

NONPARAMETRIC ANALYSIS OF REFERENCE-DEPENDENT PREFERENCES*

Revised 9 July 2026

Laura Blow

Department of Economics, University of Surrey

Vincent P. Crawford

Department of Economics and All Souls College, University of Oxford

Department of Economics, University of California, San Diego

Abstract: This paper derives nonparametric conditions for reference-dependent preferences that rationalize a consumer's demand behavior. We show that unless reference points are modelable and sensitivity is constant, a reference-dependent model can rationalize virtually any data. Assuming modelable reference points, we characterize continuous reference-dependent preferences with constant sensitivity, relaxing strong functional-structure assumptions maintained in all previous theoretical and empirical analyses of reference-dependent consumer theory. Our characterization enables more general structural econometric as well as nonparametric analyses. In this paper we use our characterization to reconsider Farber's (2005, 2008) analyses of cabdrivers' labor supply nonparametrically, using his data. Our analysis suggests that relaxing the functional-structure assumptions is empirically important. Neither a relaxed neoclassical nor a relaxed reference-dependent model is unambiguously superior in the aggregate. But drivers are highly heterogeneous, and for a significant fraction of them a relaxed reference-dependent model has a higher nonparametric measure of predictive success than a relaxed neoclassical model. (*JEL* C14, C23, D11, D12, J22)

Keywords: consumer theory, labor supply, reference-dependent preferences, revealed preference, nonparametric demand analysis, loss aversion

*Laura Blow: School of Economics, Elizabeth Fry Building (AD), University of Surrey, Guildford, Surrey, GU2 7XH, United Kingdom (email: l.blow@surrey.ac.uk); Vincent P. Crawford: All Souls College and Department of Economics, University of Oxford, Oxford OX1 4AL, United Kingdom, and Department of Economics, University of California, San Diego, 9500 Gilman Drive #0508, La Jolla, CA 92093-0508, USA (email: v2crawford@ucsd.edu). This paper is an extensively revised version of Blow, I. Crawford, and V. P. Crawford (2022) and of Blow and V. P. Crawford (2024). Our reduced authorship is by agreement with Ian Crawford, whose contributions to the current version were nonetheless extremely valuable.

1 Introduction

Kahneman and Tversky (1979) and Tversky and Kahneman (1991; “TK”) introduce a model of individual decisions in which people have preferences over gains and losses relative to a reference point. Such reference-dependence alters the domain of preferences from levels of outcomes to changes in outcomes; but it remains consistent with a complete and transitive preference ordering over changes, thus not inherently irrational. Although Kahneman and Tversky and TK focus on changes alone, Kőszegi and Rabin (2006; “KR”) and most subsequent analyses allow preferences over both levels and changes; and such reference-dependence is also not inherently irrational.

Reference-dependent consumer theory has been a workhorse model in behavioral economics since Camerer et al.’s (1997) analysis of New York City cabdrivers’ labor supply.¹ A standard neoclassical model of labor supply is analogous to a two-good model of consumer demand, with preferences over levels of leisure and earnings—black-boxing the goods earnings can buy. In such a model the elasticity of hours with respect to the wage is positive unless there are very large income effects. However, Camerer et al., taking a driver’s earnings per hour as a proxy for the wage, estimate negative wage elasticities.

To explain this anomaly, Camerer et al. propose a model in which drivers bracket narrowly, evaluating their choices day by day instead of over their entire lifetimes; and have daily earnings targets. A reference-dependent model in which the domain of preferences includes changes in earnings relative to daily targets as well as levels of earnings and leisure can sometimes reconcile such earnings-targeting with choice that is rational within the daily bracket.²

¹ See also Hardie et al.’s (1993) analysis of consumer demand. Cabdrivers are of particular interest because many choose their own hours, unlike most workers in modern economies.

² Such narrow bracketing is of course irrational from a lifetime point of view. Some have argued that earnings targeting is irrational even within the daily bracket, because it leads drivers to sacrifice levels of earnings for changes that neoclassical preferences do not respond to. In Farber’s (2008, p. 1070) words: “This [earnings targeting] is clearly nonoptimal from a

Experiments suggest that most people are loss-averse—more sensitive to changes below their targets (losses) than above them (gains). Loss aversion creates kinks in drivers’ preferences that make their daily earnings tend to bunch around their targets, possibly leading them to work less on days with higher wages. This allows a rationality-based account of Camerer et al.’s negative daily wage elasticities, without unrealistically large income effects.

In a theory paper inspired by TK’s and Camerer et al.’s analyses, KR propose a more general model of reference-dependent preferences. KR again take narrow bracketing as given and focus on explaining a consumer’s choices within the bracket. They assume that reference-dependence is active for every good and that a reference point is conformable to a consumption vector. They also assume that a consumer’s reference-dependent utility function is continuous and can be written as the sum of a consumption utility function that depends only on consumption levels and a gain-loss utility function that depends on both levels and the reference point.

Further, for simplicity, KR assume that consumption and gain-loss utility are each additively separable across goods. (This is distinct from their assumption that the reference-dependent utility function is the sum of consumption and gain-loss utility.) They also assume that gain-loss utility is determined by the good-by-good differences between realized and reference consumption utilities. Finally, in most of their analysis KR assume that the sum of consumption and gain-loss utility that determines a consumer’s demand has constant sensitivity (TK’s “sign-dependence”; KR’s A3').³

neoclassical perspective, since it implies quitting early on days when it is easy to make money and working longer on days when it is harder to make money. Utility would be higher by allocating time in precisely the opposite manner.” We stress that we use “rational” to refer to consistency of a driver’s choices in the larger domain of preferences over levels and changes.

³ Constant sensitivity is defined in Section 2. Informally, how an observation’s consumption bundle relates to its reference point puts the bundle into a gain-loss regime, such as “earnings

For instance, in Camerer et al.’s two-good labor-supply case, let I and H denote a driver’s earnings and hours on a given day (so that leisure equals 24 minus hours) and let I^r and H^r denote her/his earnings and hours targets. Assuming constant sensitivity, KR’s reference-dependent utility function then reduces to a weighted average of consumption and gain-loss utility

$$(1) \quad V(I, H | I^r, H^r) = (1 - \eta)(U_1(I) + U_2(H)) + \eta R(I, H | I^r, H^r),$$

where

$$(2) \quad R(I, H | I^r, H^r) = 1_{(I-I^r \leq 0)} \lambda (U_1(I) - U_1(I^r)) + 1_{(I-I^r > 0)} (U_1(I) - U_1(I^r)) \\ + 1_{(H-H^r \geq 0)} \lambda (U_2(H) - U_2(H^r)) + 1_{(H-H^r < 0)} (U_2(H) - U_2(H^r)).$$

Here, η ($0 \leq \eta \leq 1$) and $1 - \eta$ reflect the relative importance of consumption and gain-loss utility and λ (normally ≥ 1) is the coefficient of loss aversion.

Importantly, in the spirit of Camerer et al.’s use of a driver’s average daily earnings as a proxy for her/his earnings target, KR close their model by equating a consumer’s reference point to the good-by-good rational expectations of her/his next choice of consumption bundle.

Like Camerer et al.’s earnings-targeting model, KR’s expectations-based reference-dependent model can allow a rationality-based account of labor supply with negative wage elasticities without invoking large income effects: With perfectly anticipated changes in earnings and hours, gain-loss utility drops out of the model, which then reproduces the neoclassical prediction that higher anticipated wages increase labor supply. But with unanticipated changes, loss aversion makes daily earnings tend to bunch around its reference point, so that earnings surprises may yield a negative overall correlation.

TK’s and KR’s theoretical papers have many empirical counterparts, with applications to demand itself (Hardie et al. 1993), labor supply (Camerer et al. 1997; Oettinger 1999; Fehr and Goette 2007; Farber 2005, 2008, 2015; Crawford and Meng 2011 (“CM”); Thakral and Tô 2021; Brandon et al. 2023;

loss, hours gain” in labor supply. With constant sensitivity a consumer’s preferences over bundles can vary across regimes, independent of the reference point within a given regime.

Crawford et al. 2025), housing (Genesove and Mayer 2001; Andersen et al. 2022), and finance (Odean 1998; Barberis and Thaler 2003; Barberis 2013; Meng and Weng 2018). Even so, the jury is still out on whether reference-dependent models of consumer demand are empirically useful.

For instance, Farber (2005, 2008) analyzes a newer dataset on New York City cabdrivers. For his drivers hours worked and earnings per hour are again negatively correlated. He finds that a reference-dependent model with daily earnings targets treated as latent variables fits better than a neoclassical model. But such a model also yields unstable estimates of the targets, which Farber argues limits the usefulness of reference-dependent models of labor supply.

CM reconsider Farber’s (2005, 2008) analyses, re-analyzing his data using a two-good version of KR’s model with hours as well as earnings targets. In the spirit of KR’s rational-expectations reference points, CM model the targets via natural sample proxies, instead of treating them as latent variables. That allows CM to avoid the instability of latent-variable estimates that made Farber doubt his reference-dependent model. Modeling the targets appears to yield a useful reference-dependent model.

Given this disparity and similar issues raised by other applications, it is natural to ask whether reference-dependent models’ empirical successes (or failures) are due to reference-dependence (or its absence) per se, or are artifacts of the strong functional-structure and functional-form assumptions that have been maintained, without testing, in all applications to date.

This paper begins to answer such questions by deriving nonparametric conditions for the existence of reference-dependent preferences that can rationalize demand behavior.⁴ Like KR, we take narrow bracketing as given

⁴ There are several other nonparametric theoretical analyses of reference-dependent models, including Gul and Pesendorfer (2006); Abdellaoui et al. (2007); Ok et al. (2015); Masatioglu and Raymond (2016); Nishimura et al. (2017); Freeman (2017, 2019); and Kibris et al. (2021). All but Gul and Pesendorfer (2006) and Freeman (2017), whose contributions we discuss in Section 4, focus on different aspects of the problem than the ones we consider.

and assume that the reference-dependent utility function is continuous and can be written as the sum of a consumption utility function that depends only on levels of consumption and a gain-loss utility function that depends on both levels and a reference point, thus allowing preferences to respond to changes as well as levels. Without such additive separability, reference-dependent models are unlikely to have any nonparametrically refutable implications.

Our model is a nonparametric generalization of KR's model, which relaxes their other functional-structure assumptions and all functional-form assumptions. Like KR's model, our model encompasses TK's model and, to our knowledge, the models used in all theoretical and empirical analyses to date. Our generalizations allow a more flexible approach to a wide range of applications, from labor supply to demand, finance, housing, and beyond.

Under our assumptions, the refutable implications of a reference-dependent model of consumer demand are limited by two factors:

Proposition 1 shows that unless reference points are modelable in the sense that the available data allow them to be precisely predicted, one can construct a reference-dependent utility function to rationalize any demand data.⁵

Proposition 2 shows that even if reference points are modelable, unless sensitivity is constant one can construct a well-behaved reference-dependent utility function to rationalize any demand data, with a minor qualification.

Propositions 1 and 2 identify a grain of truth in the common belief that allowing reference-dependence destroys the parsimony of neoclassical consumer theory: For a model of reference-dependent consumer demand to

⁵ Examples of modelability include Camerer et al.'s use of average daily earnings as sample proxies for earnings targets, KR's rational-expectations model of reference-points, CM's and Farber's (2015) implementations of KR's model, Thakral and Tô's (2021) dynamic model of reference points, and Crawford et al.'s (2025) elicitation of reference points. By contrast, Kahneman and Tversky (1979) and TK are vague on how reference points are determined; and Farber (2005, 2008) and most other empirical papers estimate them as latent variables.

have useful, nonparametrically refutable implications, it must have both modelable reference points and constant sensitivity.

With constant sensitivity, KR's functional-structure assumptions raise important new issues. They then imply that the sum of consumption and gain-loss utility that determines demand must be additively separable across goods; and that its marginal rates of substitution must vary across gain-loss regimes in a particular, knife-edge way.⁶ Both restrictions are supported theoretically only as simplifications. Empirically, additive separability across goods would be a non-starter in a neoclassical model; and we are unaware of any evidence for the restrictions on marginal rates of substitution. Yet both restrictions have been maintained, without testing, in all applications to date.

Proposition 3 paves the way for positive results by characterizing continuous reference-dependent utility functions that satisfy constant sensitivity. It identifies the most general class of well-behaved reference-dependent utility functions that, given Propositions 1 and 2, can have nonparametrically refutable implications for consumer demand. It thereby shows how to relax both of KR's constant-sensitivity restrictions.

Proposition 3's characterization could be used to conduct more general structural econometric analyses in many applications, combining its flexible functional-structure assumptions with standard functional-form assumptions regarding consumption and gain-loss utility; and modelling reference points via sample proxies like Camerer et al.'s and CM's.

However, in this paper we use Proposition 3 to derive nonparametric conditions for estimating reference-dependent preferences that rationalize a

⁶ As CM noted (their Table 1, with the same notation as (1) and (2)). In labor supply marginal rates of substitution must be equal across the (gain, gain) and (loss, loss) regimes and in constant proportions (determined by λ and η) across the (gain, loss) and (loss, gain) regimes.

consumer's demand behavior. We then illustrate our conditions by using them to reconsider Farber's and CM's analyses of cabdrivers' labor supply.

Like Afriat's (1967), Diewert's (1973), and Varian's (1982) classic nonparametric analyses of neoclassical demand, our analysis makes essential use of rationality, in the broad sense of consistency of a consumer's choices across budget sets. However, we assess such consistency within the bracket and in a preference domain expanded in the disciplined way suggested by reference-dependence, to include changes in as well as levels of consumption.

Our analysis must address two issues with no counterparts in neoclassical nonparametric analyses: The levels and changes that a reference-dependent consumer values are bundled and priced together; and, even with constant sensitivity, a consumer can in effect alter her/his preferences by buying a consumption bundle in a different gain-loss regime. As a result, there appears to be no simple condition, like the generalized axiom of revealed preference ("GARP") in the neoclassical case (Afriat's Theorem, Section 3), to determine whether reference-dependent preferences can rationalize demand behavior.

We approach this difficulty in two steps. Proposition 4 uses Proposition 3's characterization to derive benchmark necessary and sufficient conditions for a reference-dependent rationalization. Those conditions are not directly applicable because, with finite data, there is normally a range of preferences that rationalize a consumer's choices within a gain-loss regime (Varian 1982, Fact 4) and the conditions rely on an unspecified choice among them.

Proposition 5 then derives directly applicable sufficient conditions by specializing Proposition 4's result, choosing rationalizing preferences in each gain-loss regime that satisfy inequalities analogous those in Afriat's Theorem. Although Proposition 5's conditions are not necessary, they should approach necessity as the data become rich enough to tie down the regime preferences.

Proposition 5 immediately suggests an algorithm for nonparametric estimation of preferences. We illustrate our results by using it to estimate neoclassical and reference-dependent models, using Farber's (2005, 2008)

data on cabdrivers' labor supply. We relax Farber's and CM's driver-homogeneity assumptions (which follow the labor supply literature) to estimate driver by driver (as in the nonparametric demand literature). We control for models' differences in flexibility using Beatty and Crawford's (2011, pp. 2786-2787) proximity-based variant of Selten and Krischker's (1983) and Selten's (1991) nonparametric measures of predictive success, which judge flexibility by the likelihood that random data would fit a model.

Although the GARP condition for a neoclassical rationalization is violated for most of Farber's drivers, our methods yield reference-dependent rationalizations for almost all of most drivers' choices. Surprisingly, by contrast with the focus on earnings-targeting in most previous work, hours-targeting is empirically more important than earnings-targeting.

Farber's drivers are highly heterogeneous. For some, reference-dependent models that relax TK's and KR's strong constant-sensitivity restrictions have significantly higher predictive success than their counterparts imposing the restrictions, suggesting that Proposition 3's generalizations are empirically important. Neither a relaxed neoclassical nor a relaxed reference-dependent model is unambiguously superior in the aggregate. But for a significant fraction of drivers, a relaxed reference-dependent model has a higher Selten measure of predictive success than a relaxed neoclassical model.

The rest of the paper is organized as follows. Section 2 introduces our model of reference-dependent preferences. Section 3 reviews the nonparametric analysis of neoclassical consumer demand. Section 4 shows that without modelable reference points and constant sensitivity, the hypothesis of reference-dependent preferences is nonparametrically irrefutable. Section 5 characterizes continuous reference-dependent preferences with constant sensitivity. Section 6 derives the resulting model's nonparametric implications. Section 7 uses Farber's (2005, 2008) data to illustrate Section 6's methods. Section 8 is the conclusion.

2 Reference-dependent Preferences

We consider reference-dependent preferences with a finite number of demand observations for a single consumer—or equivalently for a pooled group of consumers assumed to have homogeneous preferences, but we will speak of a single consumer. We index goods $k = 1, \dots, K$ and observations $t = 1, \dots, T$. We assume the consumer is a price-taker, choosing a consumption bundle $\mathbf{q} \in \mathbb{R}_+^K$ with a linear budget constraint. Her/his preferences are represented by a family of real-valued utility functions $u(\mathbf{q}, \mathbf{r})$, parameterized by an exogenous reference point $\mathbf{r} \in \mathbb{R}_+^K$, which is conformable to a K -good consumption bundle as in TK and CM.⁷ If reference points are unmodelable, the data are prices and quantities $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1, \dots, T}$, with hypothetical reference points $\{\mathbf{r}_t\}_{t=1, \dots, T}$. If reference points are modelable, the data are prices, quantities, and reference points $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}$. The context will make the interpretation of $\mathbf{r}_t$ clear. Sometimes we denote goods by scalars with superscripts, so for $k = 1, \dots, K$, $\mathbf{q} \equiv (q^1, \dots, q^K)$ and for observation $t = 1, \dots, T$, $\mathbf{q}_t \equiv (q_t^1, \dots, q_t^K)$, with analogous notation for $\mathbf{p}$, $\mathbf{p}_t$, $\mathbf{r}$, and $\mathbf{r}_t$.

To describe preferences that respond positively to changes in consumption relative to the reference point as well as to levels, we take the reference-dependent utility function $u(\mathbf{q}, \mathbf{r})$ to be strictly increasing in $\mathbf{q}$ and strictly decreasing in $\mathbf{r}$. Our specification is then at least as flexible as a general strictly increasing function of levels $\mathbf{q}$ and changes $\mathbf{q} - \mathbf{r}$. It nests the neoclassical case where preferences respond only to levels; Kahneman and Tversky's (1979) and TK's case where they respond only to changes; and cases like Camerer et al.'s (1997), Farber's (2005, 2008), KR's, and CM's where preferences respond to both. As in those papers, we take $u(\mathbf{q}, \mathbf{r})$ to be

⁷ In KR's model, which assumes rational expectations but makes no allowance for errors, only probabilistic targets can yield the unanticipated changes in outcomes that allow expectations-based reference-dependence to have any effect. CM use the fact that sampling variation causes unanticipated changes to simplify KR's targets to point expectations, as we do here.

continuous in $\mathbf{q}$ and $\mathbf{r}$; and we assume that $u(\mathbf{q}, \mathbf{r})$ is additively separable in the sense that it can be written as the sum of a consumption utility function, which depends only on levels of consumption, and a gain-loss utility function, which depends on both levels of consumption and a reference point.

The general case of preferences in the class just described allows variable sensitivity (diminishing, increasing, or a mixture). An important special case is constant sensitivity (TK's sign-dependence; KR's assumption A3'). Let $\text{sign}(\mathbf{q} - \mathbf{r})$, the vector whose k th component is $\text{sign}(q^k - r^k)$, be the good-by-good sign pattern of gains and losses. A reference point divides commodity space into gain-loss regimes throughout each of which $\text{sign}(\mathbf{q} - \mathbf{r})$ remains constant. With constant sensitivity a consumer has a separate reference-dependent utility function for each regime, with preferences over consumption bundles $\mathbf{q}$ (but not the level of utility) independent of $\mathbf{r}$ within that regime. Gain-loss regimes' utility functions can vary freely across regimes as long as the sum of consumption and gain-loss utility is continuous across regimes.

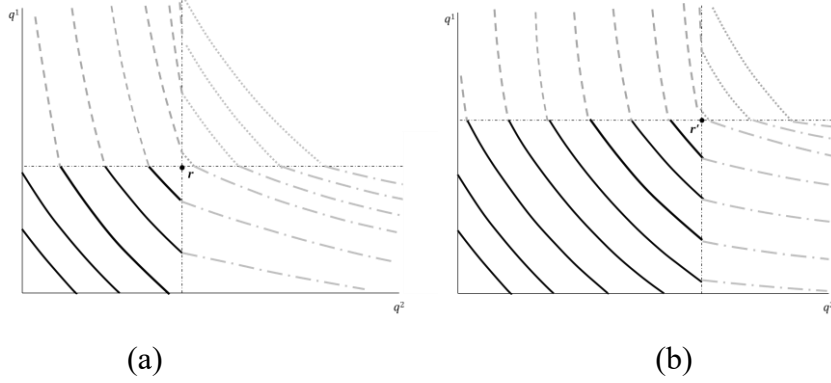
Note that each gain-loss regime's utility function must be defined for the entire commodity space, because any $\mathbf{q}$ is in the regime for some $\mathbf{r}$: Each value of $\text{sign}(\mathbf{q} - \mathbf{r})$ "switches on" a different regime utility function.

DEFINITION 1: [Preferences and utility functions with constant sensitivity.]

A reference-dependent utility function $u(\mathbf{q}, \mathbf{r})$ satisfies constant sensitivity if and only if, for any consumption bundles $\mathbf{q}$ and $\mathbf{q}^$ and reference points $\mathbf{r}$ and $\mathbf{r}^*$ such that $\text{sign}(\mathbf{q} - \mathbf{r}) = \text{sign}(\mathbf{q}^* - \mathbf{r}) = \text{sign}(\mathbf{q} - \mathbf{r}^*) = \text{sign}(\mathbf{q}^* - \mathbf{r}^*)$, $u(\mathbf{q}, \mathbf{r}) \geq u(\mathbf{q}^*, \mathbf{r})$ if and only if $u(\mathbf{q}, \mathbf{r}^*) \geq u(\mathbf{q}^*, \mathbf{r}^*)$.*

With two goods, a reference point in the interior of the commodity space divides it into four gain-loss regimes. Figure 1's panels (a) and (b) show four regime indifference maps and the associated global maps for reference points $\mathbf{r}$ and $\mathbf{r}'$. The shift from $\mathbf{r}$ to $\mathbf{r}'$ does not alter the regime maps, but as $\mathbf{r}$ varies, even locally, the shift alters how they connect across regimes, as in Figure 1.

Figure 1. A set of gain-loss regime maps with constant sensitivity and the associated global maps for alternative reference points



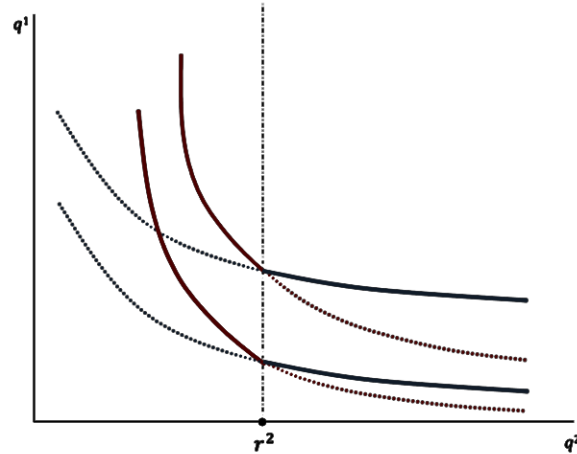
Loss aversion is a concept that has strong experimental and empirical support and figures in some of our results. Generalizing TK's (pp. 1047-1048) definition for the two-good case, Definition 2 gives a nonparametric definition of loss aversion with constant sensitivity.⁸ As loss aversion is a property of the relationship between different regimes' preferences over consumption bundles given a reference point, it is independent of the reference points themselves.

DEFINITION 2: [Preferences with constant sensitivity and loss aversion.] Assume that reference-dependent preferences and an associated utility function $u(\mathbf{q}, \mathbf{r})$ have constant sensitivity. A collection of gain-loss regime preferences over consumption bundles satisfies loss aversion if and only if, for any observation $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}$, given $\mathbf{r}_t$, the preference ordering's better-than- $\mathbf{q}_t$ set is weakly contained in each regime preference ordering's better-than- $\mathbf{q}_t$ set.

Figure 2 illustrates Definition 2's notion of loss aversion with one active reference point and two gain-loss regimes.

⁸ The idea of loss aversion is well defined with variable sensitivity, but formalizing it is more complex, and Proposition 2 will show that it is then nonparametrically irrefutable anyway.

Figure 2. Loss aversion with one active reference point
(solid curves for active parts of indifference maps,
dashed for inactive parts)



Definition 2's nesting of better-than sets is equivalent to requiring that gain-loss regimes' indifference maps satisfy a global single-crossing property: For any observation, across regimes that differ only in the gain-loss status of good i , the loss-side marginal rate of substitution between good i and any other good (generalized as needed for non-differentiable preferences) is weakly more favorable to good i than the gain-side marginal rate of substitution.

It is this single-crossing property, not the kinks in global indifference maps that it creates, that shapes loss aversion's nonparametric implications, which are testable even with finite data. Loss aversion precludes nonconvex kinks, so if the regime maps have convex better-than sets, so do the global maps.

3 Nonparametric Implications of Neoclassical Preferences

In preparation for our theoretical analysis, this section reviews Afriat's (1967), Diewert's (1973), and Varian's (1982) nonparametric analyses of consumer demand in the neoclassical case. In the revealed-preference tradition of Samuelson (1948) and Houthakker (1950), Afriat, Diewert, and Varian

show that a consumer's demand behavior can be nonparametrically rationalized by the maximization of a nonsatiated utility function if and only if the data satisfy the Generalized Axiom of Revealed Preference ("GARP"). They also show how to estimate a rationalizing utility function.

DEFINITION 3: [Rationalization with neoclassical preferences.] Preferences and an associated utility function $u(\mathbf{q})$ rationalize the data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1, \dots, T}$ if and only if $u(\mathbf{q}_t) \geq u(\mathbf{q})$ for all $\mathbf{q}$ and t such that $\mathbf{p}_t \cdot \mathbf{q} \leq \mathbf{p}_t \cdot \mathbf{q}_t$.

DEFINITION 4: [Generalized Axiom of Revealed Preference ("GARP").] $\mathbf{q}_s R \mathbf{q}_t$ implies $\mathbf{p}_t \cdot \mathbf{q}_t \leq \mathbf{p}_t \cdot \mathbf{q}_s$, where R indicates that there is some sequence of observations $\mathbf{q}_h, \mathbf{q}_i, \mathbf{q}_j, \dots, \mathbf{q}_t$ such that $\mathbf{p}_h \cdot \mathbf{q}_h \geq \mathbf{p}_h \cdot \mathbf{q}_i$, $\mathbf{p}_i \cdot \mathbf{q}_i \geq \mathbf{p}_i \cdot \mathbf{q}_j$, $\dots$, $\mathbf{p}_s \cdot \mathbf{q}_s \geq \mathbf{p}_s \cdot \mathbf{q}_t$.

AFRIAT'S THEOREM: The following statements are equivalent:

- [A] There exists a utility function $u(\mathbf{q})$ that is continuous, non-satiated, and concave, and that rationalizes the data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1, \dots, T}$.*
- [B] There exist numbers $\{U_t, \lambda_t > 0\}_{t=1}^T$ such that*

$$(3) \quad U_s \leq U_t + \lambda_t \mathbf{p}_t \cdot (\mathbf{q}_s - \mathbf{q}_t) \text{ for all } s, t \in \{1, \dots, T\}$$
- [C] The data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1, \dots, T}$ satisfy GARP.*
- [D] There exists a non-satiated utility function $u(\mathbf{q})$ that rationalizes the data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1, \dots, T}$.*

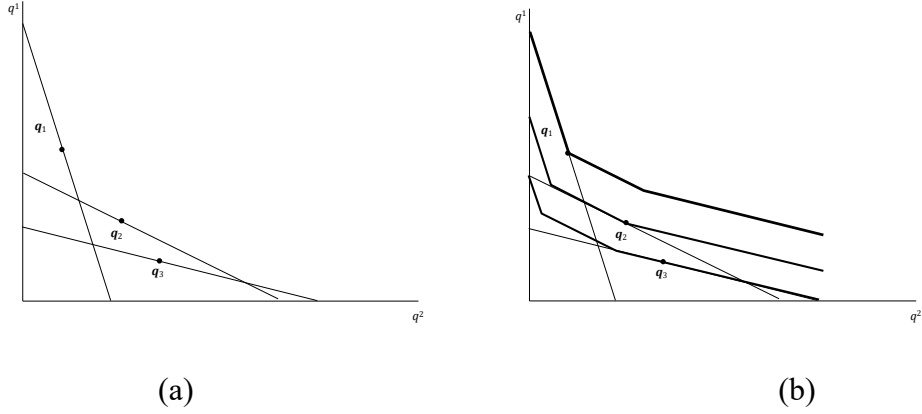
In implementing Afriat's Theorem, for given $\{U_t, \lambda_t > 0\}_{t=1}^T$ that satisfy [B]'s inequalities (3), we can take $u(\mathbf{q}) = \min_{t \in \{1, \dots, T\}} \{U_t + \lambda_t \mathbf{p}_t \cdot (\mathbf{q} - \mathbf{q}_t)\}$. With finite data there are generally many possibilities for a rationalization (Varian 1982, Fact 4 and Figure 3). However, a choice of $u(\mathbf{q})$ we call the Afriat utility function plays a central role in Proposition 5.

DEFINITION 5: [Afriat preferences and utility function.] For data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1, \dots, T}$ that satisfy GARP, or equivalently for $\{U_t, \lambda_t > 0\}_{t=1}^T$ that

satisfy condition [B] of Afriat's Theorem, the Afriat preferences follow the associated utility function $u(\mathbf{q}) = \min_{t \in \{1, \dots, T\}} \{U_t + \lambda_t \mathbf{p}_t \cdot (\mathbf{q} - \mathbf{q}_t)\}$.

Figure 3 illustrates the Afriat preferences for a three-observation dataset that satisfies GARP. Figure 3a shows the observations' budget sets and consumption bundles. Figure 3b shows the Afriat indifference map, whose marginal rates of substitution are determined by the budget lines. The Afriat utility function is piecewise linear, continuous, non-satiated, and concave.

Figure 3. Neoclassical Afriat preferences for data that satisfy GARP



4 Nonparametric Implications of Reference-dependent Preferences

This section begins our nonparametric analysis of reference-dependent preferences. Like KR, we assume that reference-dependence is active for every good and that a reference point is conformable to a consumption vector. Further, a consumer's utility function is continuous and can be written as the sum of a consumption utility function that depends only on levels and a gain-loss utility function that depends on both levels and the reference point.

Under these assumptions, whether reference-dependent preferences can rationalize demand behavior depends on whether reference points are modelable, in that they can be precisely predicted (possibly as functions of the data), and sensitivity is constant. We consider the associated cases in turn.

4.1 Reference-dependent rationalization with unmodelable reference points

With unmodelable reference points, Definition 6 allows a rationalization to choose a reference point hypothetically for each observation—the nonparametric analogue of econometrically treating targets as latent variables.

DEFINITION 6: [Rationalization with unmodelable reference points.]

Reference-dependent preferences, an associated utility function $u(\mathbf{q}, \mathbf{r})$, and hypothetical reference points $\{\mathbf{r}_t\}_{t=1,\dots,T}$, rationalize the data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,\dots,T}$ if and only if $u(\mathbf{q}_t, \mathbf{r}_t) \geq u(\mathbf{q}, \mathbf{r}_t)$ for all $\mathbf{q}$ and t such that $\mathbf{p}_t \cdot \mathbf{q} \leq \mathbf{p}_t \cdot \mathbf{q}_t$.

Proposition 1 shows that unless reference points are modelable, one can construct a reference-dependent utility function and hypothetical reference points to rationalize any demand data.

PROPOSITION 1: [Rationalization with unmodelable reference points via preferences with constant sensitivity.] For any data $\{\mathbf{p}_t, \mathbf{q}_t\}_{t=1,\dots,T}$ with unmodelable reference points, there exist reference-dependent preferences and an associated utility function $u(\mathbf{q}, \mathbf{r})$ that are continuous, increasing in $\mathbf{q}$, and decreasing in $\mathbf{r}$, and a sequence of hypothetical reference points $\{\mathbf{r}_t\}_{t=1,\dots,T}$, that rationalize the data.

Proof: Recall that we denote goods by superscripts, so that $\mathbf{q} \equiv (q^1, \dots, q^K)$, $\mathbf{q}_t \equiv (q_t^1, \dots, q_t^K)$, and so on. Let $a^k \equiv \min_{t=1,\dots,T} \{p_t^k\} > 0$ for each k and t such that $q_t^k \geq r_t^k$; and $a^k \equiv \max_{t=1,\dots,T} \{p_t^k\} > 0$ for each k and t such that $q_t^k < r_t^k$. Define the utility function $u(\mathbf{q}, \mathbf{r}) \equiv \sum_k a^k q^k + \sum_k a^k (q^k - r^k)$, which is strictly increasing in $\mathbf{q}$, strictly decreasing in $\mathbf{r}$, and satisfies constant sensitivity and Proposition 1's conditions for continuity. For observation t , set $\mathbf{r}_t = \mathbf{q}_t$ and consider any bundle $\mathbf{q} \neq \mathbf{q}_t = \mathbf{r}_t$ that (without loss of generality given strict monotonicity) exactly satisfies t 's budget constraint. For such bundles, $\sum_k p_t^k (q^k - q_t^k) = 0$ and, by the definition of the a^k ,

$\sum_k (a^k - p_t^k)(q^k - q_t^k) = \sum_k (a^k - p_t^k)(q^k - r_t^k) < 0$ and $\sum_k a^k(q^k - r_t^k) \leq 0$
and

$u(\mathbf{q}, \mathbf{r}_t) - u(\mathbf{q}_t, \mathbf{r}_t) = 2 \sum_k a^k(q^k - q_t^k) = 2 \sum_k a^k(q^k - r_t^k) \leq 0$,
so $u(\mathbf{q}, \mathbf{r})$ rationalizes the choice of $\mathbf{q}_t$. ■

Proposition 1's proof hypothesizes a reference point for each observation with $\mathbf{r}_t = \mathbf{q}_t$ and gain and loss utilities that then put the observation's consumption bundle at the kink of an approximately (to preserve strict monotonicity) Leontief indifference curve. Those preferences satisfy continuity, constant sensitivity, and Farber's and CM's functional-form assumptions, which shows that they are all nonparametrically irrefutable as well. The rationalization works entirely by varying reference points across observations, showing as directly as possible that the usefulness of reference-dependent consumer theory depends on modeling reference points.⁹

⁹ Proposition 1's proof makes clear that with unmodelable reference points, rationalization is also possible with diminishing or increasing sensitivity. Cases with limited knowledge of reference points are doubtless empirically relevant, but we have found no informative results for them. Proposition 1 is reminiscent of Varian's (1988, Theorem 1) demonstration that if only a subset of goods and their prices are observable, revealed-preference theory yields essentially no restrictions on a neoclassical consumer's behavior. As Varian notes, his analysis does not require the unobservables to be goods; they could, for instance, be demographic variables. Its intuitive connection to Proposition 1 is that in reference-dependent consumer theory unmodelability of reference points makes a subset of the variables in the domain of preferences (changes in consumption) unobservable. But substantively, Proposition 1 shows that reference-dependent preferences can rationalize any demand data even though a consumer's levels of and changes in consumption are bundled and priced together, restrictions with no counterpart in Varian's neoclassical analysis. Incidentally, it is well known that experiments that go beyond consumer choices from linear budget sets can reveal more about preferences, circumventing even non-testability results like Proposition 1's. And Van Bruggen and Heufer (2017) show that Varian's (2008) nonfalsifiability result does not hold if the unobserved prices and demands remain constant, as they normally do in laboratory settings.

As Proposition 1 suggests, econometric analyses that treat targets as latent variables may be influenced as much by how they constrain the estimation of targets and other functional-form assumptions as by reference-dependence per se. That may be why Farber's (2005, 2008) model yields unstable estimates of drivers' earnings targets, which is the basis of his argument that reference-dependence is not useful in modeling labor supply. By contrast, CM's sample-proxy model of the targets yields stable estimates with the same data.

Gul and Pesendorfer (2006) and Freeman (2017) prove results with conclusions like Proposition 1's.¹⁰ However, the preferences in Gul and Pesendorfer's proof do not satisfy KR's, CM's, and our assumption of additive separability across consumption and gain-loss utility; and they allow the strength of loss aversion to vary wildly with the cardinality of their (finite) choice set. Freeman's Observation 1 does not restrict preferences, even to be monotonic. By contrast, the rationalizing preferences in the proof of Proposition 1 are well-behaved candidates for an empirical explanation.

4.2 Reference-dependent rationalization with modelable reference points and variable sensitivity

We now show that, even if reference points are modelable, unless sensitivity is constant the hypothesis of reference-dependent preferences is refutable only via violations of Definition 4's GARP condition within subsets of observations that share exactly the same reference point. For such subsets, reference-dependent preferences reduce to neoclassical preferences. Thus, reference-dependence adds nothing empirically useful to a neoclassical model.

DEFINITION 7: [Rationalization with modelable reference points.]

Reference-dependent preferences and an associated utility function $u(\mathbf{q}, \mathbf{r})$

¹⁰ More precisely, Gul and Pesendorfer (2006) show that the choice function for reference-dependent preferences is the same as that which maximizes a complete binary relation.

rationalize the data $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}$ with modelable reference points if and only if $u(\mathbf{q}_t, \mathbf{r}_t) \geq u(\mathbf{q}, \mathbf{r}_t)$ for all $\mathbf{q}$ and $\mathbf{t}$ such that $\mathbf{p}_t \cdot \mathbf{q} \leq \mathbf{p}_t \cdot \mathbf{q}_t$.

PROPOSITION 2: [Rationalization with modelable reference points via preferences with variable sensitivity.] For any data $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}$ with modelable reference points, there exist reference-dependent preferences and an associated utility function $u(\mathbf{q}, \mathbf{r})$ that for each observation t and reference point $\mathbf{r}_t$, are continuous and strictly increasing in $\mathbf{q}$ and rationalize the data, if and only if every subset of the data whose observations share exactly the same reference point satisfies GARP.

Proof of Proposition 2: Partition the observations into subsets $\tau^j, j = 1, \dots, J$, such that if and only if two observations $\{\mathbf{p}_s, \mathbf{q}_s, \mathbf{r}_s\}$ and $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}$ have the same reference point $\mathbf{r}_s = \mathbf{r}_t$, they are in the same subset. If there exists a reference-dependent utility function with the stated properties that rationalizes the data, then the data must satisfy GARP within any such subset, by Afriat's Theorem. Conversely, suppose the data within each such subset satisfies GARP. Let $b^k \equiv \min_{t=1, \dots, T} \{p_t^k\}$, so that $0 < b^k \leq p_t^k$, and let $\mathbf{b} \equiv (b^1, \dots, b^K)$. For any subset τ^j and observation $t \in \tau^j$, let the indicator function $I_{\tau^j}(t) = 1$ if the observation $t \in \tau^j$ and $I_{\tau^j}(t) = 0$ otherwise, and let $u(\mathbf{q}, \mathbf{r}) \equiv \sum_j I_{\tau^j}(t) U^j(\mathbf{q}, \mathbf{r}_t)$, where $U^j(\mathbf{q}, \mathbf{r}_t) \equiv \min_{\rho \in \tau^j} \{U_\rho^j + \lambda_\rho^j \mathbf{p}_\rho \cdot (\mathbf{q} - \mathbf{q}_\rho)\} - \mathbf{b} \cdot \mathbf{r}_t$, which is Definition 5's Afriat utility function for observations in τ^j , with the U_ρ^j and λ_ρ^j taken from τ^j 's binding condition [B] inequalities (3) in Afriat's Theorem. If τ^j is a singleton subset, the terms in $U^j(\mathbf{q}, \mathbf{r}_t)$ follow observation t 's budget line. If not, those terms follow the minimum of τ^j 's observations' budget lines as in Figure 3b. Either way, $\mathbf{r}_t$ completely determines the $\mathbf{p}_\rho$ and $\mathbf{q}_\rho$ for all $\rho \in \tau^j$, as required to determine $U^j(\mathbf{q}, \mathbf{r}_t)$. For each $\mathbf{r}_t$, $u(\mathbf{q}, \mathbf{r}_t)$ and $U^j(\mathbf{q}, \mathbf{r}_t)$ are continuous and increasing

in $\mathbf{q}$. For any subset τ^j and observation $t \in \tau^j$ and any $\mathbf{q}$ with $\mathbf{p}_t \cdot \mathbf{q} \leq \mathbf{p}_t \cdot \mathbf{q}_t$, using τ^j 's binding [B] inequalities ((3)) for the preferences in that subset,

$$\begin{aligned} U^j(\mathbf{q}, \mathbf{r}_t) &\equiv \min_{\rho \in \tau^j} \{U_\rho^j + \lambda_\rho^j \mathbf{p}_\rho \cdot (\mathbf{q} - \mathbf{q}_\rho)\} - \mathbf{b} \cdot \mathbf{r}_t \\ &\leq U_t^j + \lambda_t^j \mathbf{p}_t \cdot (\mathbf{q}_t - \mathbf{q}_t) - \mathbf{b} \cdot \mathbf{r}_t = U_t^j - \mathbf{b} \cdot \mathbf{r}_t \equiv U^j(\mathbf{q}_t, \mathbf{r}_t). \blacksquare \end{aligned}$$

In the proof of Proposition 2, for any subset of the data whose observations share exactly the same reference point, Afriat's Theorem yields necessary and sufficient conditions for a rationalization *within that subset*. The rest of the proof shows, constructively, that preferences with variable sensitivity are flexible enough to extend such within-subset rationalizations to rationalize all other observations, roughly by following budget lines as in Figure 3b.

It may seem surprising that constant sensitivity, which is usually seen as no more than a convenient simplification, is essential for a reference-dependent model to have useful nonparametrically refutable implications. Intuitively, this resembles the ambiguity caused by income effects in neoclassical demand, where in theory even small changes in the budget set can lead to large changes in preferences. In structural analyses such large income effects are ruled out implicitly by conventional functional-form assumptions. A nonparametric analysis must rule them out explicitly, as constant sensitivity does here.¹¹

¹¹ As Proposition 2's proof shows, restricting sensitivity short of assuming that it is constant still does not yield refutable implications. The part of Proposition 2 that refers to subsets of the data whose observations share the same reference point is reminiscent of the analogous part of Varian's (1988, Theorem 1). However, Proposition 2's main substance regards the use of preferences with variable sensitivity to rationalize those and any other observations, which has no counterpart in Varian's analysis. Incidentally, unlike Proposition 1, Proposition 2 does *not* claim that $u(\mathbf{q}, \mathbf{r})$ is continuous in $\mathbf{q}$ and $\mathbf{r}$ or decreasing in $\mathbf{r}$. A rationalization might require discontinuous preferences if observations with nearby $\mathbf{r}$'s have very different budget sets. We have not tried to characterize rationalizability via a continuous $u(\mathbf{q}, \mathbf{r})$. But for data generated by continuous preferences, Proposition 2's rationalizations should converge to a continuous limiting $u(\mathbf{q}, \mathbf{r})$ as the data become rich.

5 Characterizing Reference-dependent Preferences with Constant Sensitivity and Continuity

Sections 4's results show that nonparametrically refutable implications of reference-dependence depend on the modelability of reference points and constant sensitivity. We maintain those assumptions from now on.

To prepare for Section 6's nonparametric analysis of rationalization, Proposition 3 characterizes continuous reference-dependent utility functions that satisfy constant sensitivity. It thereby relaxes the constant-sensitivity functional-structure assumptions maintained in KR's analysis and identifies the most general class of continuous reference-dependent utility functions that can have nonparametrically refutable implications for consumer demand.

In particular, Proposition 3 shows that KR's assumption that gain-loss utility is determined by the good-by-good differences between realized and reference utilities is necessary as well as sufficient for a reference-dependent utility function to be continuous. It also derives quite precise functional-structure restrictions: The gain-loss utility function must be additively separable across gain-loss regimes, across $\mathbf{q}$ and $\mathbf{r}$, and across goods within each regime, with good-by-good responses to reference points that exactly mirror their responses to the components of consumption, and with gain-loss component utility functions that are the same for each good k in all regimes.

By contrast, when the reference-dependent utility function is additively separable across consumption and gain-loss utility, continuity does not restrict the consumption utility function, which is constant across gain-loss regimes by definition. Thus KR's further assumption that gain-loss utility is governed by exactly the same function that governs consumption utility is not necessary for continuity. As a result, Proposition 3's characterization enables us in Propositions 4 and 5, with nonparametric flexibility, to relax KR's constant-sensitivity restrictions that the sum of consumption and gain-loss utility that determines a consumer's demand is additively separable across goods, and on how that sum's marginal rates of substitution vary across gain-loss regimes.

Suppose that preferences and a reference-dependent utility function $u(\mathbf{q}, \mathbf{r})$ satisfy: additive separability across consumption and gain-loss utility; constant sensitivity; and continuity in $\mathbf{q}$ and $\mathbf{r}$; with the number of goods $K \geq 2$ and reference-dependence active for all K goods;¹² and, for any $\mathbf{r}$, with preferences over $\mathbf{q}$ differentiable in the interior of each gain-loss regime, and marginal rates of substitution that differ across regimes throughout commodity space.

Let $G(\mathbf{q}, \mathbf{r})$ be a vector of binary numbers of length K with k th component 1 if $q^k \geq r^k$ and 0 otherwise. The gain-loss regime indicator $I_g(\mathbf{q}, \mathbf{r}) = 1$ if $\mathbf{g} = G(\mathbf{q}, \mathbf{r})$ and 0 otherwise; and the gain-loss indicators $G_+^k(\mathbf{q}, \mathbf{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\mathbf{q}, \mathbf{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise.

PROPOSITION 3: [Preferences and utility functions with continuity and constant sensitivity.] Suppose there are $K \geq 2$ goods, with reference-dependence active for all K goods, and that a reference-dependent preference ordering and an associated utility function have additively separable consumption utility and gain-loss utility components. Then the ordering satisfies constant sensitivity if and only if an associated utility function $u(\mathbf{q}, \mathbf{r})$ can be written, for some consumption utility function $U(\cdot)$ and gain-loss regime utility functions $V_g(\cdot, \cdot)$ and $v_g(\cdot)$, as

$$(4) \quad u(\mathbf{q}, \mathbf{r}) \equiv U(\mathbf{q}) + \sum_{\mathbf{g}} I_{\mathbf{g}}(\mathbf{q}, \mathbf{r}) V_{\mathbf{g}}(v_{\mathbf{g}}(\mathbf{q}), \mathbf{r}).$$

Suppose further that the induced preferences over $\mathbf{q}$ are differentiable in the interior of each regime, with marginal rates of substitution that differ across regimes throughout commodity space. Then the ordering satisfies constant sensitivity and continuity if and only if it is representable by a utility function

¹² In riskless environments with convex budget sets, if $K = 1$ all increasing preferences are observationally equivalent, so reference-dependence cannot be empirically meaningful. And, as Proposition 3's wording suggests, its assumptions don't tie down the functional structure for goods for which reference-dependence is inactive. As we seek general characterizations, Propositions 4 and 5 take Proposition 3's conclusion, not its assumptions, as their premises.

$u(\mathbf{q}, \mathbf{r})$ that can be written, for some consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$ (with the indicator functions $G_+^k(\cdot, \cdot)$ and $G_-^k(\cdot, \cdot)$ doing the work of the indicator $I_g(\cdot, \cdot)$), as

(5) $u(\mathbf{q}, \mathbf{r}) \equiv U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\mathbf{q}, \mathbf{r})\{v_-^k(q^k) - v_-^k(r^k)\}]$.

Conversely, any combination of induced regime preferences over $\mathbf{q}$ is consistent with continuity and constant sensitivity for some gain-loss utility functions.

Proof of Proposition 3: The “if” part of each claim is immediate. The “only if” part regarding (4) follows from Definition 1 via the standard characterization of additively separable preferences (Debreu 1960, Section 3). To prove the “only if” part regarding (5), note that $u(\mathbf{q}, \mathbf{r})$ in (4) is continuous if and only if

$$(6) \quad V_g(v_g(\mathbf{q}), \mathbf{r}) = V_{g'}(v_{g'}(\mathbf{q}), \mathbf{r})$$

for any $\mathbf{q}, \mathbf{r}$, and i with $q^i = r^i$ and any gain-loss regimes $\mathbf{g}$ and $\mathbf{g}'$ that differ in the gain-loss status of good i . But (6) can hold under those conditions only if each regime's $V_g(v_g(\mathbf{q}), \mathbf{r})$ is additively separable in the components of $\mathbf{q}$ and, for component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$, $k = 1, \dots, K$,

$$(7) \quad \sum_g I_g(\mathbf{q}, \mathbf{r}) V_g(v_g(\mathbf{q}), \mathbf{r}) \equiv \sum_k [G_+^k(\mathbf{q}, \mathbf{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\mathbf{q}, \mathbf{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

First suppose that (6) is satisfied for some $\mathbf{q}, \mathbf{r}$, and i with $q^i = r^i$. Using the hypothesized differentiability in the interior of each gain-loss regime, if $\partial V_g(v_g(\mathbf{q}), \mathbf{r})/\partial q^j \neq 0$, (6) implies that $\partial V_{g'}(v_{g'}(\mathbf{q}), \mathbf{r})/\partial q^j \neq 0$ as well. Adding $U(\mathbf{q})$ to each side of (6), partially differentiating each side with respect to q^j and then q^i , with $r^i = q^i$, and taking ratios would then show that the marginal rates of substitution between goods i and j are equal across regimes $\mathbf{g}$ and $\mathbf{g}'$ for all $q^i = r^i$, a contradiction. Thus with $q^i = r^i$, $\partial V_g(v_g(\mathbf{q}), \mathbf{r})/\partial q^j \equiv \partial V_{g'}(v_{g'}(\mathbf{q}), \mathbf{r})/\partial q^j \equiv 0$ for any $j \neq i$, and standard characterization results show that for a regime $\mathbf{g}$, $V_g(v_g(\mathbf{q}), \mathbf{r})$ is additively separable across the components of $\mathbf{q}$. Given that, changing the gain-loss status of a good j with $q^i = r^i$ would violate (6) and therefore continuity, unless for some functions $w_+^k(\cdot)$ and $w_-^k(\cdot)$, $k = 1, \dots, K$,

$$\sum_g I_g(\mathbf{q}, \mathbf{r}) V_g(v_g(\mathbf{q}), \mathbf{r}) \equiv \sum_k [G_+^k(\mathbf{q}, \mathbf{r}) w_+^k(q^k, \mathbf{r}) + G_-^k(\mathbf{q}, \mathbf{r}) w_-^k(q^k, \mathbf{r})].$$

Finally, unless the $w_+^k(\cdot, \cdot)$ and $w_-^k(\cdot, \cdot)$ are also additively separable in $\mathbf{r}$, with good-by-good responses to reference points that exactly mirror their good-by-good responses to bundles as in (7) (with $w_+^k(q^k, \mathbf{r}) \equiv \{v_+^k(q^k) - v_+^k(r^k)\}$ and $w_-^k(q^k, \mathbf{r}) \equiv \{v_-^k(q^k) - v_-^k(r^k)\}$), for some $\mathbf{q}$, $\mathbf{r}$, and k , changing q^k and r^k with $r^k = q^k$ would induce different changes in $V_g(v_g(\mathbf{q}), \mathbf{r})$ and $V_{g'}(v_{g'}(\mathbf{q}), \mathbf{r})$, violating (6) and continuity. The contradiction establishes our claim regarding (7) and completes the proof of (5). A similar argument shows that any combination of induced regime preferences over $\mathbf{q}$ is consistent with continuity and constant sensitivity for some gain-loss utility functions. ■

The heart of Proposition 3's proof uses simple variational arguments to show that a continuous gain-loss utility function must be additively separable across gain-loss regimes, across $\mathbf{q}$ and $\mathbf{r}$, and across goods within each regime, with good-by-good responses to reference points that exactly mirror their responses to the components of consumption, and gain-loss component utility functions (the $v_+^k(\cdot)$ and $v_-^k(\cdot)$) that are the same for each k in all regimes.

Proposition 3's characterization (5) plays a central role in Proposition 4's and 5's conditions for a rationalization with modelable reference points and constant sensitivity. With constant sensitivity a consumer's preferences over $\mathbf{q}$ and optimal choice of $\mathbf{q}$ are independent of $\mathbf{r}$ within a gain-loss regime. However, the maximized value of $u(\mathbf{q}, \mathbf{r})$ varies with $\mathbf{r}$ even within a regime. (5)'s terms in $v_+^k(r^k)$ and $v_-^k(r^k)$ ensure continuity of $u(\mathbf{q}, \mathbf{r})$ despite such variation, by subtracting a regime-by-regime "loss cost". Because the loss costs depend on $\mathbf{r}$, which is exogenous, not on $\mathbf{q}$, a consumer faces a menu of fixed regime charges, which influence her/his incentive to "defect" from an observation's consumption bundle to bundles in other regimes. This incentive constraint figures in Proposition 4's and 5's conditions for a rationalization.

6 Nonparametric Implications of Reference-Dependent Preferences with Modelable Reference Points, Constant Sensitivity, and Continuity

This section uses Proposition 3's characterization of continuous reference-dependent utility functions that satisfy constant sensitivity to derive nonparametric conditions for a reference-dependent rationalization.

With modelable reference points and constant sensitivity, observations' consumption bundles can be objectively sorted into gain-loss regimes. By Afriat's Theorem (Section 3), GARP for each regime's observations is required for the existence of preferences that preclude defections from an observation's bundle to affordable bundles within the same regime, and thus necessary for a rationalization. However, it is not sufficient, for two reasons.

First, the regime utility functions that rationalize the consumer's choices within each regime must satisfy Proposition 3's restriction that their gain-loss component utility functions (the $v_+^k(\cdot)$ and $v_-^k(\cdot)$) are the same across all regimes. GARP regime-by-regime does not ensure that that is possible. Second, the rationalizing regime utility functions must prevent defections from an observation's consumption bundle to affordable bundles in other regimes, in which the consumer's preferences may differ. This involves Section 5's loss costs, which are determined by the rationalizing regime utility functions.

Another difficulty in deriving conditions for a rationalization is that there is normally a range of rationalizing gain-loss regime utility functions, as in the neoclassical case (Varian 1982, Fact 4 and Figure 3). Choosing among them involves complex trade-offs, because a choice that lowers the gain from defecting *from* bundles in a regime raises the gain from defecting *to* them.

Propositions 4 and 5 approach these difficulties in two steps.

Proposition 4 uses Proposition 3's characterization of continuous reference-dependent utility functions with constant sensitivity to derive benchmark necessary and sufficient conditions for a rationalization, conditional on the choice of rationalizing consumption and gain-loss component utilities (each

evaluated observation by observation). Because Proposition 4's conditions are conditional on that flexible choice, they are not directly applicable.

Proposition 5 then derives directly applicable sufficient conditions based on rationalizing regime utility functions analogous to Definition 3's Afriat utility functions. Because there are normally multiple rationalizing regime utility functions, Proposition 5's sufficient conditions are not necessary; but with rich enough data they should be asymptotically necessary, as explained below.

Let $\Gamma(g; \mathbf{r})$ be the set of $\mathbf{q}$ in regime g for $\mathbf{r}$. Let $\Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g) \equiv \{t \in \{1, \dots, T\} \mid \mathbf{q}_t \in \Gamma(g; \mathbf{r}_t)\}$ be the set of t with $\mathbf{q}_t$ in regime g for $\mathbf{r}_t$.

PROPOSITION 4: [Rationalization with modelable reference points via preferences and utility functions with constant sensitivity.] Suppose that reference-dependent preferences and an associated utility function are defined over $K \geq 2$ goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function satisfies Proposition 3's (6). Consider data $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}$ with modelable reference points. Then the statements [A] and [B] are equivalent:

[A] There exists a continuous reference-dependent utility function $u(\mathbf{q}, \mathbf{r})$ that satisfies constant sensitivity; is strictly increasing in $\mathbf{q}$ and strictly decreasing in $\mathbf{r}$; and that rationalizes the data $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}$.

[B] Each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over consumption bundles, with continuous, strictly increasing consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$, such that, for any regime g and any pair of observations $\sigma, \tau \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)$ (with the indicator functions $G_+^k(\cdot, \cdot)$ and $G_-^k(\cdot, \cdot)$ again doing the work of $I_g(\cdot, \cdot)$),

$$(8) \quad U(\mathbf{q}_\sigma) + \sum_k [G_+^k(\mathbf{q}_\sigma, \mathbf{r}_\tau) v_+^k(q_\sigma^k) + G_-^k(\mathbf{q}_\sigma, \mathbf{r}_\tau) v_-^k(q_\sigma^k)] \\ \leq U(\mathbf{q}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_-^k(q_\tau^k)] + \lambda_\tau \mathbf{p}_\tau \cdot (\mathbf{q}_\sigma - \mathbf{q}_\tau)$$

and for each observation $\{\mathbf{p}_\tau, \mathbf{q}_\tau, \mathbf{r}_\tau\}_{t=1,\dots,T}$ with $\tau \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1,\dots,T}; g)$ and each $\mathbf{q} \in \Gamma(g'; \mathbf{r}_\tau)$ with $g' \neq g$ for which $\mathbf{p}_\tau \cdot \mathbf{q} \leq \mathbf{p}_\tau \cdot \mathbf{q}_\tau$,

$$(9) \ U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau) \{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\mathbf{q}, \mathbf{r}_\tau) \{v_-^k(q^k) - v_-^k(r_\tau^k)\}] \\ \leq U(\mathbf{q}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].$$

The proof of Proposition 4 is in the Supplemental Appendix. That [B] implies [A] is immediate. That [A] implies (8)'s conditions precluding defections within a given gain-loss loss regime is only a translation of the standard Afriat inequalities into the language of Proposition 3. That [A] implies (9)'s conditions precluding defections across regimes is more difficult because it requires linking Section 5's loss costs to things that can be estimated from the data, not only at given points but as functions of $\mathbf{r}$. The proof shows by calculation that this can be done.

Proposition 4's necessary and sufficient conditions for a rationalization could be used to test the feasibility of satisfying the necessary conditions, following Varian (1982), by writing a nonlinear programming problem to check for the existence of solutions of its inequality constraints.

Figures 4 and 5 illustrate Proposition 4. In each case the entire dataset violates GARP, with observation 1's consumption bundle chosen in 1's budget set over observation 2's bundle, and vice versa. In each case the observations' reference points put their bundles in different gain-loss regimes, in which constant sensitivity allows different preferences. And in each case each regime's single observation trivially satisfies GARP within its regime.

Figure 4 illustrates how it is possible for reference-dependence to yield a rationalization despite a failure of GARP. Figures 4a-b depict Afriat and non-Afriat rationalizing regime preferences. In each case condition (9) is satisfied, so that a rationalization is possible.

Figure 5 shows that there are some failures of GARP that reference-dependence cannot overcome. In Figure 5a Afriat rationalizing regime utilities do not satisfy (9) and Figure 5b shows that there can be no choice of

rationalizing regime utilities (Afriat or not) for which (9) is satisfied, so that a rationalization is impossible. A rationalization in Figure 5b would require regime preferences that connect a loss-regime indifference curve through observation 1's bundle to a gain-regime curve that cuts into observation 2's budget set and stays outside observation 1's budget set, thus passing northeast of 2's bundle; and also loss- and gain-regime indifference curves satisfying the analogous conditions interchanging observations 1 and 2. Such curves are inconsistent with the optimality of each observation's consumption bundle.

Figure 4. Rationalizing data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for loss maps, dashed lines for gains maps)

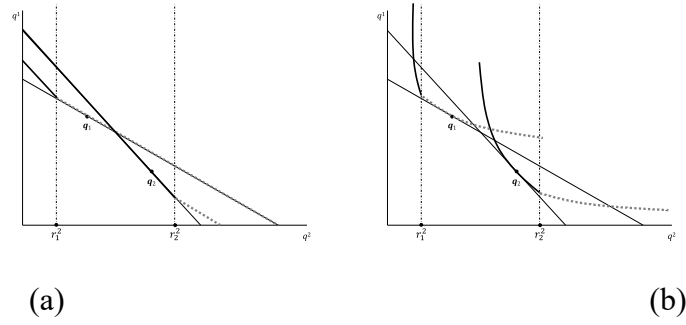
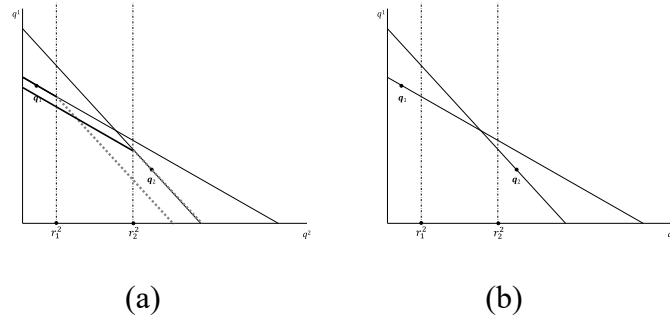


Figure 5. Failing to rationalize data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for (a)'s loss maps, dashed lines for (a)'s gains maps)



The difference between Figure 4's and Figure 5's examples can be understood in terms of loss aversion (Definition 2). The change in rationalizing Afriat preferences across Figure 4a's gain-loss regimes is consistent with loss aversion, but the analogous change in Figure 5a is not.

A Corollary shows that if the rationalizing regime preferences satisfy loss aversion, Proposition 4's conditions (9) precluding cross-regime defections must be satisfied, so that its conditions (8) are sufficient for a rationalization.

Recall that the gain-loss indicator functions $G_+^k(\mathbf{q}, \mathbf{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\mathbf{q}, \mathbf{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise; and that $\theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g) \equiv \{t \in \{1, \dots, T\} \mid \mathbf{q}_t \in \Gamma(g; \mathbf{r}_t)\}$ is the set of observation indicators t for which $\mathbf{q}_t$ is in regime g for $\mathbf{r}_t$.

COROLLARY: *[Rationalization with modelable reference points via preferences and utility functions with constant sensitivity that satisfy a condition weaker than loss aversion.] Suppose that reference-dependent preferences and an associated utility function are defined over $K \geq 2$ goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function therefore satisfies Proposition 3's (5). Consider data $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}$ with modelable reference points. If each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over consumption bundles, with continuous, strictly increasing consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$, such that, for any regime g and any pair of observations $\sigma, \tau \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)$ for which $\mathbf{p}_\tau \cdot \mathbf{q}_\sigma \leq \mathbf{p}_\tau \cdot \mathbf{q}_\tau$ (with the indicator functions $G_+^k(\cdot, \cdot)$ and $G_-^k(\cdot, \cdot)$ doing the work of a regime indicator function $I_g(\cdot, \cdot)$),*

$$\begin{aligned} & U(\mathbf{q}_\sigma) + \sum_k [G_+^k(\mathbf{q}_\sigma, \mathbf{r}_\tau) v_+^k(q_\sigma^k) + G_-^k(\mathbf{q}_\sigma, \mathbf{r}_\tau) v_-^k(q_\sigma^k)] \\ & \leq U(\mathbf{q}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_-^k(q_\tau^k)] + \lambda_\tau \mathbf{p}_\tau \cdot (\mathbf{q}_\sigma - \mathbf{q}_\tau), \end{aligned}$$

and there are no observations for which $\mathbf{q}_t$ is not on the boundary of the convex hull of $\mathbf{q}_t$'s upper contour set for the associated candidate global

preference ordering for $\mathbf{r}_t$, then the consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$ rationalize the data.

The proof of Proposition 4's Corollary is in the Supplemental Appendix. It is best understood by recalling that loss-averse preferences have better-than sets that are weakly contained in the better-than sets of each of their gain-loss regime preferences (Definition 2 and Figure 2). That immediately implies that rationalizing regime preferences preclude cross-regime as well as within-regime defections. The proof also suggests that, like the convexity of neoclassical preferences, loss aversion is nonparametrically untestable.

It may be surprising that loss aversion, which is empirically well supported but not usually seen as essential for the *existence* of a reference-dependent rationalization, figures in the Corollary's sufficient conditions. Further, the Corollary's final "no observations for which $\mathbf{q}_t$ is *not* on the boundary" condition rules out bunching of consumption bundles in regions of commodity space where the rationalizing regime preferences violate loss aversion, and so is vacuously satisfied for preferences that satisfy loss aversion. To our knowledge restrictions on bunching are unusual in a nonparametric analysis.

The Supplemental Appendix's Figure A1 shows that, contrary to what Figures 4 and 5 may suggest, when GARP is satisfied within each gain-loss regime, loss aversion is not necessary for a rationalization.

As already noted, Proposition 4's necessary and sufficient conditions for a rationalization are not directly applicable because they are conditional on the choice of rationalizing gain-loss regime utility functions. Proposition 5 derives directly applicable sufficient conditions by specifying rationalizing regime utility functions in the style of the regime's Afriat utility functions (Definition 5). (The Supplemental Appendix's Figure A2 shows, however, that with finite data a rationalization may require non-Afriat rationalizing regime utilities.)

Proposition 5's conditions include inequalities like (3) in Afriat's Theorem or Proposition 4's (8), which prevent defections from an observation's

consumption bundle to affordable bundles in the same gain-loss regime, and inequalities like (9), which prevent defections to affordable bundles in other regimes. In each case Proposition 5's conditions also enforce Proposition 3's restrictions (5) on the component gain-loss utility functions.

PROPOSITION 5: [Sufficient conditions for rationalization with modelable reference points, via reference-dependent preferences and utility function with constant sensitivity and continuity.] The following conditions are sufficient for the existence of continuous reference-dependent preferences and utility function with constant sensitivity $u(\mathbf{q}, \mathbf{r})$ that rationalize data with modelable reference points $\{\mathbf{p}_t, \mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}$: There exist numbers U_t , v_{t+}^k , v_{t-}^k , and $\lambda_t > 0$ for each $k = 1, \dots, K$ and $t = 1, \dots, T$ such that:

[A] For any gain-loss regime g and any pair of observations $\sigma, \tau \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)$ (with the indicator functions $G_+^k(\cdot, \cdot)$ and $G_-^k(\cdot, \cdot)$ again doing the work of $I_g(\cdot, \cdot)$),

$$(10) \quad U_\sigma + \sum_k [G_+^k(\mathbf{q}_\sigma, \mathbf{r}_\tau) v_{\sigma+}^k + G_-^k(\mathbf{q}_\sigma, \mathbf{r}_\tau) v_{\sigma-}^k] \\ \leq U_\tau + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_{\tau+}^k + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_{\tau-}^k] + \lambda_\tau \mathbf{p}_\tau \cdot (\mathbf{q}_\sigma - \mathbf{q}_\tau).$$

[B] For observations σ, τ , $q_\sigma^k \geq q_\tau^k$ for $k = 1, \dots, K$, $U_\sigma \geq U_\tau$; and for observations σ, τ and any $k = 1, \dots, K$, $q_\sigma^k \geq q_\tau^k$, $v_{\sigma+}^k \geq v_{\tau+}^k$, and $v_{\sigma-}^k \geq v_{\tau-}^k$.

[C] For any pair of regimes g and $g' \neq g$, observation $\tau \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)$, and bundle $\mathbf{q} \in \Gamma(g'; \mathbf{r}_\tau)$ for which $\mathbf{p}_\tau \cdot \mathbf{q} \leq \mathbf{p}_\tau \cdot \mathbf{q}_\tau$,

$$(11) \quad \min_{\rho \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g')} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{q} - \mathbf{q}_\rho)\} \\ - \min_{\rho \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g')} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{r}_\tau - \mathbf{q}_\rho)\} \\ \leq \min_{\rho \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{q}_\tau - \mathbf{q}_\rho)\} \\ - \min_{\rho \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau) v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{r}_\tau - \mathbf{q}_\rho)\}.$$

Proof of Proposition 5: Given choices of U_t , v_{t+}^k , v_{t-}^k , and λ_t , $t = 1, \dots, T$, that satisfy [A] and [B], let $u^g(\mathbf{q}, \mathbf{r})$ denote the rationalizing Afriat regime utility

function for regime g , including (5)'s loss costs, which exists by Afriat's Theorem. For $\mathbf{q} \in \Gamma(g; \mathbf{r})$, using (8) as in the proof of Afriat's Theorem:

$$\begin{aligned}
u^g(\mathbf{q}, \mathbf{r}) - U(\mathbf{r}) &\equiv U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\mathbf{q}, \mathbf{r})\{v_-^k(q^k) - v_-^k(r^k)\}] - U(\mathbf{r}) \\
(12) \quad &\equiv \{U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r})v_+^k(q^k) + G_-^k(\mathbf{q}, \mathbf{r})v_-^k(q^k)]\} \\
&\quad - \{U(\mathbf{r}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r})v_+^k(r^k) + G_-^k(\mathbf{q}, \mathbf{r})v_-^k(r^k)]\} \\
&\equiv \min_{\rho \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r})v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r})v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{q} - \mathbf{q}_\rho)\} \\
&\quad - \min_{\rho \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r})v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r})v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{r} - \mathbf{q}_\rho)\},
\end{aligned}$$

where the last identities follow as in Afriat's Theorem in the neoclassical case. The rationalizing reference-dependent utility function, including loss costs, is then $u(\mathbf{q}, \mathbf{r}) \equiv U(\mathbf{q}) + \sum_g I_g(\mathbf{q}, \mathbf{r}) u^g(\mathbf{q}, \mathbf{r})$. By construction, $u(\mathbf{q}, \mathbf{r})$ is continuous, strictly increasing in $\mathbf{q}$, and strictly decreasing in $\mathbf{r}$.

For observations $\sigma, \tau \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)$ in the same gain-loss regime g , with $\mathbf{p}_\tau \cdot \mathbf{q}_\sigma \leq \mathbf{p}_\tau \cdot \mathbf{q}_\tau$, loss costs cancel out and (11) reduces to the usual Afriat inequalities expressed in Proposition 3's language. Thus by Afriat's Theorem, [A] prevents defections to affordable bundles in the same regime.

For gain-loss regimes g and $g' \neq g$, observation $\tau \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)$, and bundle $\mathbf{q} \in \Gamma(g'; \mathbf{r}_\tau)$ with $\mathbf{p}_\tau \cdot \mathbf{q} \leq \mathbf{p}_\tau \cdot \mathbf{q}_\tau$,

$$\begin{aligned}
&u(\mathbf{q}, \mathbf{r}_\tau) - U(\mathbf{r}_\tau) \\
(13) \quad &\equiv U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\mathbf{q}, \mathbf{r}_\tau)\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\mathbf{r}_\tau) \\
&\equiv \{U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau)v_+^k(q^k) + G_-^k(\mathbf{q}, \mathbf{r}_\tau)v_-^k(q^k)]\} \\
&\quad - \{U(\mathbf{r}_\tau) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau)v_+^k(r_\tau^k) + G_-^k(\mathbf{q}, \mathbf{r}_\tau)v_-^k(r_\tau^k)]\} \\
&\equiv \min_{\rho \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g')} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{q} - \mathbf{q}_\rho)\} \\
&\quad - \min_{\rho \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g')} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{r}_\tau - \mathbf{q}_\rho)\} \\
&\leq \min_{\rho \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{q}_\tau - \mathbf{q}_\rho)\} \\
&\quad - \min_{\rho \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_\tau)v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{r}_\tau - \mathbf{q}_\rho)\} \\
&\equiv \left\{ U(\mathbf{q}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau)v_+^k(q_\tau^k) + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau)v_-^k(q_\tau^k)] \right\} \\
&\quad - \left\{ U(\mathbf{r}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau)v_-^k(r_\tau^k)\} \right\} \\
&\equiv u(\mathbf{q}_\tau, \mathbf{r}_\tau) - U(\mathbf{r}_\tau),
\end{aligned}$$

which prevents defections across regimes. ■

The proof of Proposition 5 closely parallels the proof of Proposition 4. Again the main difficulty is linking Section 5's loss costs to things that can be estimated from the data, not only at given points but as functions of $\mathbf{r}$. Proposition 5's proof shows by calculation that this can be done.

In the neoclassical case, Mas-Colell (1978) and Forges and Minelli (2009) study the limit of the rationalizing preferences as the data become "rich" in the sense that as $T \rightarrow \infty$ they include budget sets as close as desired to any possible budget set. They show that the range of convexified rationalizing preferences (if they exist) collapses on Definition 5's Afriat preferences.¹³

Mas-Colell's and Forges and Minelli's result cannot be immediately applied, gain-loss regime by regime, in the reference-dependent case with constant sensitivity, because of our cross-regime "no-defection" constraints and Proposition 3's continuity-driven constraints that each gain-loss component utility function (the $v_+^k(\cdot)$ and $v_-^k(\cdot)$) must be the same for each k in all regimes. Even so, we conjecture that in the limit as the data become rich, now in the sense of including {reference point $\times$ budget set} combinations as close as desired to any possible combination, Proposition 5's sufficient conditions are asymptotically necessary. Put another way, if the Afriat regime preferences used in Proposition 5's cross-regime constraints [C] do not yield a reference-dependent rationalization, then in the limit Proposition 4's cross-regime constraints [B] are close enough to Proposition 5's constraints [C] that no component gain-loss utility functions that satisfy Proposition 4's cross-regime constraints can yield a reference-dependent rationalization.

Propositions 4, its Corollary, and Proposition 5 somewhat refocus the view of reference-dependent consumer demand from structural models. Constant sensitivity, usually seen only as a convenient simplification, is essential for reference-dependent models to have any nonparametrically refutable implications. And loss aversion, usually seen as empirically well-established

¹³ Also requiring richness of consumption bundles would rule out non-convex preferences.

but not essential to the existence of a reference-dependent rationalization, now plays a role in at least the Corollary's sufficient conditions for existence.

7 Empirical Illustration

Although Proposition 3's characterization of reference-dependent preferences in that case would be well suited to a structural econometric analysis, we instead illustrate our methods by nonparametrically reconsidering Farber's and CM's econometric analyses, using sample proxies like CM's for the targets.¹⁴ Proposition 5's sufficient conditions for a reference-dependent rationalization with modelable reference points and constant sensitivity suggest methods for recovering rationalizing preferences when they exist.

We consider preferences over levels of and changes in earnings and leisure. With two goods, GARP (Definition 4) reduces to the Weak Axiom of Revealed Preference ("WARP"). WARP is then necessary and sufficient for a neoclassical rationalization. In this section we use WARP for GARP.

DEFINITION 6: [Weak Axiom of Revealed Preference ("WARP").] $\mathbf{q}_s R \mathbf{q}_t$ and $\mathbf{q}_s \neq \mathbf{q}_t$ implies not $\mathbf{q}_t R \mathbf{q}_s$, where R indicates that there is some sequence of observations $\mathbf{q}_h, \mathbf{q}_i, \mathbf{q}_j, \dots, \mathbf{q}_t$ such that $\mathbf{p}_h \cdot \mathbf{q}_h \geq \mathbf{p}_h \cdot \mathbf{q}_i$, $\mathbf{p}_i \cdot \mathbf{q}_i \geq \mathbf{p}_i \cdot \mathbf{q}_j$, $\dots$, $\mathbf{p}_s \cdot \mathbf{q}_s \geq \mathbf{p}_s \cdot \mathbf{q}_t$.

We relax Camerer et al.'s (1997), Farber's (2005, 2008), and CM's assumption that drivers have homogeneous preferences, instead allowing unrestricted heterogeneity of preferences. (Our theory covers both cases, distinguished only by whether the data are pooled across drivers.)

¹⁴ Our choice is motivated by curiosity regarding the robustness of Farber's and CM's structural analyses and by the computational difficulty of nonparametric estimation using more recent datasets, which are much larger than Farber's. We are also curious about the robustness of Camerer et al.'s analysis, but their data are no longer fully available.

To guide future work, we consider expectations-based and recent-experience-based models of reference points, each using sample proxies. Like CM, but unlike Camerer et al. and Farber, we also consider reference-dependence in earnings alone, in hours alone, and in both earnings and hours.

Section 7.1 and the Supplemental Appendix review Farber's (2005, 2008) data. Section 7.2 outlines the models of reference-dependent preferences we consider. Section 7.3 discusses Selten and Krischker's (1983), Selten's (1991), and Beatty and Crawford's (2011) notions of predictive success. Section 7.4 describes our estimation procedure. Section 7.5 reports our results.

7.1 Data

Like CM, we use Farber's (2005, 2008) data.¹⁵ Farber collected 593 trip sheets for 13461 trips by 21 drivers between June 1999 and May 2001. Each sheet records the driver's name, hack number, date, each fare's start time and location, each fare's end time and location, and the fare paid. Nine sheets duplicate the day and driver, so there are only 584 shifts. To avoid arbitrariness, and because our methods make some allowance for sample size, in addition to the 15 drivers Farber and CM studied we include the 6 with samples of 10 or fewer shifts they excluded. We note however that for samples this small, the gap between Proposition 5's sufficient conditions for a rationalization and the necessary condition may be large. Also, as will be seen, drivers with small samples are much more likely to satisfy the WARP condition for a neoclassical rationalization, possibly by chance.

The Supplemental Appendix's Table B1 reports descriptive statistics driver by driver. The values are the same as those in Farber's (2005) Table B1, except for the hourly wage variable and the Afriat efficiencies in the last two columns. Our earnings and wage variables differ from Farber's and CM's in

¹⁵ The datasets are posted at https://www.aeaweb.org/aer/data/june08/20030605_data.zip, and https://www.aeaweb.org/aer/data/aug2011/20080780_data.zip. The CPI data are posted at <https://data.bls.gov/timeseries/CUURS12ASA0>, under the years 1999-2001.

two ways, which affect the Afriat efficiencies. First, we use the NY/NJ urban CPI to control for price level changes in the sample period. Second, Farber's and CM's wage variable is income per hour spent working, with working time defined as the sum of time driving with a passenger and time waiting for the next passenger. However, as waiting time varies randomly from shift to shift with weather, the flow of customers, etc., and is not directly linked to earnings, it appears largely exogenous. Accordingly, we define the wage as earnings per hour driving, treating waiting time as an exogenous fixed cost. This seems a natural choice in a model where earnings targeting is day by day.

Our redefinition of the wage matters more in our nonparametric analysis than it would in Farber's and CM's structural analyses. Drivers' waiting times range from about 25-40% of their shifts. If we included waiting in driving time, shift-to-shift wage variation would make a driver's observations' budget lines pivot around their common zero-hours end, they would never cross, he would trivially satisfy WARP, and a nonparametric analysis would give only a meaningless recapitulation of his data. By contrast, treating waiting times as a fixed cost allows a driver's budget lines to cross. The Supplemental Appendix's Figure A1 shows that with our wage definition drivers' budget lines cross frequently, making WARP a meaningful restriction and allowing a nonparametric analysis to provide a meaningful interpretation of the data.

The Supplemental Appendix's Table B2's last column reports each driver's Afriat efficiency index. The index is 1 for a driver whose data satisfy WARP but is otherwise less than 1. Only 7 of Farber's 21 drivers satisfy WARP. Except for drivers 12 (sample size 13), 14 (sample size 17), and 17 (sample size 10), the drivers with exact neoclassical fits (2, 3, 6, 9, 11, 13, and 15) are the ones with the smallest sample sizes among the 21 drivers. Except for drivers 2 (sample size 14), 9 (sample size 19), and 17 (sample size 10), those drivers are the same as the six (3, 6, 11, 13, 15, and 17) Farber and CM excluded due to sample sizes ≤ 10 .

We stress that such small sample sizes may greatly widen the gap between Proposition 5's sufficient conditions for a rationalization and the necessary conditions. Also, small samples make it easier to satisfy WARP by chance, so drivers' data may simply be too under-powered to reject the neoclassical model. We return to the issue of correcting for power to reject in Section 7.3.

7.2 Alternative models of reference-dependent preferences

Our reference-dependent models vary in three dimensions.¹⁶ The first distinguishes models based on proxied rational expectations from those based on recent experience. Our expectations-based models are sample averages of a driver's choices, excluding the current shift. Our experience-based models are one-shift lags. For each kind of model we consider both unconditional models and models that condition on Farber's and CM's variables that shift demand and influence waiting time: weather (rain, snow, or dry) and time of day (day or night). This yields 18 different *kinds* of reference-point model.

The second dimension distinguishes three *forms* of reference-dependence: with respect to hours, earnings, or both hours and earnings.

The third dimension distinguishes reference-dependent or neoclassical models that do or do not impose additive separability across goods.

7.3 Nonparametric notions of predictive success

The simplest possible measure of a model's predictive success is its pass rate. A model's pass rate for driver i , denoted $r^i \in [0,1]$, is defined as the maximal proportion of the driver's observations that are consistent with the model. A related measure replaces r^i with a model's proximity π^i , defined as one minus the Euclidean distance, rescaled as a proportion of the maximum possible distance, between i 's set of observations and the set of sets of observations that fit the model exactly (Beatty and Crawford 2011, pp. 2786-2787). Like r^i , $\pi^i \in [0,1]$, with higher values for more successful models.

¹⁶ We explicitly compare only static models of reference points, but our theory covers cases where reference points are dynamic, as long as they are modelable.

However, neither measure is adequate for comparing models of varying flexibility. Reference-dependent models are more flexible than neoclassical models and must have pass rates and proximities at least as high. This accounts for much of the profession's skepticism about their parsimony. Even neoclassical models can be highly restrictive or without nonparametric content depending on the number of observations and whether budget lines cross.

To control for flexibility, Selten and Krischker (1983) and Selten (1991) penalize a model's pass rate for flexibility using what they call the model's "area", $a^i \in [0,1]$. The area is the size of the set of all model-consistent sets of observations for driver i , relative to the size of the set of all feasible sets of observations of the same size, or equivalently the probability that uniformly random data are consistent with the model. Noting that successful models have small values of a^i and large values of r^i , Selten and Krischker define a measure of predictive success, $m(r^i, a^i) \equiv r^i - a^i \in [-1,1]$.¹⁷ As $m \rightarrow 1$ a model's restrictions grow tighter but behavior satisfies them: a highly successful model. As $m \rightarrow -1$ a model's restrictions become looser but behavior fails to satisfy them: a pathologically bad model. As $m \rightarrow 0$ a model's compliance approaches random: a harmless but useless model.

Selten and Krischker's all-or-nothing pass rate r^i is not discriminating enough for our application, in which drivers with more than a few trips have little chance of satisfying even a reference-dependent model exactly. Accordingly, we replace their pass rate r^i with Beatty and Crawford's proximity measure π^i , following Beatty and Crawford in continuing to penalize it via Selten and Krischker's area. Thus our proposed measure is $n(\pi^i, a^i) \equiv \pi^i - a^i \in [-1,1]$. Like Selten and Krischker's measure, $n(\pi^i, a^i)$ levels the playing field between more- and less-flexible models in a well-

¹⁷ Selten (1991) shows that three axioms, *monotonicity* $m(1,0) > m(0,1)$; *equivalence of trivial theories* $m(1,1) = m(0,0)$; and *aggregability* $m(\lambda r_1 + (1-\lambda)r_2, \lambda a_1 + (1-\lambda)a_2) = \lambda m(r_1, a_1) + (1-\lambda)m(r_2, a_2)$ for $\lambda \in [0,1]$, characterize the measure $m(r^i, a^i) \equiv r^i - a^i$.

defined, objective way. Both measures are similar in spirit to the adjusted R^2 or the Akaike Information Criteria in structural econometrics, which penalize model fit and likelihood for a model's number of free parameters. From now on we use "Selten measure" loosely for Beatty and Crawford's proximity-based measure of predictive success.

7.4 Estimation procedure

We estimate driver by driver and model by model.¹⁸ For each model we fix whether preferences are additively separable across goods and the form and kind of reference-dependence.¹⁹ The details are in our replication package.

Proposition 5 immediately suggests an estimation procedure:²⁰

- (i) Use the observations' modeled reference points to sort their consumption bundles into gain-loss regimes.
- (ii) Pooling the data from all regimes, use linear programming to find Afriat numbers U_t , v_{t+}^k , v_{t-}^k , and $\lambda_t > 0$ for each $k = 1, \dots, K$ and $t = 1, \dots, T$ that satisfy [A]'s Afriat inequalities (10).

¹⁸ Rather than nesting and estimating the form and kind of reference-dependence we condition on them and compare the resulting models. Nesting and estimating would be computationally complex, in part because the Afriat regime preferences are not invariant to merging regimes.

¹⁹ With regard to additive separability across goods, Debreu's (1960) necessary and sufficient "double cancellation" condition shows that with two goods the Afriat rationalizing regime preferences preclude it in gain-loss regimes with more than one observation. We therefore use Varian's (1983, Theorem 6) linear program, modifying inequalities like Proposition 5's to require separability. For proximities and Selten measures, we design and implement an efficient search algorithm using the fact that the proximity for a separable model cannot exceed that for its non-separable counterpart. Details and code are in our replication package.

²⁰ This description ignores the choice of rationalizing regime preferences for drivers who are reference-dependent on less than K dimensions, or who are neoclassical. But Propositions 3-5 continue to hold, *mutatis mutandis*, for such preferences and our arguments extend to them.

- (iii) Use the fact that for each observation in a regime, (10) can hold with equality for another observation in the regime, to choose numbers so that for observation t in regime g , the rationalizing Afriat utilities are given as in (12) in the proof of Proposition 5:
- (iv) $U_t = u^g(\mathbf{q}_t, \mathbf{r}_t) \equiv \min_{\rho \in \theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)} \{U_\rho + \sum_k [G_+^k(\mathbf{q}_\rho, \mathbf{r}_t) v_{\rho+}^k + G_-^k(\mathbf{q}_\rho, \mathbf{r}_t) v_{\rho-}^k] + \lambda_\rho \mathbf{p}_\rho \cdot (\mathbf{q}_t - \mathbf{q}_\rho)\}.$
- (v) Use (ii)'s Afriat utilities U_t , v_{t+}^k , and v_{t-}^k to check that [B]'s monotonicity restrictions are satisfied.
- (vi) Use (iii)'s rationalizing Afriat utilities to check, regime by regime and observation by observation, that [C]'s conditions (11) are satisfied by scanning along the budget surface.

Proposition 5's conditions (10) involve linear inequalities in a finite number of variables; and its conditions (11) involve nonlinear inequalities in a continuum of $\mathbf{q}$ values. However, both sets of inequalities are finitely parameterized by the U_t , v_{t+}^k , v_{t-}^k , and λ_t that satisfy [A]'s (10).

As with Diewert's (1973) and Varian's (1982) methods for the neoclassical case, our procedure rests on inequality restrictions that can be checked directly in the data, without estimating econometric models of unobservable objects such as demand or labor supply curves. It also largely avoids the need for the auxiliary statistical assumptions that structural econometric approaches require for consistent estimation. Measurement error is an exception, but it too can be handled nonparametrically (Varian 1985).

For computational efficiency, instead of using the estimation procedure directly suggested by Proposition 5, or analogues of Varian's (1982, Appendix I) methods for finding solutions to (10), we use linear programming methods analogous to Diewert's (1973, Section 3) methods for the neoclassical case.

We estimate Selten and Kriskcher's area a^i by checking the conditions for a rationalization repeatedly for random data, as in Beatty and Crawford.²¹ For a neoclassical model we use WARP or, for models that impose additive separability across goods, Varian's (1983, Theorem 6) conditions.²² For a reference-dependent model we use Proposition 5's conditions [A]-[C], with Varian's (footnote 21) modifications for additive separability across goods.

When a model of either type does not fit exactly for driver i , we define its proximity π^i as the Euclidean distance, rescaled as a proportion of the maximum possible distance, between driver i 's set of observations and the set of sets of observations that fit exactly, with the latter estimated in the process of estimating the area a^i (Beatty and Crawford 2011, pp. 2786-87). However, the conditions for fitting a model exactly greatly increase in stringency with the number of observations, and for the drivers with the seven largest sample sizes of the 21 (1, 4, 10, 16, 18, 20, and 21; sample sizes 39 to 70), repeated sampling (up to 20,000 times) yielded no passes. For such drivers we set $\pi^i = 0$, as if we found passing observations only at the maximum possible distance.

Given Propositions 5's gap between the sufficient and necessary conditions for a reference-dependent rationalization, which precludes precise estimation of proximities and Selten measures for reference-dependent models, we bound them as follows.²³ Imposing Proposition 5's within-regime conditions [A] ((10)) and monotonicity conditions [B], but not its cross-regime "no defection" conditions

²¹ We calculate the area by numerical (Monte Carlo) integration over the budget sets. New sets of choices that satisfy the budget constraints are repeatedly drawn and the conditions of interest are tested for each draw. The area is the proportion of those draws that satisfy the conditions. The area estimate converges as the square root of the number of draws. We draw until the uncertainty of the estimate is confined to the fifth decimal place.

²² Because we only need WARP, not GARP, this is easily implemented for the non-additively separable model using R's *igraph* package. Details and code are in our Replication files.

²³ Such bounds are unnecessary for a neoclassical model because GARP is necessary and sufficient for a rationalization without regard to Varian's (1982, Fact 4).

[C] ((11)), yields an approximate (though neither exactly necessary or sufficient) upper bound on the proximity. Because conditions [A] and [B] assume the Afriat gain-loss regime utilities, even if [C] is also satisfied they are sufficient but not necessary, and the true proximity could be higher than the upper bound; and without [C] they are not sufficient either. Similarly, imposing [A]-[C] yields an approximate lower bound on the proximity, which again because they assume the Afriat regime utilities could also be higher than the true lower bound. Approximate lower and upper bounds on a reference-dependent model's Selten measure follow similarly.

7.5 Results

We now summarize our estimation results. To create a comprehensible path through the large number of models considered, we proceed sequentially. We first compare neoclassical and reference-dependent models that impose or relax additive separability across goods. This comparison so strongly favors relaxing separability that that can be seen by looking at aggregate summaries.

Next, relaxing additive separability across goods, we compare reference-dependent models that differ in the kind and form of reference-dependence. Although reference-dependent models differ significantly from neoclassical models for many drivers, the kind and form of reference-dependence make little difference, as we show again via aggregate summaries.

Finally, continuing to relax additive separability across goods, we compare neoclassical and reference-dependent models more comprehensively, first via aggregate summaries and then driver by driver.

7.5a *Additive separability across goods*

Additive separability across goods has been assumed in all previous theoretical and empirical work on this topic, but it lacks theoretical or empirical justification, and Proposition 3's characterization of reference-dependent preferences with constant sensitivity shows that it is unnecessary.

Figures 6-9 give the empirical cumulative distribution functions ("CDFs") of proximities and Selten measures for neoclassical and reference-dependent

models that impose or relax additive separability across goods.²⁴ Each CDF pools over all 21 drivers. For reference-dependent models each CDF also pools over all 18 kinds and three forms of reference-dependent model.

Figure 6: Empirical CDFs of Proximities for Neoclassical Models
Imposing and Relaxing Additive Separability Across Goods

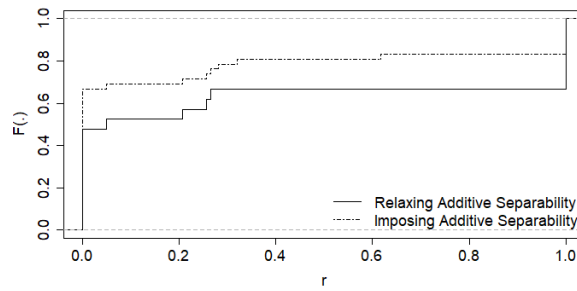
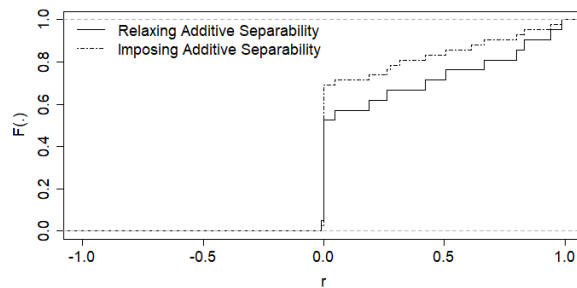


Figure 7: Empirical CDFs of Selten Measures for Neoclassical Models
Imposing and Relaxing Additive Separability Across Goods



Figures 6-7 show that neoclassical models that relax additive separability across goods have higher proximities and Selten measures than models that impose it. These aggregate summaries don't show for how many drivers

²⁴ These comparisons also relax KR's constant-sensitivity constraints on how marginal rates of substitution vary across gain-loss regimes. A reference-dependent model must have at least as high a proximity as its neoclassical counterpart, but its Selten measure could be higher or lower. There is a minor exception for experience-based reference-point models, in which we lose one observation (two for models that condition on something) due to the construction of the lag. This can yield a slightly higher upper proximity bound for the neoclassical model.

relaxing additive separability improves a neoclassical model's fit enough to justify the added flexibility but the gap in Selten measures is large enough to confirm that relaxing separability is preferable for neoclassical models.

Figures 8-9 show that reference-dependent models that relax additive separability across goods also have significantly higher proximities and Selten measures than models that impose it. Figure 9's gap in Selten measures is large enough to confirm that relaxing separability is also preferable for reference-dependent models. From now on, we set aside models that impose additive separability across goods and focus on models that relax it.

Figure 8: Empirical CDFs of Proximities for Reference-dependent Models Imposing and Relaxing Additive Separability Across Goods

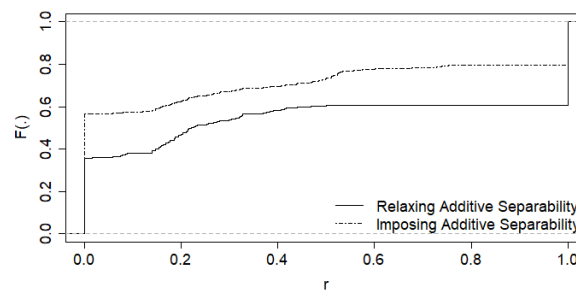
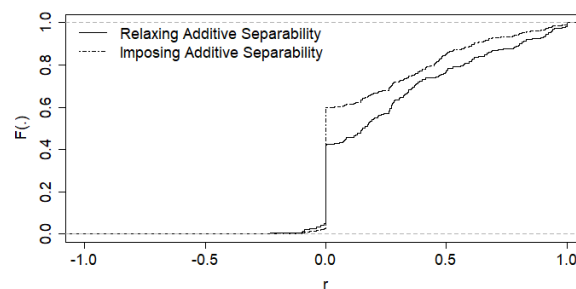


Figure 9: Empirical CDFs of Selten Measures for Reference-dependent Models Imposing and Relaxing Additive Separability Across Goods



7.5b. Reference-point models

Table B.6's paired tests (discussed further in Section 7.5c) show that in these data there are comparatively small differences among models' kinds and

forms of reference-dependence. Expectations-based models usually have higher Selten measures than experience-based models, and unconditioned expectations-based models have measures almost as high as conditioned ones; although expectations-based models that are conditioned on day/night usually have even higher Selten measures. Expectations-based models with hours- and earnings-targeting have measures approximately as high as such models with only hours-targeting and, surprisingly, somewhat higher measures than such models with only earnings-targeting.

Accordingly, from now on we focus on unconditioned expectations-based models (also relaxing additive separability across goods). But we also report results for unconditioned experience-based reference-point models, in each case considering all three forms of reference-dependence.

7.5c Comparing neoclassical and reference-dependent models

Figures 10 and 11 give the empirical CDFs of proximities and Selten measures for neoclassical versus expectations-based reference-dependent models, pooling over drivers and kinds and forms of reference-dependent model. In these aggregate summaries, neoclassical models have higher Selten measures than reference-dependent models for measure values from 0 to 0.5, but slightly lower Selten measures for values from 0.5 to 1.0, so the comparison is inconclusive, we believe due mainly to driver heterogeneity.

Figure 10: Empirical CDFs of Proximities
for Neoclassical and Reference-dependent Models

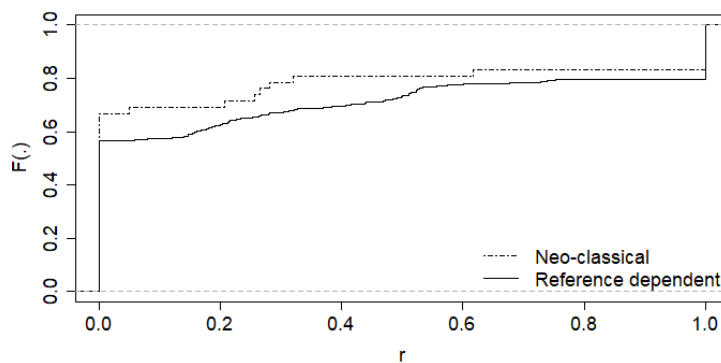
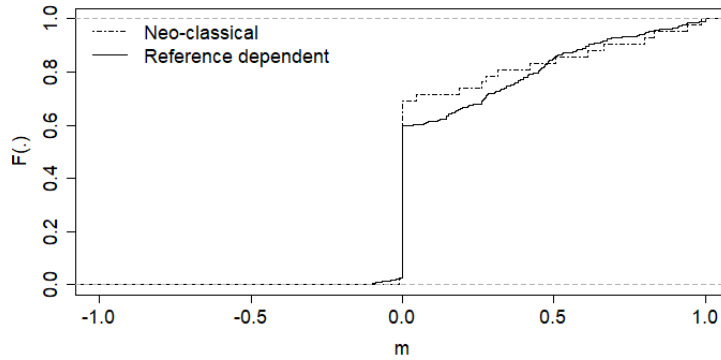


Figure 11: Empirical CDFs of Selten Measures
for Neoclassical and Reference-dependent Models



Figures 12-15 give driver-by-driver plots for neoclassical and expectations-based and experience-based reference-dependent models' proximities and Selten measures. (The Supplemental Appendix's Tables B2-B5 give the precise numerical values behind the plots.) Each figure has separate plots for different forms of reference-dependence, with a separate "spoke" for each driver. Figures 12's and 14's proximity plots are centered at -0.25, for clarity a tick below the lowest possible value of 0; with outer rims at the highest possible value of 1. The solid lines trace proximities for the neoclassical model. The shaded areas depict Section 7.3's approximate bounds on the proximities for the reference-dependent models. Figures 13's and 15's Selten measure plots are also centered at -0.25, with outer rims at the highest possible value of 1. The solid lines trace measures for the neoclassical model.

Figure 12. Proximities for neoclassical and unconditional expectations-based reference-dependent models

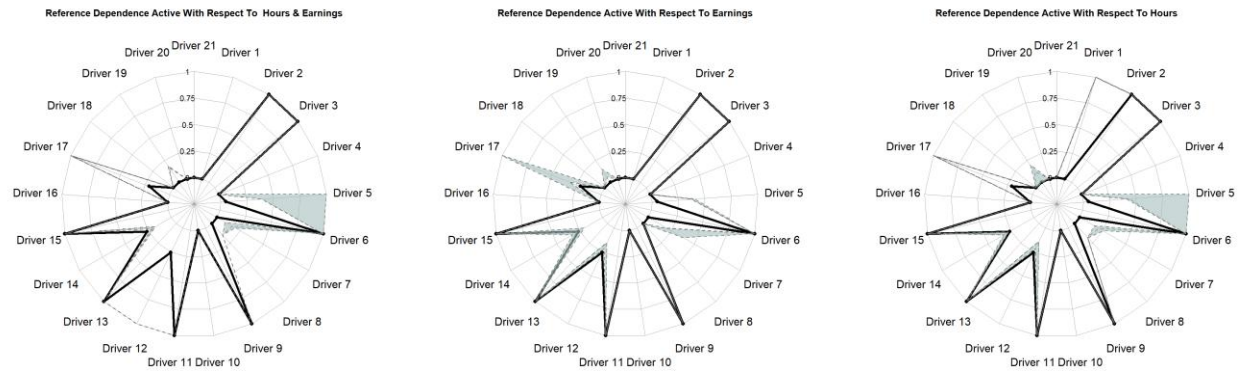


Figure 13. Selten measures for neoclassical and unconditional expectations-based reference-dependent models

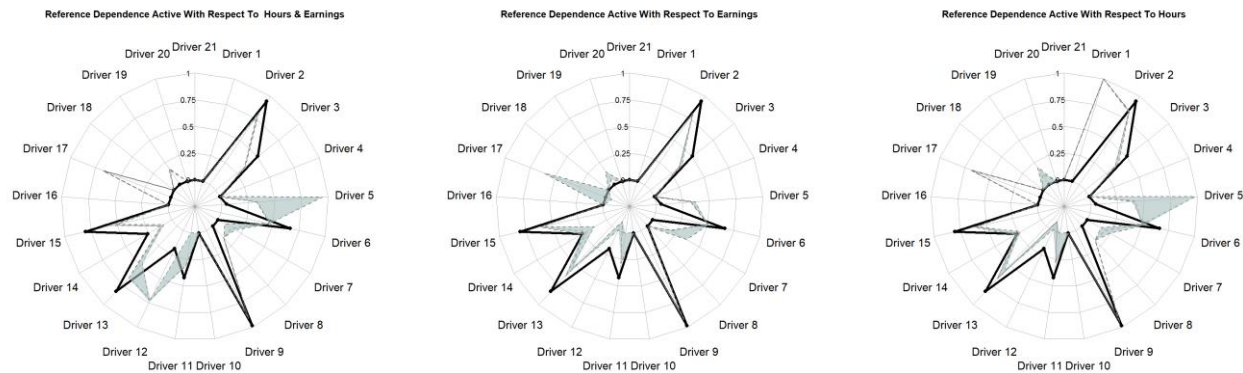


Figure 14. Proximities for neoclassical and unconditional experience-based reference-dependent models

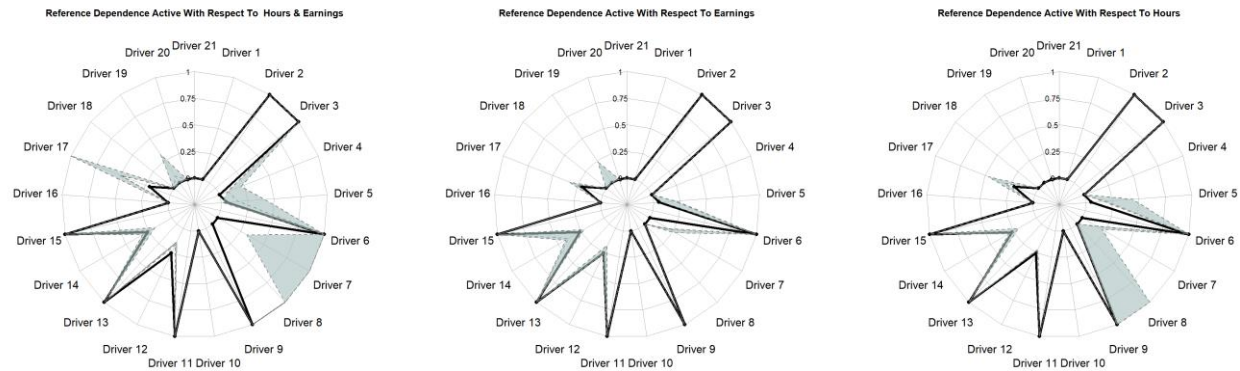
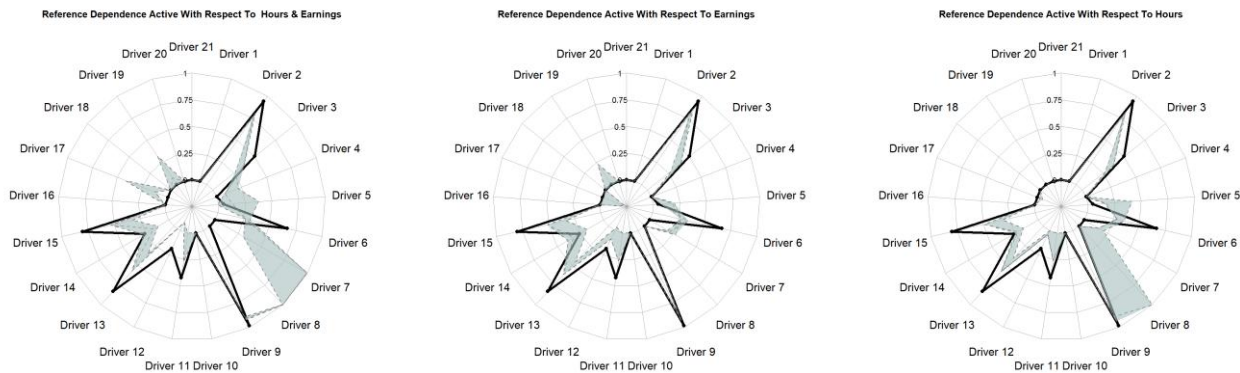


Figure 15. Selten measures for neoclassical and unconditional experience-based reference-dependent models



Overall, the qualitative model comparisons differ only slightly across forms of reference-dependence. Our discussion focuses on models with reference-dependence in both hours and earnings, with plots in the left-most panels.

Drivers are heterogeneous, and neither model has uniformly higher Selten measures. In Figure 13, the expectations-based reference-dependent model has the same bounded measure as the neoclassical model (thus possibly higher, Section 7.3) for seven of 21 drivers (1, 4, 10, 16, 18, 20, and 21); an unambiguously higher measure for six (5, 7, 8, 12, 17, and 19); and an unambiguously lower measure for eight (2, 3, 6, 9, 11, 13, 14, and 15). Similarly, in Figure 15, the experience-based reference-dependent model has the same (possibly higher) bounded measure as the neoclassical model for six drivers (1, 10, 16, 18, 20, and 21); a higher measure for four (4, 8, 17, and 19); a lower measure for nine (2, 3, 6, 9, 11, 12, 13, 14, and 15); and ambiguous bounds for two (5 and 7).

However, not all drivers' comparisons are equally informative. With our CPI adjustment, all but one of the six drivers Farber and CM excluded due to small (≤ 10) sample sizes (3, 6, 11, 13, 15, and 17) has an exact neoclassical fit, so that a neoclassical model necessarily has a higher Selten measure than its more flexible reference-dependent counterpart. That favors a neoclassical model, but it might only be an artifact of overfitting.

For the seven drivers with the largest samples (1, 4, 10, 16, 18, 20, and 21), we are computationally unable to estimate the set of sets of observations that fit exactly (Monte Carlo simulation found no random datasets that satisfy the conditions for either model). For those drivers we set the proximities to 0 for both models, so that a neoclassical model again necessarily has a higher Selten measure. That does not truly favor a neoclassical over a reference-dependent model.

For the eight remaining drivers (2, 5, 7, 8, 9, 12, 14, and 19), an expectations-based model with reference-dependence in hours and earnings has a higher Selten measure for five drivers (5, 7, 8, 12, and 19) and a neoclassical model has a higher measure for three (2, 9, and 14).²⁵

We test whether the mean difference in Selten measures between the neoclassical and reference-dependent models is significantly different from zero using paired *t*-tests, pooling drivers, following Demuynck (2015), who shows that differences in such measures across models are asymptotically normal when each dataset is treated as an independent draw. The Supplemental Appendix's Table B6 reports the results for unconditional expectations-based and experience-based reference-dependent models with reference-dependence active in hours alone, earnings alone, and both hours and

²⁵ Similarly, for the experience-based model, the model with reference-dependence in hours and earnings has a Selten measure that is higher than for the neoclassical model for four drivers, (7, 8, 14, and 19), and lower for four drivers, (2, 5, 9, and 12).

earnings. We report results both for all 21 drivers and for the subset for whom we were able to estimate proximities and Selten measures. The Supplemental Appendix's Figures B2 and B3 show the driver-by-driver differences for the hours-and-earnings specification.

For the expectations-based models, the mean differences in Selten measures are positive for both the hours-and-earnings and hours-only specifications (0.029 and 0.051, respectively), indicating that the reference-dependent model outperforms the neoclassical model on average. However, neither difference is statistically significant at the 5% level ($p = 0.58$ and $p = 0.45$). The earnings-only specification shows a smaller, negative difference (-0.010), which is again statistically insignificant ($p = 0.82$). These results are qualitatively unchanged when the seven drivers for whom we are unable to estimate proximities and Selten measures are excluded.

For the experience-based models, the hours-and-earnings specification again shows a positive but insignificant mean difference (0.038, $p = 0.63$). The earnings-only and hours-only specifications show negative mean differences, with the hours-only specification reaching significance (-0.064 , $p = 0.04$; -0.112 , $p = 0.03$ for the non-degenerate subset), indicating that for this specification a neoclassical model has significantly higher predictive success than a reference-dependent model.

Overall, neither model is unambiguously superior but ruling out reference-dependence a priori is far from justified.

8 Conclusion

This paper presents a nonparametric analysis of the theory of consumer demand or, equivalently, labor supply with reference-dependent preferences. Our nonparametric model closely follows KR's structural model, maintaining their assumption that preferences are additively separable across components of consumption and gain-loss utility but relaxing KR's other assumptions on functional structure, as well as all assumptions on functional form.

Under our assumptions, the refutable implications of a reference-dependent model of consumer demand are limited by two factors: Unless reference points are precisely modelable and the standard notion of sensitivity is constant, a reference-dependent model can rationalize virtually any data.

With constant sensitivity, KR's functional-structure assumptions raise important new issues. They then imply that the sum of consumption and gain-loss utility that determines demand must be additively separable across goods; and that its marginal rates of substitution must vary across gain-loss regimes in a particular, knife-edge way (CM, Table 1). Those restrictions are largely unsupported but have been maintained, without testing, in all applications to date.

Assuming modelable reference points, we pave the way to positive results by characterizing continuous reference-dependent preferences that satisfy constant sensitivity. We identify the most general class of well-behaved reference-dependent utility functions that can have nonparametrically refutable implications for consumer demand, relaxing KR's constant-sensitivity restrictions.

Our characterization could be used to conduct more general structural econometric analyses in a wide range of applications, combining our flexible functional-structure assumptions with standard functional-form assumptions regarding consumption and gain-loss utility; and modelling reference points via sample proxies like Camerer et al.'s and CM's.

However, in this paper we continue our nonparametric analysis, using our characterization to derive sufficient conditions for a reference-dependent rationalization, which conditions should be asymptotically necessary with rich enough data. Our analysis makes essential use of rationality, but within the daily bracket and over a preference domain expanded in the disciplined way suggested by reference-dependence. It requires substantial generalizations of neoclassical nonparametric analyses, because levels of and changes in a good's consumption are by definition bundled and priced together, and because a reference-dependent consumer can, in effect, alter her/his preferences by buying a consumption bundle in a different gain-loss regime.

We illustrate the use of our conditions by reconsidering Farber's (2005, 2008) and Crawford and Meng's (2011) analyses of cabdrivers' labor supply, using Farber's data. Although the GARP condition for a neoclassical rationalization is violated for most of Farber's drivers, our methods yield reference-dependent rationalizations for almost all of most drivers' choices. For some drivers, reference-dependent models that relax KR's strong constant-sensitivity restrictions have higher Selten measures of predictive success than their counterparts imposing the restrictions, suggesting that our characterization's generalization is empirically important. By contrast with the focus on earnings-targeting in previous work, hours-targeting is more important than earnings-targeting.

Neither a relaxed neoclassical nor a relaxed reference-dependent model is unambiguously superior in the aggregate. But Farber's drivers are highly heterogeneous, and for a significant fraction of them a relaxed reference-dependent model has a significantly higher Selten measure of predictive success than a relaxed neoclassical model. This suggests that reference-dependent models of consumer demand are a useful addition to the consumer-demand toolkit, which might allow parsimonious, rationality-based explanations of otherwise puzzling behavior in labor supply, consumer demand, housing, finance, and beyond.

Acknowledgements

We are especially grateful to Ian Crawford, whose contributions to the current version were extremely valuable. We are also grateful for valuable advice to Abi Adams, Richard Blundell, Alec Brandon, Colin Camerer, David Cooper, Stefano DellaVigna, Ernst Fehr, David Freeman, Georgios Gerasimou, Botond Köszegi, John List, Juanjuan Meng, Robert Moffit, Adam Sanjurjo, Ricardo Serrano-Padial, Matthew Rabin, Joel Sobel, and Richard Thaler.

Funding

Blow's work was supported by the European Research Council under grant ERC-2009-StG-240910-ROMETA and the Economic and Social Research Council's Centre for Microeconomic Analysis of Public Policy at the Institute for Fiscal Studies under grant RES-544-28-5001. Crawford's work was supported by All Souls College, the University of Oxford, and the European Research Council under grant ERC-2013-AdG 339179-BESTDECISION. The contents reflect only our views and not those of the E.R.C. or the European Commission, and the European Union is not liable for any use that may be made of the information contained herein.

Conflict of interest

The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

References

- Abdellaoui, Mohammed, Han Bleichrodt, and Corina Paraschiv. 2007. "Loss Aversion under Prospect Theory: A Parameter-Free Measurement." *Management Science* 53 (10): 1659-1674.
- Afriat, Sidney N. 1967. "The Construction of a Utility Function from Expenditure Data." *International Economic Review* 8 (1): 76-77.
- Andersen, Steffen, Cristian Badarinza, Lu Liu, Julie Marx, and Tarun Ramadorai. 2022. "Reference Dependence in the Housing Market." *American Economic Review*, 112 (10): 3398–3440.
- Barberis, Nicholas. 2013. "Thirty Years of Prospect Theory in Economics: A Review and Assessment." *Journal of Economic Perspectives*, 27 (1): 173-196.
- Barberis, Nicholas, and Richard Thaler. 2003. "A Survey of Behavioral Finance." Chapter 18 in G.M. Constantinides, M. Harris and R. Stulz, eds., *Handbook of the Economics of Finance*, Elsevier Science B.V.: 1051-1121.

- Beatty, Timothy, and Ian Crawford. 2011. "How Demanding is the Revealed Preference Approach to Demand?" *American Economic Review* 101 (6): 2782-2795.
- Blow, Laura, Ian Crawford, and Vincent P. Crawford. 2022. "Meaningful Theorems: Nonparametric Analysis of Reference-dependent Preferences," 19 January 2022; <http://econweb.ucsd.edu/~v2crawford/19Jan22BlowCrawCrawMain.pdf>.
- Brandon, Alec, Colin F. Camerer, John A. List, Ian Muir, and Jenny Wang. 2023. "Evaluating the Evidence of Daily Income Targeting with Experimental and Observational Data," 8 April 2023; <https://drive.google.com/file/d/1-IVFHudnbxwG7x92okTaV49D2SsMLRlM/view?usp=sharing>.
- Camerer, Colin, Linda Babcock, George Loewenstein, and Richard Thaler. 1997. "Labor Supply of New York City Cabdrivers: One Day at a Time." *Quarterly Journal of Economics* 112 (2): 407-441.
- Crawford, Vincent P., Miao Jin, Juanjuan Meng, and Lan Yao. 2025. "Expectations-based Reference-Dependence and Labor Supply: Eliciting Cabdrivers' Expectations in the Field." *Journal of Economic Behavior and Organization*, 239: 107259.
- Crawford, Vincent P., and Juanjuan Meng. 2011. "New York City Cabdrivers' Labor Supply Revisited: Reference-Dependent Preferences with Rational-Expectations Targets for Hours and Income." *American Economic Review* 101 (5): 1912-1932.
- Debreu, Gerald. 1960. "Topological Methods in Cardinal Utility." 16-26 in Kenneth J. Arrow, Samuel Karlin, and Patrick Suppes (eds.). *Mathematical Methods in the Social Sciences*, Stanford, CA: Stanford University Press.
- Demuyne, Thomas. 2015. "Statistical Inference for Measures of Predictive Success," *Theory and Decision* 79: 689-699.
- Diewert, W. Erwin. 1973. "Afriat and Revealed Preference Theory." *Review of Economic Studies* 40 (3): 419-426.
- Farber, Henry S. 2005. "Is Tomorrow Another Day? The Labor Supply of New York City Cabdrivers." *Journal of Political Economy* 113 (1): 46-82.
- Farber, Henry S. 2008. "Reference-Dependent Preferences and Labor Supply: The Case of New York City Taxi Drivers." *American Economic Review* 98 (3): 1069-1082.
- Farber, Henry S. 2015. "Why You Can't Find a Taxi in the Rain and Other Labor Supply Lessons from Cab Drivers." *Quarterly Journal of Economics* 130: 1975-2026.
- Fehr, Ernst, and Lorenz Goette. 2007. "Do Workers Work More if Wages are Higher? Evidence from a Randomized Field Experiment." *American Economic Review* 97 (1): 298-317.

- Forges, Françoise, and Enrico Minelli. 2009. "Afriat's Theorem for General Budget Sets." *Journal of Economic Theory* 144 (1): 135-145.
- Freeman, David. 2017. "Preferred Personal Equilibrium and Simple Choices." *Journal of Economic Behavior and Organization* 143: 165–172.
- Freeman, David. 2019. "Expectations-Based Reference-Dependence and Choice under Risk." *Economic Journal* 129 (622): 2424–2458.
- Genesove, David and Christopher Mayer. 2001. "Loss Aversion and Seller Behavior: Evidence from the Housing Market." *Quarterly Journal of Economics* 116 (4): 1233-1260.
- Gul, Faruk, and Wolfgang Pesendorfer. 2006. "The Revealed Preference Implications of Reference Dependent Preferences." <https://www.princeton.edu/~pesendor/refdep.pdf> .
- Hardie, Bruce G. S., Eric J. Johnson, and Peter S. Fader. 1993. "Modeling Loss Aversion and Reference Dependence Effects on Brand Choice," *Marketing Science*, 12 (4): 378-394.
- Houthakker, Hendrik. S. 1950. "Revealed Preference and the Utility Function." *Economica* 17 (66): 159-174.
- Kahneman, Daniel, and Amos Tversky. 1979. "Prospect Theory: An Analysis of Decision under Risk." *Econometrica* 47 (2): 263-292.
- Kibris, Özgür, Yusufcan Masatlioglu, and Elchin Suleymanov. 2021. "A Theory of Reference Point Formation." *Economic Theory*, 75 (3): 137-166.
- Kőszegi, Botond, and Matthew Rabin. 2006. "A Model of Reference-Dependent Preferences." *Quarterly Journal of Economics* 121 (4): 1133-1165.
- Masatlioglu, Yusufcan, and Collin Raymond. 2016. "A Behavioral Analysis of Stochastic Reference Dependence." *American Economic Review*, 106 (9): 2760-2782.
- Mas-Colell, Andreu. 1978. "On Revealed Preference Analysis." *Review of Economic Studies* 45 (1): 121-131.
- Meng, Juanjuan and and Xi Weng. 2018. "Can Prospect Theory Explain the Disposition Effect? A New Perspective on Reference Points." *Management Science* 64 (7): 3331-3351.
- Nishimura, Hiroki, Efe A. Ok, and John K.-H. Quah. 2017. "A Comprehensive Approach to Revealed Preference Theory." *American Economic Review*, 107 (4): 1239-1263.
- Odean, Terence. 1998. "Are Investors Reluctant to Realize Their Losses?" *Journal of Finance*, 53 (5): 1775–1798.
- Oettinger, Gerald S. 1999. "An Empirical Analysis of the Daily Labor Supply of Stadium Vendors." *Journal of Political Economy* 107 (2): 360-392.

- Ok, Efe A., Pietro Ortoleva, and Gil Riella. 2015. "Revealed (P)Reference Theory." *American Economic Review* 105 (1): 299-321.
- Samuelson, Paul A. 1948. "Consumption Theory in Terms of Revealed Preference." *Economica* 15 (60): 243-253.
- Selten, Reinhard. 1991. "Properties of a Measure of Predictive Success." *Mathematical Social Sciences* 21 (2): 153-167.
- Selten, Reinhard, and Wilhelm Krischker. 1983. "Comparison of Two Theories for Characteristic Function Experiments." 259-264 in Reinhard Tietz (ed.). *Aspiration Levels in Bargaining and Economic Decision Making*. Berlin: Springer.
- Thakral, Neil, and Linh T. Tô. 2021. "Daily Labor Supply and Adaptive Reference Points." *American Economic Review* 111 (8): 2417–2443.
- Tversky, Amos, and Daniel Kahneman. 1991. "Loss Aversion in Riskless Choice: A Reference-Dependent Model." *Quarterly Journal of Economics* 106 (4): 1039-1061.
- Uzawa, Hirofumi. 1960. "Preferences and Rational Choice in the Theory of Consumption." 1229-148 in Kenneth Arrow, Samuel Karlin, and Patrick Suppes (eds.). *Mathematical Methods in the Social Sciences*. Stanford, CA: Stanford University Press.
- Van Bruggen, Paul, and Jan Heufer. 2017. "Afriat in the Lab." *Journal of Economic Theory* 169: 546–550.
- Varian, Hal R. 1982. "The Nonparametric Approach to Demand Analysis." *Econometrica* 50 (4): 945-973.
- Varian, Hal R. 1983. "Nonparametric Tests of Consumer Behaviour." *Review of Economic Studies* 50 (1): 99-110.
- Varian, Hal R. 1985. "Non-Parametric Analysis of Optimizing Behavior with Measurement Error." *Journal of Econometrics* 30 (1-2): 445-458.
- Varian, Hal R. 1988. "Revealed Preference with a Subset of Goods," *Journal of Economic Theory* 46 (1): 179-185.

Supplemental Appendix

A. Proofs of Proposition 4 and its Corollary

Proof of Proposition 4: That [B] implies [A] is immediate. To prove that [A] implies [B], take the rationalizing regime preferences represented by $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$, which satisfy (8). Use Proposition 3 to write the condition preventing defections from the bundle of observation $\tau \in \Theta(\{\mathbf{q}_t, \mathbf{r}_t\}_{t=1, \dots, T}; g)$ in regime g to a bundle $\mathbf{q} \in \Gamma(g'; \mathbf{r}_\tau)$ in regime $g' \neq g$ for $\mathbf{r}_\tau$ with $\mathbf{p}_\tau \cdot \mathbf{q} \leq \mathbf{p}_\tau \cdot \mathbf{q}_\tau$:

$$\begin{aligned}
 u(\mathbf{q}, \mathbf{r}_\tau) - U(\mathbf{r}_\tau) &\equiv U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau) \{v_+^k(q^k) - v_+^k(r_\tau^k)\} \\
 &\quad + G_-^k(\mathbf{q}, \mathbf{r}_\tau) \{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\mathbf{r}_\tau) \\
 &\equiv \{U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau) v_+^k(q^k) + G_-^k(\mathbf{q}, \mathbf{r}_\tau) v_-^k(q^k)]\} \\
 &\quad - \{U(\mathbf{r}_\tau) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau) v_+^k(r_\tau^k) + G_-^k(\mathbf{q}, \mathbf{r}_\tau) v_-^k(r_\tau^k)]\} \\
 &\leq \{U(\mathbf{q}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) v_-^k(q_\tau^k)]\} \\
 &\quad - \{U(\mathbf{r}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_+^k(r_\tau^k) - v_+^k(q_\tau^k)\} + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_-^k(r_\tau^k) - v_-^k(q_\tau^k)\}]\} \\
 &\equiv U(\mathbf{q}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}] \\
 &\equiv u(\mathbf{q}_\tau, \mathbf{r}_\tau) - U(\mathbf{r}_\tau).
 \end{aligned}$$

The central inequality can then be rearranged to yield (9). ■

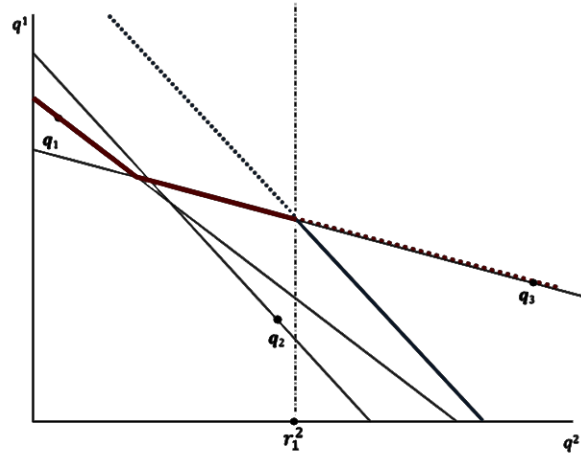
Proof of Proposition 4's Corollary: As in Proposition 4, by Afriat's Theorem, the hypothesized combination of preferences over bundles with consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$ prevent defections from any observation's consumption bundle to any affordable bundle in the same own gain-loss regime. If the hypothesized preferences are such that there are no observations t for which $\mathbf{q}_t$ is *not* on the boundary of the convex hull of the better-than- $\mathbf{q}_t$ set for the candidate global preference ordering given $\mathbf{r}_t$, then we can assume that they satisfy loss aversion without loss of generality. The candidate global ordering can then be replaced by a convexified ordering whose better-than- $\mathbf{q}_t$ sets are the convex hulls of the candidate global ordering, without changing any observation's optimal bundle. Definition 2 then implies that $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$ also prevent defections from any observation's bundle to any affordable bundle in a different regime. Alternatively, consider a defection from $\mathbf{q}_\tau \in \Gamma(g; \mathbf{r}_\tau)$ to some $\mathbf{q} \in \Gamma(g'; \mathbf{r}_\tau)$ with $g' \neq g$ and $\mathbf{p}_\tau \cdot \mathbf{q} \leq \mathbf{p}_\tau \cdot \mathbf{q}_\tau$. If $\mathbf{q}$ were in regime g , we would have, by Afriat's Theorem,

$$\begin{aligned}
 &U(\mathbf{q}) + \sum_k [G_+^k(\mathbf{q}, \mathbf{r}_\tau) \{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\mathbf{q}, \mathbf{r}_\tau) \{v_-^k(q^k) - v_-^k(r_\tau^k)\}] \\
 &\leq U(\mathbf{q}_\tau) + \sum_k [G_+^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\mathbf{q}_\tau, \mathbf{r}_\tau) \{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].
 \end{aligned}$$

Given that $\mathbf{q}$ is actually in regime g' , the interpretation of loss aversion in terms of marginal rates of substitution implies that the left-hand side is lower or at least no higher than if $\mathbf{q}$ were in regime g . That prevents defections from $\mathbf{q}_\tau$ to affordable bundles in different regimes. ■

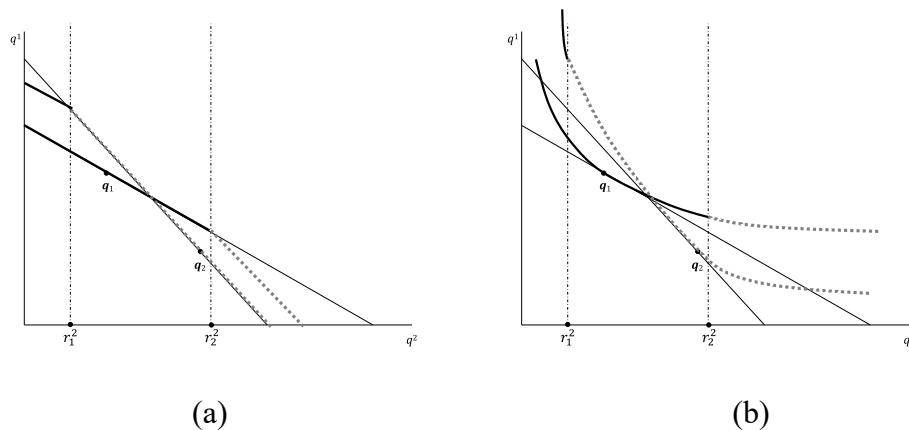
Figure A1 shows that when GARP is satisfied within each gain-loss regime, although the Corollary shows that loss aversion is then sufficient for a reference-dependent rationalization, it is not necessary. In Figure A1 the entire dataset violates GARP, the Afriat gain-loss regime preferences violate loss aversion, but the data satisfy the Corollary's final conditions, thus allowing a rationalization. Only reference point $\mathbf{r}_1$ is shown and observation 1 is in the good-2 loss regime. Assume that $\mathbf{r}_2 = [0, 0]$, so that observation 2's budget set is entirely in the good-2 gain regime; and that $\mathbf{r}_3 = [0, m]$, where m is large enough that observation 3's budget set is entirely in the good-2 loss regime. The Afriat regime preferences yield a candidate for global preferences that make all three observations' consumption bundles optimal: Observations 2's and 3's budget sets are entirely in their regimes (good-2 gain and good-2 loss, respectively), so their bundles' optimality in their regimes suffices for global optimality. Observation 1's bundle is optimal for its good-2 loss regime preferences and Corollary 1 ensures that its bundle's optimality extends to its entire budget set.

Figure A1. Rationalizing data that violate GARP when preferences violate loss aversion but satisfy the Corollary's sufficient conditions for a rationalization
(solid curves for active parts of indifference maps, dashed for inactive parts)



Proposition 5 states conditions under which the data can be rationalized by a reference-dependent model with regime utility functions that satisfy Afriat's inequalities.²⁶ As other choices might also suffice, its sufficient conditions are not necessary. Figure A2 shows that a rationalization may *require* non-Afriat rationalizing regime utilities. For example, the Afriat regime preferences in Figure A2a do not yield a rationalization but the non-Afriat regime preferences in Figure A2b do.

Figure A2. A rationalization may require non-Afriat rationalizing regime preferences
(solid lines for the loss map, dashed for the gain map)



B. Supplementary Notes on Data and Model Comparisons

Like CM, we use Farber's (2005, 2008) data.²⁷ As explained in the text, Farber collected 593 trip sheets for 13461 trips by 21 drivers between June 1999 and May 2001. Each sheet records the driver's name, hack number, date, each fare's start time and location, each fare's end time and location, and the fare paid. Nine sheets duplicate the day and driver, so there are only 584 shifts. Because our methods make some allowance for sample size, in addition to the 15 drivers Farber and CM studied we include the 6 with samples of 10 or fewer shifts they excluded.

Table B1 reports descriptive statistics driver by driver. The values are the same as those in Farber's (2005) Table B1, except for the hourly wage variable and the Afriat efficiencies in the last two columns. Our earnings and wage variables differ from Farber's and CM's in two ways, which affect the Afriat efficiencies. First, we use the NY/NJ urban CPI to control for price level changes in

²⁶ Varian's (1982, Fact 4) bounds for the neoclassical case don't imply that all rationalizing preferences are convex, and examples not reproduced here show that requiring convexity in Proposition 4 would involve a loss of generality. Proposition 5 avoids this issue difficulty by using the Afriat regime preferences, which are convex by construction.

²⁷ The datasets are posted at https://www.aeaweb.org/aer/data/june08/20030605_data.zip, and https://www.aeaweb.org/aer/data/aug2011/20080780_data.zip. The CPI data are posted at <https://data.bls.gov/timeseries/CUURS12ASA0>, under the years 1999-2001.

the sample period. Second, Farber's and CM's wage variable is income per hour spent working, with working time defined as the sum of time driving with a passenger and time waiting for the next passenger. As explained in the text, we treat waiting time as an exogenous fixed cost and define the wage as earnings per hour driving.

Table B1: Descriptive statistics, driver by driver

	T	Working Hours	Driving Hours	Waiting Hours	Break Hours	Earnings (\$/CPI)	Wage (\$/hr)	Afriat Efficiency
Driver 1	39	6.85	4.32	2.53	0.90	153.01	36.41	0.9952
Driver 2	14	3.89	2.78	1.11	2.41	95.98	34.68	1
Driver 3	6	6.66	4.61	2.05	0.74	160.07	36.19	1
Driver 4	40	6.28	4.52	1.76	0.39	145.89	33.02	0.9978
Driver 5	23	6.46	3.98	2.48	2.11	144.00	38.12	0.9971
Driver 6	6	8.62	6.48	2.14	2.42	202.71	33.49	1
Driver 7	24	6.47	4.42	2.05	0.74	159.50	36.69	0.9991
Driver 8	37	7.78	5.13	2.64	0.86	170.33	34.23	0.9897
Driver 9	19	7.17	5.47	1.70	0.54	158.82	30.61	1
Driver 10	45	6.35	3.90	2.45	1.65	129.68	33.83	0.9954
Driver 11	6	7.15	5.22	1.93	0.71	182.40	35.50	1
Driver 12	13	6.15	4.03	2.13	0.55	155.57	39.44	0.9972
Driver 13	10	7.03	4.72	2.31	0.53	153.99	33.26	1
Driver 14	17	7.06	4.49	2.57	0.64	157.15	37.37	0.9930
Driver 15	8	10.82	7.64	3.17	0.19	217.29	29.92	1
Driver 16	70	6.84	4.56	2.28	0.93	163.56	37.72	0.9936
Driver 17	10	5.88	3.71	2.17	0.54	137.28	39.10	0.9946
Driver 18	72	8.53	5.84	2.69	0.60	194.88	35.07	0.9849
Driver 19	33	6.91	4.63	2.29	0.97	155.65	36.01	0.9870
Driver 20	46	7.10	4.80	2.30	0.67	148.76	32.73	0.9842
Driver 21	46	5.32	3.66	1.66	0.24	123.57	35.62	0.9915

Table B1's last column reports each driver's Afriat efficiency index. The index is 1 for a driver whose data satisfy WARP; otherwise less than 1. Only seven of Farber's 21 drivers satisfy WARP. Except for drivers 12 (sample size 13), 14 (sample size 17), and 17 (sample size 10), the drivers with exact neoclassical fits (2, 3, 6, 9, 11, 13, and 15) are the ones with the smallest sample sizes among the 21 drivers. Except for drivers 2 (sample size 14), 9 (sample size 19), and 17 (sample size 10), those drivers are the same as the six (3, 6, 11, 13, 15, and 17) Farber and CM excluded due to small (≤ 10) sample sizes. Small samples make it easier to satisfy WARP by chance, and those drivers' data may simply be too under-powered to reject the neoclassical model. Discuss the issue of correcting for power to reject in Section 7.3.

Figure B1 shows that with our wage definition drivers' budget lines cross frequently, making WARP a meaningful restriction and allowing a nonparametric analysis to provide a meaningful interpretation of the data.

Tables B2-B5 give the precise numerical values behind the driver-by-driver plots for neoclassical and expectations-based and experience-based reference-dependent models' proximities and Selten

measures in Figures 18-21. The Selten measures are Section 7.4's lower bounds, the estimates imposing Proposition 5's full conditions [A]-[C].

Figure B1. Farber's (2005, 2008) drivers' hours and earnings choices

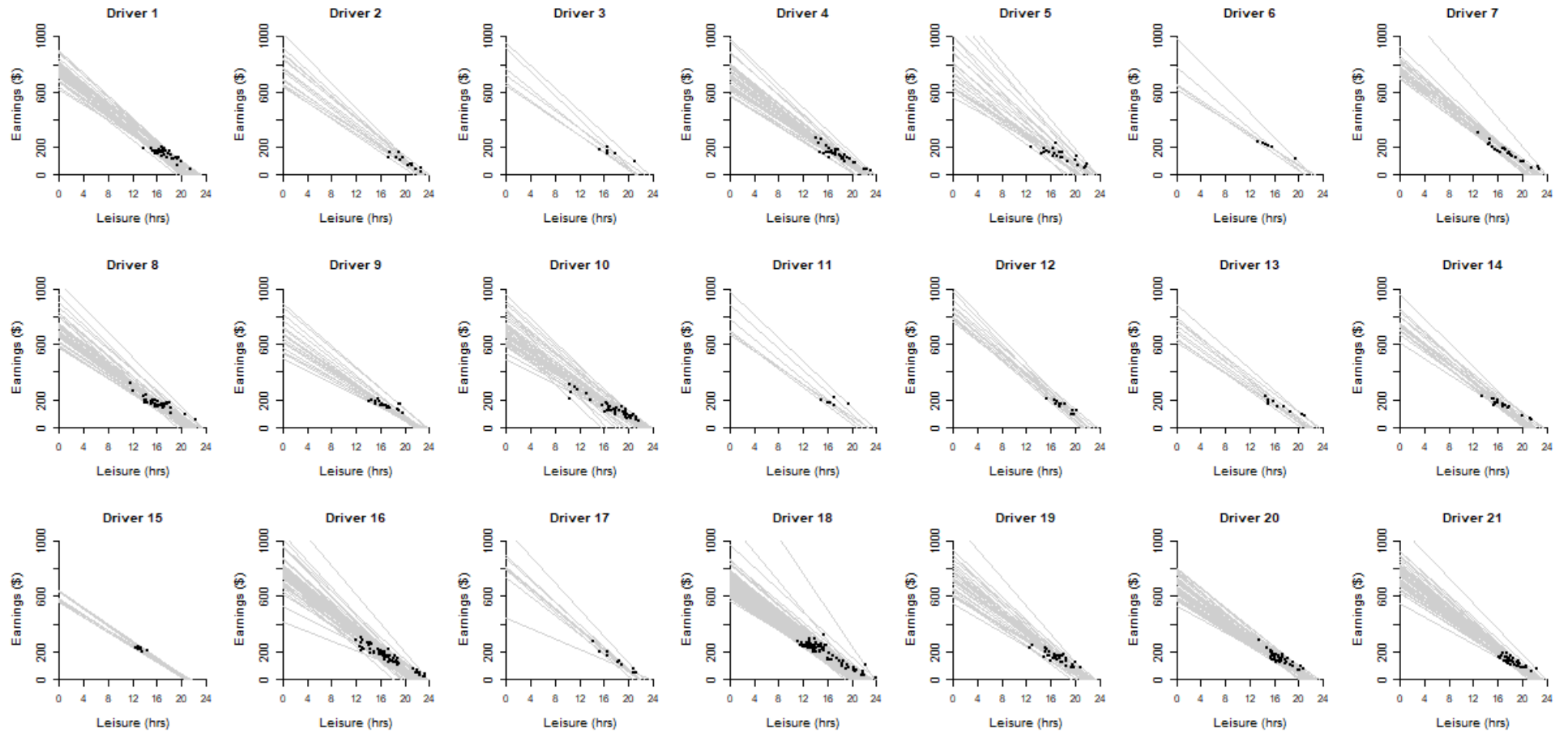


Table B2. Proximities for neoclassical and unconditional expectations-based
reference-dependent models

	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	1
Driver 2	14	1	1	1	1
Driver 3	6	1	1	1	1
Driver 4	40	0	0	0	0
Driver 5	23	0.048492	0.384315	0.406994	0.43469
Driver 6	6	1	1	1	1
Driver 7	24	0	0.162525	0.203046	0.15848
Driver 8	37	0	0.146842	0	0.17589
Driver 9	19	1	1	1	1
Driver 10	45	0	0	0	0
Driver 11	6	1	1	1	1
Driver 12	13	0.256557	1	0.15315	0.13958
Driver 13	10	1	1	1	1
Driver 14	17	0.265625	0.172794	0.280798	0.27437
Driver 15	8	1	1	1	1
Driver 16	70	0	0	0	0
Driver 17	10	0.207382	1	1	1
Driver 18	72	0	0	0	0
Driver 19	33	0	0.185454	0.057727	0
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0

Table B3. Selten measures for neoclassical and unconditional expectations-based
reference-dependent models

	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	1
Driver 2	14	0.942	0.788	0.803	0.82
Driver 3	6	0.505	0.343	0.344	0.361
Driver 4	40	0	0	0	0
Driver 5	23	0.047492	0.328315	0.367994	0.416699
Driver 6	6	0.665	0.466	0.499	0.497
Driver 7	24	0	0.148525	0.193046	0.145481
Driver 8	37	0	0.144842	0	0.174891
Driver 9	19	0.987	0.912	0.913	0.941
Driver 10	45	0	0	0	0
Driver 11	6	0.421	0.303	0.287	0.279
Driver 12	13	0.187557	0.714	-0.09685	-0.09742
Driver 13	10	0.832	0.659	0.632	0.669
Driver 14	17	0.261625	0.084794	0.217798	0.222375
Driver 15	8	0.8	0.504	0.604	0.594
Driver 16	70	0	0	0	0
Driver 17	10	-0.01062	0.677	0.604	0.685
Driver 18	72	0	0	0	0
Driver 19	33	0	0.178454	0.056727	-0.002
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0

Table B4. Proximities for neoclassical and unconditional experience-based reference-dependent models

	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	0
Driver 2	14	1	1	1	1
Driver 3	6	1	1	1	1
Driver 4	40	0	0.078039	0	0
Driver 5	23	0.048492	0.016492	0.212637	0.187298
Driver 6	6	1	1	1	1
Driver 7	24	0	1	0.278643	0.157611
Driver 8	37	0	1	0	0
Driver 9	19	1	1	1	1
Driver 10	45	0	0	0	0
Driver 11	6	1	1	1	1
Driver 12	13	0.256557	0.13873	0.265933	0.226169
Driver 13	10	1	1	1	1
Driver 14	17	0.265625	0.313394	0.416434	0.251828
Driver 15	8	1	1	1	1
Driver 16	70	0	0	0	0
Driver 17	10	0.207382	0.500883	0.326962	0.304039
Driver 18	72	0	0	0	0
Driver 19	33	0	0.322158	0	0
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0

Table B5. Selten measures for neoclassical and unconditional experience-based
reference-dependent models

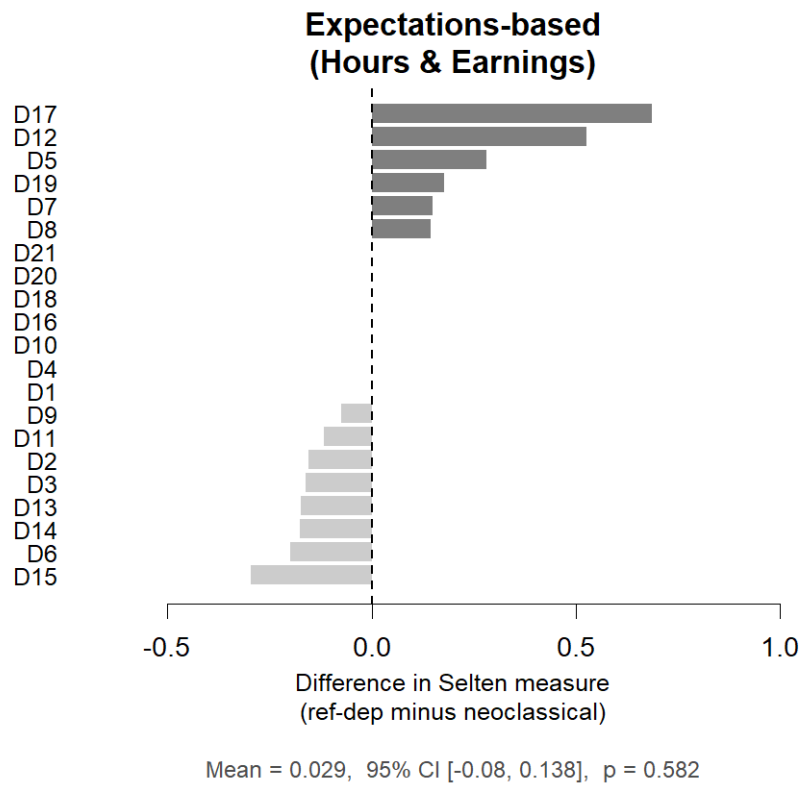
	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	0
Driver 2	14	0.942	0.804	0.856	0.814
Driver 3	6	0.505	0.373	0.401	0.381
Driver 4	40	0	0.077039	0	0
Driver 5	23	0.047492	-0.00751	0.197637	0.175298
Driver 6	6	0.665	0.267	0.274	0.285
Driver 7	24	0	0.994	0.276643	0.153611
Driver 8	37	0	1	0	0
Driver 9	19	0.987	0.924	0.935	0.938
Driver 10	45	0	0	0	0
Driver 11	6	0.421	0.271	0.261	0.261
Driver 12	13	0.187557	-0.08827	0.081933	0.036169
Driver 13	10	0.832	0.575	0.616	0.577
Driver 14	17	0.261625	0.270394	0.381434	0.219828
Driver 15	8	0.8	0.527	0.505	0.49
Driver 16	70	0	0	0	0
Driver 17	10	-0.01062	0.124883	-0.02104	-0.03996
Driver 18	72	0	0	0	0
Driver 19	33	0	0.321158	0	0
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0

Table B6: Paired tests for differences in Selten measures (reference-dependent minus neoclassical)

Specification	Sample	n	Mean diff	SE	95% CI	p	Sig.
<i>Expectations-based models</i>							
Hours & earnings	All 21	21	0.029	0.052	[-0.080, 0.138]	0.58	No
	Non-zero	14	0.044	0.079	[-0.127, 0.214]	0.59	No
Earnings only	All 21	21	-0.010	0.043	[-0.100, 0.080]	0.82	No
	Non-zero	13	-0.016	0.071	[-0.170, 0.137]	0.82	No
Hours only	All 21	21	0.051	0.067	[-0.088, 0.190]	0.45	No
	Non-zero	15	0.071	0.093	[-0.129, 0.272]	0.46	No
<i>Experience-based models</i>							
Hours & earnings	All 21	21	0.038	0.078	[-0.124, 0.200]	0.63	No
	Non-zero	15	0.053	0.109	[-0.182, 0.288]	0.64	No
Earnings only	All 21	21	-0.042	0.032	[-0.108, 0.025]	0.20	No
	Non-zero	12	-0.073	0.055	[-0.193, 0.048]	0.21	No
Hours only	All 21	21	-0.064	0.029	[-0.124, -0.004]	0.04	Yes
	Non-zero	12	-0.112	0.046	[-0.214, -0.011]	0.03	Yes

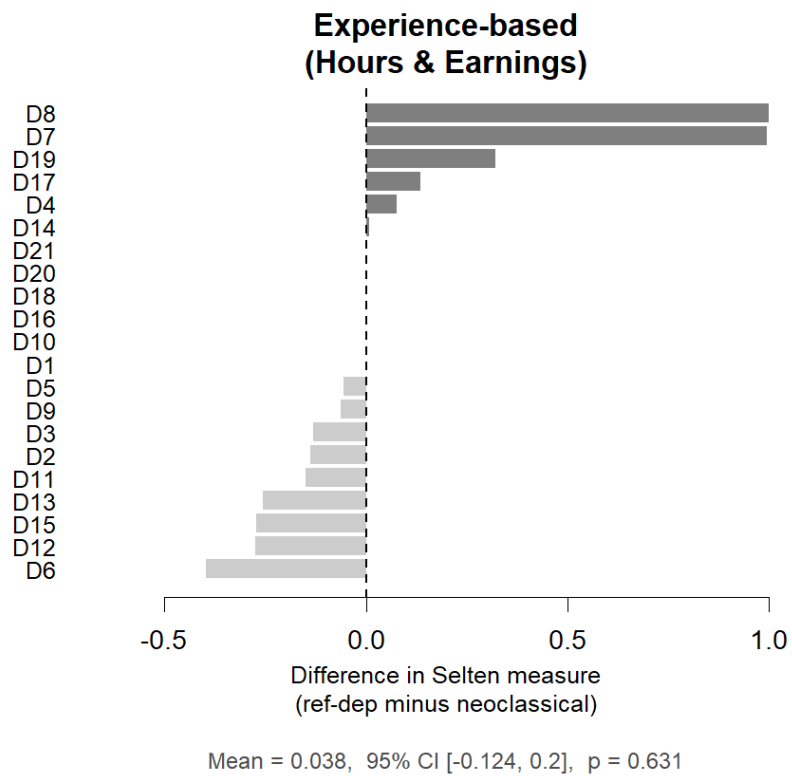
Notes: Each row reports a paired *t*-test of the null hypothesis that the mean difference in Selten measures (reference-dependent minus neoclassical) is zero. “All 21” includes all drivers, with the difference coded as zero for drivers whose Selten measures are bounded rather than precisely estimated (see text); this is conservative. “Non-zero” excludes those drivers. Highlighted rows are significant at the 5% level.

Figure B2. Driver-by-driver differences in Selten measures: unconditional expectations-based reference-dependent model minus neoclassical (hours and earnings).



Notes: Positive values indicate higher predictive success for the reference-dependent model. Drivers with zero differences have bounded Selten measures (see text). The mean difference and 95% confidence interval are reported at the bottom of the figure.

Figure B3. Driver-by-driver differences in Selten measures: unconditional experience-based reference-dependent model minus neoclassical (hours and earnings).



Notes: See Figure B2.