherine. *Strategic military deception*. New York: Pergamon Press, 1981.

Dao, James and Schmitt, Eric. "Pentagon Readies Efforts to Sway Sentiment Abroad." *New York Times*, February 19, 2002, p. A1.

Farrell, Joseph. "Communication, Coordination and Nash Equilibrium." *Economics Letters*, 1988, *27*(3), pp. 209–14; see also "Erratum," July 1990, *33*(3), p. 299.

______. "Meaning and Credibility in Cheap-Talk Games." *Games and Economic Behavior*, October 1993, *5*(4), pp. 514–31.

Farrell, Joseph and Gibbons, Robert. "Cheap Talk with Two Audiences." *American Economic Review*, December 1989, *79*(5), pp. 1214–23.

Farrell, Joseph and Rabin, Matthew. "Cheap Talk." *Journal of Economic Perspectives*, Summer 1996, *10*(3), pp. 103–18.

Goffman, Erving. *Strategic interaction*. Philadelphia, PA: University of Pennsylvania Press, 1969.

Harrison, Gordon. *Cross-channel attack*. Washington, DC: Department of the Army, Center of Military History Publication 7-4, 1951; available online at http://www.army.mil/cmh-pg/books/wwii/7-4/7-4_cont.htm.

Hendricks, Kenneth and McAfee, R. Preston. "Feints." Unpublished manuscript, University of Texas, 2001 (revised 2002).

Kemp, Anthony. *D-day and the invasion of Normandy*. New York: Harry N. Abrams, 1994.

Kreps, David and Wilson, Robert. "Reputation and Imperfect Information." *Journal of Economic Theory*, August 1982, *27*(2), pp. 253–79.

Krugman, Paul. "Reckonings: Truth and Lies." *New York Times*, August 28, 2001, p. A15.

Loury, Glenn. "Self-Censorship in Public Discourse: A Theory of 'Political Correctness' and Related Phenomena." *Rationality and Society*, October 1994, *6*(4), pp. 428–61.

Milgrom, Paul and Roberts, John. "Predation, Reputation, and Entry Deterrence." *Journal of Economic Theory*, August 1982, *27*(2), pp. 280–312.

Morgan, John and Stocken, Phillip. "An Analysis of Stock Recommendations." *RAND Journal of Economics*, Spring 2003, *34*(1).

Prange, Gordon. *Miracle at Midway*. New York: McGraw-Hill, 1982.

Rabin, Matthew. "A Model of Pre-Game Communication." *Journal of Economic Theory*, August 1994, *63*(2), pp. 370–91.

Reny, Philip. "Backward Induction, Normal-Form Perfection and Explicable Equilibria." *Econometrica*, May 1992, *60*(3), pp. 627–49.

Royko, Mike. "Put up or Shut up; And Stop Those Lips." *Chicago Tribune*, November 3, 1988, p. 3; available online at http://weber.ucsd.edu/~vcrawfor/Royko.html.

Selten, Reinhard. "Features of Experimentally Observed Bounded Rationality." *European Economic Review*, May 1998, *42*(3–5), pp. 413–36.

Shimoji, Makoto and Watson, Joel. "Conditional Dominance, Rationalizability, and Game Forms." *Journal of Economic Theory*, December 1998, *83*(2), pp. 161–95.

Sobel, Joel. "A Theory of Credibility." *Review of Economic Studies*, October 1985, *52*(4), pp. 557–73.

Stahl, Dale and Wilson, Paul. "On Players' Models of Other Players: Theory and Experimental Evidence." *Games and Economic Behavior*, July 1995, *10*(1), pp. 218–54.

Thomas, Lowell. *With Lawrence in Arabia*. London: Hutchinson and Co., Ltd., c. 1924.

Von Neumann, John and Morgenstern, Oskar. *Theory of games and economic behavior*. New York: John Wiley and Sons, 1944 (1st Ed.); 1953 (3rd Ed.).