NONPARAMETRIC ANALYSIS OF REFERENCE-DEPENDENT PREFERENCES*

Revised 31 January 2026

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Abstract: This paper derives nonparametric conditions for reference-dependent preferences that rationalize a consumer's demand behavior. We show that unless reference points are modelable and sensitivity is constant, a reference-dependent model can rationalize virtually any data. Assuming modelable reference points, we characterize continuous reference-dependent preferences with constant sensitivity, relaxing strong functional-structure assumptions maintained in all previous studies of reference-dependent demand. Our characterization enables more general structural econometric as well as nonparametric analyses. We use it to re-analyze Farber's (2005, 2008) data on cabdrivers' labor supply. For most drivers, relaxing the functional-structure assumptions increases a nonparametric measure of predictive success. For many drivers a relaxed reference-dependent model has greater success than a neoclassical model. (*JEL* C14, C23, D11, D12, J22)

Keywords: consumer theory, labor supply, reference-dependent preferences, revealed preference, nonparametric demand analysis, loss aversion

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1 Introduction

Kahneman and Tversky (1979) and Tversky and Kahneman (1991; "TK") introduce a model of individual decisions in which people have preferences over gains and losses relative to a reference point. Such reference-dependence alters the domain of preferences from levels of outcomes to changes in outcomes; but it remains consistent with a complete and transitive preference ordering over changes, thus not inherently irrational. Although Kahneman and Tversky and TK focus on changes alone, Kőszegi and Rabin (2006; "KR") and most subsequent analyses allow preferences over both levels and changes; and such reference-dependence is also not inherently irrational.

Reference-dependent consumer theory has been a workhorse model in behavioral microeconomics since Camerer et al.'s (1997) analysis of New York City cabdrivers' labor supply.[1] A standard neoclassical model of labor supply is analogous to a model of consumer demand, with preferences over levels of leisure and earnings—black-boxing the goods earnings can buy. In such a model the elasticity of hours with respect to the wage is positive unless there are very large income effects. However, Camerer et al., taking a driver's earnings per hour as a proxy for the wage, estimate negative wage elasticities.

To explain this anomaly, Camerer et al. propose a model in which drivers bracket narrowly, evaluating their choices day by day instead of over their working lifetimes; and have daily earnings targets. A reference-dependent model in which the domain of preferences includes changes in earnings relative to daily targets as well as levels of earnings and leisure can reconcile such income-targeting with choice that is rational within the daily bracket.[2]

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[1] See also Hardie et al.'s (1993) analysis of consumer demand. Cabdrivers are of particular interest because many choose their own hours, unlike most workers in modern economies.

[2] Such narrow bracketing is of course irrational from a lifetime point of view. Some have argued that earnings-targeting is irrational even within the daily bracket, because it leads drivers to trade off levels of earnings for changes that neoclassical preferences do not respond to. In Farber's (2008, p. 1070) words: "This [earnings-targeting] is clearly nonoptimal from a

For, experiments suggest that most people are loss-averse—more sensitive to changes below their targets (losses) than above them (gains). Loss aversion creates kinks in drivers' preferences that make their daily earnings tend to bunch around their targets, possibly leading them to work less on days with higher wages. This allows a rationality-based account of Camerer et al.'s negative daily wage elasticities, without unrealistically large income effects.

In a theory paper inspired by TK's and Camerer et al.'s analyses, KR propose a more general model of reference-dependent preferences. KR take narrow bracketing as given. Within the bracket, KR assume that a person's utility is additively separable across neoclassical consumption utility and reference-dependent "gain-loss" utility. They also assume that reference-dependence is active for every good and , as a convenient simplification, that gain-loss utility is determined, additively separably across goods, by the good-by-good differences between realized and reference consumption utilities. Finally, in the spirit of Camerer et al.'s proxying drivers' targets via average daily earnings, KR close their model by equating a consumer's reference points to her/his good-by-good rational expectations of future consumption.

KR's expectations-based reference-dependent model, like Camerer et al.'s earnings-targeting model, can give a rationality-based account of labor supply with negative wage elasticities without invoking large income effects: With perfectly anticipated changes in earnings and hours, gain-loss utility drops out of their model, which then reproduces the neoclassical prediction that higher *anticipated* wages increase labor supply. But with unanticipated changes in earnings, loss aversion makes daily earnings tend to bunch around its reference point, and earnings surprises can yield a negative overall correlation.

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neoclassical perspective, since it implies quitting early on days when it is easy to make money and working longer on days when it is harder to make money. Utility would be higher by allocating time in precisely the opposite manner." However, our use of "rational" refers to the consistency of a driver's choices in the larger domain of preferences over levels and changes.

TK's and KR's papers have spawned numerous empirical applications of reference-dependent consumer theory: to consumer demand itself (Hardie et al. 1993), labor supply (Camerer et al. 1997; Oettinger 1999; Fehr and Goette 2007; Farber 2005, 2008, 2015; Crawford and Meng 2011, "CM"; Thakral and Tô 2021; Anderson et al. 2023; Brandon et al. 2023; Crawford et al. 2025), housing (Genesove and Mayer 2001, Andersen er al. 2022), and finance (Odean 1998; Barberis and Thaler 2003; Barberis 2013; Meng and Weng 2018). However, the jury is still out on whether reference-dependent models of consumer demand are empirically useful, particularly in labor supply.

This makes it natural to ask whether the empirical successes (or failures) of reference-dependent models of consumer demand are due to reference-dependence (or its absence) per se or are artifacts of the strong functional-structure and functional-form assumptions that, to our knowledge, have been maintained without testing in all applications to date.

This paper begins to answer such questions by deriving nonparametric conditions for the existence of reference-dependent preferences that can rationalize consumer demand behavior, taking daily bracketing as given.[3]

TK, KR, and the empirical papers all assume—in our view naturally—that the reference-dependent utility function that represents preferences can be written as the sum of a consumption utility function that depends only on levels of consumption and a gain-loss utility function that depends on both levels of consumption and a reference point. Without that assumption reference-dependent models are unlikely to have any nonparametrically refutable implications; and we maintain it.

[3] There are several other nonparametric theoretical analyses of reference-dependent models, including Gul and Pesendorfer (2006); Abdellaoui et al. (2007); Ok et al. (2015); Masatioglu and Raymond (2016); Nishimura et al. (2017); Freeman (2017, 2019); and Kibris et al. (2021). All but Gul and Pesendorfer (2006) and Freeman (2017), whose contributions are discussed in Section 4, focus on different aspects of the problem than the ones we consider.

TK, KR, and the empirical papers also assume that gain-loss utility is determined, additively separably across goods, by the good-by-good differences between realized and reference consumption utilities—thus linking gain-loss utility to consumption utility in a particular way. They also assume (except in KR's most general model) that the sum of consumption and gain-loss utility that determines a consumer's demand has constant sensitivity.[4]

These functional-structure assumptions have two strong implications: The sum of consumption and gain-loss utility that determines consumer demand must be additively separable across goods.[5] And its marginal rates of substitution vary across gain-loss regimes in a particular, knife-edge way.[6]

Assuming additive separability across goods would be a non-starter in a neoclassical demand analysis. For reference-dependent preferences, neither assumption is supported by theory. We show how to relax and test both assumptions, which our empirical illustration suggests is important.

Under our assumptions, the empirically refutable implications of reference-dependent models of consumer demand are limited by two factors. Proposition 1 shows that unless reference points are modelable, in the sense that the available data allow them to be precisely predicted, one can construct a

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[4] TK's sign-dependence; KR's A3′. Constant sensitivity is formally defined in Section 2. Informally, how an observation's consumption bundle relates to the observation's reference point puts the bundle into one of several gain-loss regimes, such as "earnings loss, hours gain" in labor supply. With constant sensitivity a consumer's gain-loss utility function can vary freely across regimes, with the preferences over bundles determined by consumption plus gain-loss utility independent of the reference point within a given regime.

[5] Additive separability across goods is distinct from KR's and our assumption that the utility function is additively separable across consumption and gain-loss utility.

[6] CM's Table 1. In labor supply, for instance, the marginal rates of substitution are equal across the (gain, gain) and (loss, loss) regimes and in constant proportions across the (gain, loss) and (loss, gain) regimes.

reference-dependent utility function to rationalize *any* demand data.[7] Proposition 2 then shows that even if reference points are modelable, unless sensitivity is constant one can construct a well-behaved reference-dependent utility function to rationalize any demand data, with a minor qualification.

Propositions 1 and 2 identify a grain of truth in the widespread belief that allowing reference-dependence destroys the parsimony of neoclassical consumer theory: Without both modelable reference points and constant sensitivity, the hypothesis of reference-dependent preferences is nonparametrically irrefutable, with Proposition 2's minor qualification.

Proposition 3 paves the way for positive results by characterizing continuous reference-dependent utility functions that satisfy constant sensitivity. Assuming modelable reference points but relaxing TK's and KR's functional-form and functional structure assumptions, it identifies the most general class of reference-dependent utility functions with nonparametrically refutable implications for consumer demand.

Empirically, Proposition 3's characterization could be used to conduct a more general structural econometric analysis of reference-dependent demand, with conventional assumptions on the functional structure and forms of consumption and gain-loss utility, using sample proxies like Camerer et al.'s and CM's for the targets. Several papers cited above provide suitable datasets.

In this paper, instead, we continue by deriving nonparametric conditions for reference-dependent preferences that rationalize consumer demand behavior and using them to assess our generalization's empirical importance.

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[7] Examples of modelability include Camerer et al.'s use of average daily earnings as sample proxies for earnings targets, KR's rational-expectations model of reference-points, CM's and Farber's (2015) implementations of KR's model, Thakral and Tô's (2021) dynamic model of reference points, and Crawford et al.'s (2025) elicitation of them. By contrast, Kahneman and Tversky (1979) and TK take no clear position on how reference points are determined; and Farber (2005, 2008) and most other empirical papers estimate them as latent variables.

Like Afriat's (1967), Diewert's (1973), and Varian's (1982) classic nonparametric analyses of neoclassical demand, our nonparametric analysis makes essential use of rationality in the sense of consistency of choices across budget sets. However, as in most reference-dependent analyses, consistency is evaluated within the bracket (in this case daily) and over a preference domain expanded in the disciplined way suggested by reference-dependence.

Our analysis raises issues beyond those resolved by the classic analyses because levels of and changes in a good's consumption are bundled and priced together and a reference-dependent consumer can, in effect, change her/his preferences by buying a bundle in a different gain-loss regime. And, although Afriat's Theorem shows that demand data are rationalizable via neoclassical preferences if and only if the generalized axiom of revealed preference is satisfied ("GARP"; Section 3), with reference-dependent preferences there is no simple condition to determine whether demand data are rationalizable.

We approach these difficulties in two steps. Proposition 4 uses Proposition 3's characterization of reference-dependent utility functions with constant sensitivity and continuity to derive benchmark necessary and sufficient conditions for a reference-dependent rationalization. Proposition 4's conditions are not directly applicable because with finite data there is normally a range of preferences that rationalize a consumer's choices within a gain-loss regime (Varian 1982, Fact 4 and Figure 3), and those conditions rely on an unspecified choice of such preferences. Proposition 5 then derives directly applicable sufficient conditions based on a particular choice of rationalizing regime preferences. Proposition 5's conditions are not necessary, but they should approach necessity as the data become rich enough to precisely determine the rationalizing preferences within each gain-loss regime.

Proposition 5 immediately suggests an algorithm for nonparametric estimation. We illustrate our results by adapting it to estimate neoclassical and reference-dependent models, re-analyzing Farber's (2005, 2008) data on cabdrivers' labor supply. We relax Farber's and CM's driver homogeneity assumptions (which follow the labor supply literature) to estimate driver by driver (as in the nonparametric demand literature). We control for models' differences in flexibility using Beatty and Crawford's (2011, pp. 2786-2787) proximity-based variant of Selten and Krischker's (1983) and Selten's (1991) nonparametric measures of predictive success, which judge flexibility by the likelihood that random data would fit a model.

Farber's drivers are highly heterogeneous. For most drivers, reference-dependent models that relax TK's and KR's constant-sensitivity restrictions of additive separability across goods and on how marginal rates of substitution vary across gain-loss regimes have significantly higher Selten measures of predictive success than their counterparts imposing the restrictions, suggesting that Proposition 3's generalizations are empirically important.

Although the GARP condition for a neoclassical rationalization is violated for most drivers, our methods yield reference-dependent rationalizations for almost all of most drivers' choices. For many, a reference-dependent model has a Selten measure as high or higher than a neoclassical model. And in contrast to the focus on earnings-targeting in most previous theoretical and empirical work, hours-targeting is more important than earnings-targeting.

The rest of paper is organized as follows. Section 2 introduces our model of reference-dependent preferences. Section 3 reviews the classic nonparametric analyses of neoclassical consumer demand. Section 4 shows that without both modelable reference points and constant sensitivity, the hypothesis of reference-dependent preferences is nonparametrically irrefutable. Section 5 characterizes the structure of reference-dependent preferences, assuming constant sensitivity and continuity. Section 6 derives the model's nonparametric implications for demand. Section 7 uses using Farber's (2005, 2008) data to illustrate Section 6's methods. Section 8 is the conclusion.

2 Reference-dependent Preferences

We consider reference-dependent preferences with a finite number of demand observations for a single consumer—or equivalently for a pooled group of consumers assumed to have homogeneous preferences, but we will speak of a single consumer. We index goods $k = 1,\ldots, K$ and observations $t = 1,\ldots, T$. We assume the consumer is a price-taker, choosing a consumption bundle $\boldsymbol{q} \in \mathbb{R}_+^K$ with a linear budget constraint. Her/his preferences are represented by a family of real-valued utility functions $u(\boldsymbol{q}, \boldsymbol{r})$, parameterized by an exogenous reference point $\boldsymbol{r} \in \mathbb{R}_+^K$, which is conformable to a K–good consumption bundle as in TK and CM.[8] If reference points are unmodelable, the data are prices and quantities $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$, with hypothetical reference points $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$. If reference points are modelable, the data are prices, quantities, and reference points $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$. The context will make the interpretation of $\boldsymbol{r}_t$ clear. Sometimes we denote goods by scalars with superscripts, so for $k = 1,\ldots, K$, $\boldsymbol{q} \equiv (q^1, \ldots, q^K)$ and for observation $t = 1,\ldots, T$, $\boldsymbol{q_t} \equiv (q_t^1, \ldots, q_t^K)$, with analogous notation for $\boldsymbol{p}$, $\boldsymbol{p_t}$, $\boldsymbol{r}$, and $\boldsymbol{r_t}$.

To describe preferences that respond positively to changes in consumption relative to the reference point, as well as to levels, we take the reference-dependent utility function $u(\boldsymbol{q}, \boldsymbol{r})$ to be strictly increasing in $\boldsymbol{q}$ and strictly decreasing in $\boldsymbol{r}$. Our specification is then at least as flexible as a general strictly increasing function of levels $\boldsymbol{q}$ and changes $\boldsymbol{q} - \boldsymbol{r}$. It nests the neoclassical case where preferences respond only to levels; Kahneman and Tversky's (1979) and TK's case where they respond only to changes; and cases like Camerer et al.'s (1997), Farber's (2005, 2008), KR's, and CM's where preferences respond to both. As in those papers, we take $u(\boldsymbol{q}, \boldsymbol{r})$ to be

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[8] In KR's theoretical model, which makes no allowance for errors, only probabilistic targets make possible the unanticipated changes in outcomes that allow expectations-based reference-dependence to have any effect. CM use the fact that sampling variation causes unanticipated changes to simplify KR's probabilistic targets to point expectations, as we do here.

continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$; and we assume that $u(\boldsymbol{q},\boldsymbol{r})$ is additively separable in the sense that it can be written as the sum of a consumption utility function, which depends only on levels of consumption, and a gain-loss utility function, which depends on both levels of consumption and a reference point.

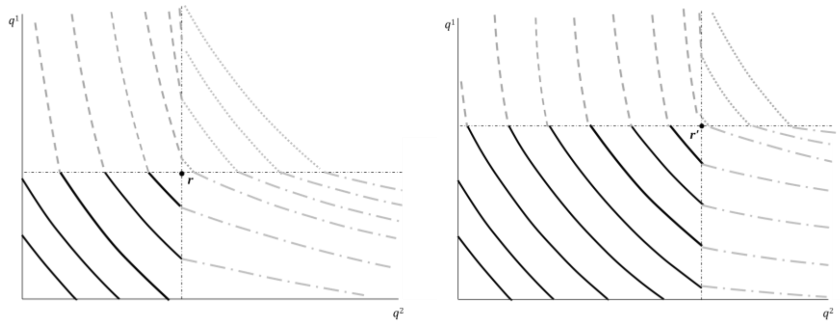
We call the general case of preferences in the class just described variable sensitivity. An important special case is constant sensitivity (TK's sign-dependence; KR's assumption A3′). Let $\text{sign}(\boldsymbol{q}-\boldsymbol{r})$, the vector whose kth component is $\text{sign}(q^k - r^k)$, be the good-by-good sign pattern of gains and losses. A reference point divides commodity space into gain-loss regimes throughout each of which $\text{sign}(\boldsymbol{q}-\boldsymbol{r})$ remains constant. With constant sensitivity a consumer has a separate reference-dependent utility function for each regime, with preferences over consumption bundles $\boldsymbol{q}$ (but not the level of utility) independent of $\boldsymbol{r}$ within that regime. Gain-loss regimes' utility functions can vary freely across regimes as long as the sum of consumption and gain-loss utility is continuous across regimes.

Note that each gain-loss regime's utility function must be defined for the entire commodity space, because any $\boldsymbol{q}$ is in the regime for some $\boldsymbol{r}$: Each value of $\text{sign}(\boldsymbol{q}-\boldsymbol{r})$ "switches on" a different regime utility function.

DEFINITION 1: [Preferences and utility functions with constant sensitivity.] A reference-dependent utility function $u(\boldsymbol{q},\boldsymbol{r})$ *satisfies constant sensitivity if and only if, for any consumption bundles* $\boldsymbol{q}$ *and* $\boldsymbol{q}^*$ *and reference points* $\boldsymbol{r}$ *and* $\boldsymbol{r}^*$ *such that* $sign(\boldsymbol{q}-\boldsymbol{r}) = sign(\boldsymbol{q}^*-\boldsymbol{r}) = sign(\boldsymbol{q}-\boldsymbol{r}^*) = sign(\boldsymbol{q}^*-\boldsymbol{r}^*)$*,* $u(\boldsymbol{q},\boldsymbol{r}) \geq u(\boldsymbol{q}^*,\boldsymbol{r})$ *if and only if* $u(\boldsymbol{q},\boldsymbol{r}^*) \geq u(\boldsymbol{q}^*,\boldsymbol{r}^*)$*.*

With two goods, a reference point in the interior of the commodity space divides it into four gain-loss regimes. Figure 1's panels (a) and (b) show four regime indifference maps and the associated global maps for reference points $\boldsymbol{r}$ and $\boldsymbol{r}'$. The shift from $\boldsymbol{r}$ to $\boldsymbol{r}'$ does not alter the regime maps, but as $\boldsymbol{r}$ varies, even locally, the shift alters how they connect across regimes, as in Figure 1.

Figure 1. A set of gain-loss regime maps with constant sensitivity and the associated global maps for alternative reference points



(a) (b)

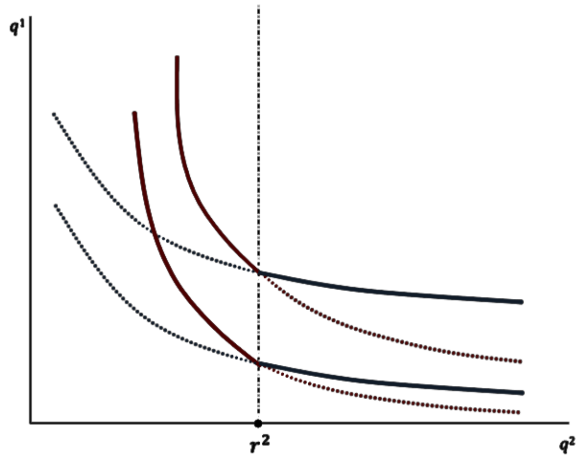
Loss aversion is a concept that has strong experimental and empirical support and figures in some of our results. Generalizing TK's (pp. 1047-1048) definition for the two-good case, Definition 2 gives a nonparametric definition of loss aversion with constant sensitivity.[9]

DEFINITION 2: [Preferences with constant sensitivity and loss aversion.] Assume that reference-dependent preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *have constant sensitivity. A collection of gain-loss regime preferences over consumption bundles satisfies loss aversion if and only if, for any observation* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}$*, given* $\boldsymbol{r}_t$*, the preference ordering's better-than-*$\boldsymbol{q}_t$ *set is weakly contained in each regime preference ordering's better-than-*$\boldsymbol{q}_t$ *set.*

Figure 2 illustrates Definition 2's notion of loss aversion with one active reference point and two gain-loss regimes. As loss aversion is a property of the relationship between different regimes' preferences over consumption bundles given a reference point, it is independent of the reference points themselves.

[9] The idea of loss aversion is well defined with variable sensitivity, but formalizing it is more complex, and Proposition 2 will show that it is then nonparametrically irrefutable anyway.

Figure 2. Loss aversion with one active reference point
(solid curves for active parts of indifference maps,
dashed for inactive parts)



Definition 2's nesting of better-than sets is equivalent to requiring that gain-loss regimes' indifference maps satisfy a global single-crossing property: For any observation, across regimes that differ only in the gain-loss status of good *i*, the loss-side marginal rate of substitution between good *i* and any other good (generalized as needed for non-differentiable preferences) is weakly more favorable to good *i* than the gain-side marginal rate of substitution.

It is this single-crossing property, not the kinks in global indifference maps that it creates, that shapes loss aversion's nonparametric implications, which are testable with finite data. Loss aversion precludes nonconvex kinks, so if the regime maps have convex better-than sets, so do the global maps.

3 Nonparametric Implications of Neoclassical Preferences

In preparation for our nonparametric analysis, this section reviews Afriat's (1967), Diewert's (1973), and Varian's (1982) nonparametric analyses of consumer demand in the neoclassical case where preferences respond only to levels of consumption. In the revealed-preference tradition of Samuelson (1948) and Houthakker (1950), Afriat, Diewert, and Varian show that a consumer's demand behavior can be nonparametrically rationalized by the maximization of a nonsatiated utility function if and only if the data satisfy the Generalized Axiom of Revealed Preference ("GARP"). They also show how to construct a rationalizing utility function.

DEFINITION 3: [Rationalization with neoclassical preferences.] Preferences and an associated utility function $u(\boldsymbol{q})$ *rationalize the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *if and only if* $u(\boldsymbol{q}_t) \geq u(\boldsymbol{q})$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

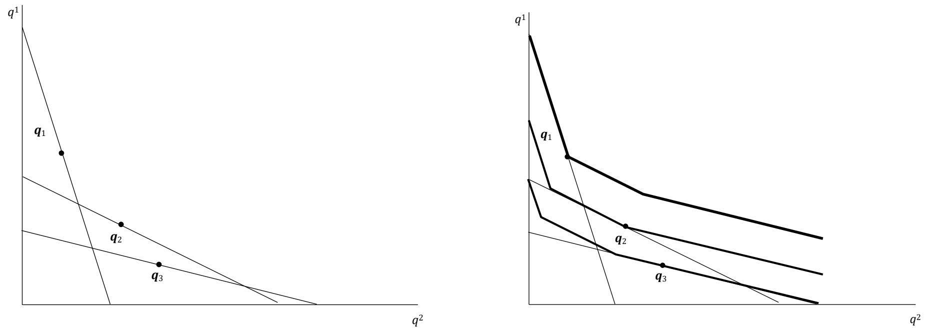
DEFINITION 4: [Generalized Axiom of Revealed Preference ("GARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *implies* $\boldsymbol{p}_t \cdot \boldsymbol{q}_t \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_s$*, where R indicates that there is some sequence of observations* $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \dots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i$, $\boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \dots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

AFRIAT'S THEOREM: The following statements are equivalent:
[A] There exists a utility function $u(\boldsymbol{q})$ *that is continuous, non-satiated, and concave, and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.
[B] There exist numbers $\{U_t, \lambda_t > 0\}_{t=1}^{T}$ *such that*
(1) $U_s \leq U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q}_s - \boldsymbol{q}_t)$ *for all* $s, t \in \{1, \dots, T\}$
[C] The data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *satisfy GARP.*
[D] There exists a non-satiated utility function $u(\boldsymbol{q})$ *that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.

In implementing Afriat's Theorem, for given $\{U_t, \lambda_t > 0\}_{t=1}^{T}$ that satisfy [B]'s inequalities (1), we can take $u(\boldsymbol{q}) = min_{t\in\{1,\dots,T\}}\{U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q} - \boldsymbol{q}_t)\}$. With finite data there are generally many possibilities for a rationalization (Varian 1982, Fact 4 and Figure 3). However, a choice of $u(\boldsymbol{q})$ **we** call the Afriat utility function plays a central role in Proposition 5.

DEFINITION 5: [Afriat preferences and utility function.] For data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *that satisfy GARP, or equivalently for* $\{U_t, \lambda_t > 0\}_{t=1}^{T}$ *that* *satisfy condition [B] of Afriat's Theorem, the Afriat preferences follow the associated utility function* $u(\boldsymbol{q}) = min_{t\in\{1,\ldots,T\}}\{U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q} - \boldsymbol{q}_t)\}$.

Figure 3. Neoclassical Afriat preferences for data that satisfy GARP



(a) (b)

Figure 3 illustrates the Afriat preferences for a three-observation dataset that satisfies GARP. Figure 3a shows the observations' budget sets and consumption bundles. Figure 3b shows the Afriat indifference map, whose marginal rates of substitution are determined by the budget lines. The Afriat utility function is piecewise linear, continuous, non-satiated, and concave.

4 Nonparametric Implications of Reference-Dependent Preferences

This section begins our nonparametric analysis of reference-dependent preferences. We assume that the reference-dependent utility function is additively separable in the sense that it can be written as the sum of a consumption utility function and a gain-loss utility function. Then, whether reference-dependent preferences can rationalize demand behavior is limited by whether sensitivity is constant and reference points are precisely modelable.

4.1 Reference-dependent rationalization with unmodelable reference points

With unmodelable reference points, Definition 6 allows a reference point to be chosen hypothetically for each observation—the nonparametric analogue of Farber's (2005, 2008) econometric treatment of targets as latent variables.

DEFINITION 6: [Rationalization with unmodelable reference points.] Reference-dependent preferences, an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$*, and hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\dots,T}$*, rationalize the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *if and only if* $u(\boldsymbol{q}_t, \boldsymbol{r}_t) \geq u(\boldsymbol{q}, \boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

Proposition 1 shows that unless reference points are modelable in the sense that they can be precisely predicted (possibly as functions of the data), one can construct a reference-dependent utility function to rationalize any demand data (even if the data violate Definition 4's necessary and sufficient GARP condition for the existence of a neoclassical rationalization,).

PROPOSITION 1: [Rationalization with unmodelable reference points via preferences with variable or constant sensitivity.] For any data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *with unmodelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q}, \boldsymbol{r})$ *that are continuous, increasing in* $\boldsymbol{q}$*, and decreasing in* $\boldsymbol{r}$*, and a sequence of hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\dots,T}$*, that rationalize the data.*

Proof: Recall that we denote goods by superscripts, so that $\boldsymbol{q} \equiv (q^1, \dots, q^K)$, $\boldsymbol{q_t} \equiv (q_t^1, \dots, q_t^K)$, and so on. Let $a^k \equiv min_{t=1,\dots,T}\{p_t^k\} > 0$ for each *k* and *t* such that $q_t^k \geq r_t^k$; and $a^k \equiv max_{t=1,\dots,T}\{p_t^k\} > 0$ for each *k* and *t* such that $q_t^k < r_t^k$. Define the utility function $u(\boldsymbol{q}, \boldsymbol{r}) \equiv \sum_k a^k q^k + \sum_k a^k (q^k - r^k)$, which is strictly increasing in $\boldsymbol{q}$, strictly decreasing in $\boldsymbol{r}$, and satisfies constant sensitivity and Proposition 1's conditions for continuity. For observation *t*, set $\boldsymbol{r}_t = \boldsymbol{q}_t$ and consider any bundle $\boldsymbol{q} \neq \boldsymbol{q}_t = \boldsymbol{r}_t$ that (without loss of generality given strict monotonicity) exactly satisfies t's budget constraint. For such bundles, $\sum_k p_t^k\,(q^k - q_t^k) = 0$ and, by the definition of the a^k,

(2) $\sum_k (a^k - p_t^k)(q^k - q_t^k) = \sum_k (a^k - p_t^k)(q^k - r_t^k) < 0$ and $\sum_k a^k (q^k - r_t^k) \leq 0$

and

(3) $u(\boldsymbol{q}, \boldsymbol{r}_t) - u(\boldsymbol{q}_t, \boldsymbol{r}_t) = 2\sum_k a^k (q^k - q_t^k) = 2\sum_k a^k (q^k - r_t^k) \leq 0$,

so $u(\boldsymbol{q}, \boldsymbol{r})$ rationalizes the choice of $\boldsymbol{q}_t$. Similarly for variable sensitivity.[10] ∎

The proof of Proposition 1 hypothesizes a reference point for each observation with $\boldsymbol{r}_t = \boldsymbol{q}_t$ and preferences that, with those reference points, put the observation's consumption bundle at the kink of an approximately Leontief indifference curve (approximately to preserve strict monotonicity). Those preferences satisfy continuity, constant sensitivity, and Farber's, KR's, and CM's functional-form assumptions, which shows that those assumptions are nonparametrically irrefutable as well. Because the rationalization works entirely by varying reference points across observations, it shows as directly as possible that the empirical usefulness of reference-dependent consumer theory depends on modeling reference points.

Implicitly, Proposition 1 also shows that analyses that treat targets as latent variables may be influenced as much by how they constrain the estimation of targets as by reference-dependence. This may be why Farber's (2005, 2008) model yields unstable estimates of earnings targets, which is the basis of his argument that reference-dependence is not useful in modeling labor supply. By contrast, CM's sample-proxy model of the targets yields stable estimates.

Gul and Pesendorfer (2006) and Freeman (2017) prove results with conclusions like Proposition 1's.[11] However, the preferences in Gul and Pesendorfer's proof do not satisfy KR's and our assumption of additive

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[10] The 2 in (3) comes from the separate components of the hypothesized $u(\boldsymbol{q}, \boldsymbol{r})$ function.

[11] More precisely, Gul and Pesendorfer show that the choice function for reference-dependent preferences is the same as that which maximizes a complete binary relation.

separability across consumption and gain-loss utility, and they allow the strength of loss aversion to vary wildly with the cardinality of their (finite) choice set. Freeman's Observation 1 does not restrict preferences, even to be monotonic. By contrast, the rationalizing preferences in the proof of Proposition 1 are credible candidates for an empirical explanation.

Cases with limited knowledge of reference points are plausible and may be empirically relevant, but we have found no informative results for them.

4.2 Reference-dependent rationalization with modelable reference points and variable sensitivity

Our next result shows that even if reference points are modelable, unless sensitivity is constant the hypothesis of reference-dependent preferences is refutable only via violations of Definition 4's GARP condition within subsets of observations that share *exactly* the same reference point. For such subsets, reference-dependent preferences reduce to neoclassical preferences. Thus, reference-dependence adds nothing empirically useful to a neoclassical model.

Put another way, our result shows that constant sensitivity, which is usually seen as no more than a convenient simplification, is essential for a reference-dependent model to have useful nonparametrically refutable implications.

DEFINITION 7: [Rationalization with modelable reference points.] Reference-dependent preferences and an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$ *rationalize the data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points if and only if* $u(\boldsymbol{q}_t,\boldsymbol{r}_t) \geq u(\boldsymbol{q},\boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

PROPOSITION 2:[12] *[Rationalization with modelable reference points via preferences with variable sensitivity.] For any data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ *with*

[12] As Proposition 2's proof shows, restricting sensitivity short of assuming that it is constant, such as by assuming diminishing sensitivity, still does not yield refutable implications. Unlike Proposition 1, Proposition 2 does *not* claim that $u(\boldsymbol{q},\boldsymbol{r})$ is continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$ or decreasing

modelable reference points, there exist reference-dependent preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *that for each observation t and reference point* $\boldsymbol{r}_t$, *are continuous and strictly increasing in* $\boldsymbol{q}$ *and that rationalize the data, if and only if every subset of the data whose observations share exactly the same reference point satisfies GARP.*

Proof: Partition the observations into subsets $\tau^j, j = 1, \ldots, J$, such that if and only if two observations $\{\boldsymbol{p}_s, \boldsymbol{q}_s, \boldsymbol{r}_s\}$ and $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}$ have the same reference point $\boldsymbol{r}_s = \boldsymbol{r}_t$, they are in the same subset. If there exists a reference-dependent utility function with the stated properties that rationalizes the data, then the data must satisfy GARP within any such subset, by Afriat's Theorem. Conversely, suppose the data within each such subset satisfies GARP. Let $b^k \equiv min_{t=1,\ldots,T}\{p_t^k\}$, so that $0 < b^k \leq p_t^k$, and let $\boldsymbol{b} \equiv (b^1, \ldots, b^K)$. For any subset τ^j and observation $t \in \tau^j$, let the indicator function $I_{\tau^j}(t) = 1$ if the observation $t \in \tau^j$ and $I_{\tau^j}(t) = 0$ otherwise, and let $u(\boldsymbol{q}, \boldsymbol{r}) \equiv \sum_j I_{\tau^j}(t) U^j(\boldsymbol{q}, \boldsymbol{r}_t)$, where $U^j(\boldsymbol{q}, \boldsymbol{r}_t) \equiv min_{\rho \in \tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} - \boldsymbol{b} \cdot \boldsymbol{r}_t$, which is Definition 5's Afriat utility function for observations in τ^j, with the U_ρ^j and λ_ρ^j taken from τ^j's binding condition B) inequalities (1) in Afriat's Theorem. If τ^j is a singleton subset, the terms in $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ follow observation *t*'s budget line. If not, those terms follow the minimum of τ^j's observations'

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in $\boldsymbol{r}$. A rationalization might require discontinuous preferences if observations with nearby $\boldsymbol{r}$**'s** have very different budget sets. We have not tried to characterize rationalizability via a continuous $u(\boldsymbol{q}, \boldsymbol{r})$. But for data generated by continuous preferences, Proposition 2's rationalizations should converge to a continuous limiting $u(\boldsymbol{q}, \boldsymbol{r})$ as the data become rich. The lack of refutable implications without constant sensitivity resembles the ambiguity of neoclassical consumer demand, where in theory small changes in income can lead to large changes in preferences and demand. In structural analyses such large income effects are ruled out implicitly by conventional functional-form assumptions. A nonparametric analysis must rule them out explicitly, as here via constant sensitivity.

budget lines, as in Figure 3b. Either way, $\boldsymbol{r}_t$ completely determines the $\boldsymbol{p}_\rho$ and $\boldsymbol{q}_\rho$ for all $\rho \in \tau^j$, as required to determine $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$. For each $\boldsymbol{r}_t$, $u(\boldsymbol{q}, \boldsymbol{r}_t)$ and $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ are continuous and increasing in $\boldsymbol{q}$. For any subset $\tau^j$and observation $t \in \tau^j$ and any $\boldsymbol{q}$ with $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$, using τ^j's binding condition B) inequalities (1) for the preferences in that subset,

$$(4) \qquad U^j(\boldsymbol{q}, \boldsymbol{r}_t) \equiv min_{\rho \in \tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} - \boldsymbol{b} \cdot \boldsymbol{r}_t$$
$$\leq U_t^j + \lambda_t^j \boldsymbol{p}_t \cdot (\boldsymbol{q}_t - \boldsymbol{q}_t) \ - \boldsymbol{b} \cdot \boldsymbol{r}_t = U_t^j - \boldsymbol{b} \cdot \boldsymbol{r}_t \equiv U^j(\boldsymbol{q}_t, \boldsymbol{r}_t). \ \blacksquare$$

5 Characterizing Reference-dependent Preferences with Constant Sensitivity and Continuity

Sections 4's results show that nonparametrically refutable implications of reference-dependence depend on the modelability of reference points and constant sensitivity. In this section, to prepare for Section 6's analysis of rationalization in that case, we characterize reference-dependent utility functions with constant sensitivity and continuity.

Suppose that preferences and a reference-dependent utility function $u(\boldsymbol{q}, \boldsymbol{r})$ satisfy: additive separability across consumption and gain-loss utility; constant sensitivity; and continuity in $\boldsymbol{q}$ and $\boldsymbol{r}$; with the number of goods $K \geq 2$ and reference-dependence active for all K goods;[13] and, for any $\boldsymbol{r}$, with preferences over $\boldsymbol{q}$ differentiable in the interior of each gain-loss regime, and marginal rates of substitution that differ across regimes throughout commodity space.

Let $G(\boldsymbol{q}, \boldsymbol{r})$ be a vector of binary numbers of length K with kth component 1 if $q^k \geq r^k$ and 0 otherwise. The gain-loss regime indicator $I_{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{r}) = 1$ if

[13] In riskless environments with convex budget sets, if $K = 1$ all monotone preferences are observationally equivalent, so reference-dependence cannot be empirically meaningful. And, as Proposition 3's wording suggests, its assumptions don't tie down the functional structure for goods for which reference-dependence is inactive. As we seek general characterizations, Propositions 4 and 5 take Proposition 3's conclusion, not its assumptions, as their premises.

$\boldsymbol{g} = G(\boldsymbol{q},\boldsymbol{r})$ and 0 otherwise; and the gain-loss indicators $G_+^k(\boldsymbol{q},\boldsymbol{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\boldsymbol{q},\boldsymbol{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise.

PROPOSITION 3: [Preferences and utility functions with continuity and constant sensitivity.] Suppose there are K ≥ 2 goods, with reference-dependence active for all K goods, and that a reference-dependent preference ordering and an associated utility function have additively separable consumption utility and gain-loss utility components. Then the ordering satisfies constant sensitivity if and only if an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$ *can be written, for some consumption utility function* $U(\cdot)$ *and gain-loss regime utility functions* $V_{\boldsymbol{g}}(\cdot,\cdot)$ *and* $v_{\boldsymbol{g}}(\cdot)$*, as*

(5) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}).$$

Suppose further that the induced preferences over $\boldsymbol{q}$ *are differentiable in the interior of each regime, with marginal rates of substitution that differ across regimes throughout commodity space. Then the ordering satisfies constant sensitivity and continuity if and only if it is representable by a utility function* $u(\boldsymbol{q},\boldsymbol{r})$ *that can be written, for some consumption utility function* $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *doing the work of the indicator* $I_{\boldsymbol{g}}(\cdot,\cdot)$*), as*

(6) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

Conversely, any combination of induced regime preferences over $\boldsymbol{q}$ *is consistent with continuity and constant sensitivity for some gain-loss utility functions.*

Proof: The "if" part of each claim is immediate. The "only if" part regarding (5) follows from Definition 1 via the standard characterization of additively separable preferences (Debreu 1960, Section 3). To prove the "only if" part regarding (6), note that $u(\boldsymbol{q},\boldsymbol{r})$ in (5) is continuous if and only if

(7) $$V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big) = V_{\boldsymbol{g}'}\big(v_{\boldsymbol{g}'}(\boldsymbol{q}),\boldsymbol{r}\big)$$

for any $\boldsymbol{q}$, $\boldsymbol{r}$, and *i* with $q^i = r^i$ and any gain-loss regimes $\boldsymbol{g}$ and $\boldsymbol{g}'$ that differ in the gain-loss status of good i. But (7) can hold under those conditions only if each regime's $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ is additively separable in the components of $\boldsymbol{q}$ and, for component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$, $k = 1, \ldots, K$,

(8) $$\textstyle\sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big) \equiv \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

First suppose that (7) is satisfied for some $\boldsymbol{q}$, $\boldsymbol{r}$, and *i* with $q^i = r^i$. Using the hypothesized differentiability in the interior of each gain-loss regime, if $\partial V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \neq 0$, (7) implies that $\partial V_{\boldsymbol{g}'}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \neq 0$ as well. Adding $U(\boldsymbol{q})$ to each side of (7), partially differentiating each side with respect to q^j and then q^i, with $r^i = q^i$, and taking ratios would then show that the marginal rates of substitution between goods *i* and *j* are equal across regimes $\boldsymbol{g}$ and $\boldsymbol{g}'$ for all $q^i = r^i$, a contradiction. Thus with $q^i = r^i$, $\partial V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \equiv \partial V_{\boldsymbol{g}'}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \equiv 0$ for any $j \neq i$, and standard characterization results show that for a regime $\boldsymbol{g}$, $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ is additively separable across the components of $\boldsymbol{q}$. Given that, changing the gain-loss status of a good *j* with $q^i = r^i$ would violate (7) and therefore continuity, unless for some functions $w_+^k(\cdot)$ and $w_-^k(\cdot)$, $k = 1,\ldots, K$,

(9) $$\textstyle\sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}) \equiv \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\, w_+^k(q^k,\boldsymbol{r}) + G_-^k(\boldsymbol{q},\boldsymbol{r}) w_-^k(q^k,\boldsymbol{r})].$$

Finally, unless the $w_+^k(\cdot,\cdot)$ and $w_-^k(\cdot,\cdot)$ are also additively separable in $\boldsymbol{r}$, with good-by-good responses to reference points that exactly mirror their good-by-good responses to bundles as in (8) (with $w_+^k(q^k,\boldsymbol{r}) \equiv \{v_+^k(q^k) - v_+^k(r^k)\}$ and $w_-^k(q^k,\boldsymbol{r}) \equiv \{v_-^k(q^k) - v_-^k(r^k)\}$), for some $\boldsymbol{q}$, $\boldsymbol{r}$, and *k*, changing q^k and r^k with $r^k = q^k$ would induce different changes in $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ and $V_{\boldsymbol{g}'}\big(v_{\boldsymbol{g}'}(\boldsymbol{q}),\boldsymbol{r}\big)$, violating (7) and continuity. The contradiction establishes our claim regarding (8) and completes the proof of (6). A similar argument shows that any combination of induced regime preferences over $\boldsymbol{q}$ is consistent with continuity and constant sensitivity for some gain-loss utility functions. ■

Proposition 3's class of reference-dependent utility functions generalizes the constant-sensitivity functional-structure assumptions maintained in TK's and KR's theoretical analyses and all previous empirical studies.

In particular, a gain-loss utility function must be additively separable across gain-loss regimes, across $\boldsymbol{q}$ and $\boldsymbol{r}$, and across goods within each regime, with good-by-good responses to reference points that exactly mirror their responses to the components of consumption. Thus, KR's assumption that gain-loss utility is determined by good-by-good differences between realized and reference utilities is necessary as well as sufficient for continuity. However, KR's further assumption that gain-loss utility is governed by the same function that governs consumption utility is *not* necessary for continuity. When the utility function is additively separable across consumption and gain-loss utility, continuity does not restrict the consumption utility function, which is constant across gain-loss regimes by definition. Thus, with nonparametric flexibility, Proposition 3's characterization enables us to relax the restrictions that the sum of consumption and gain-loss utility that determines a consumer's demand is additively separable across goods, and on how that sum's marginal rates of substitution vary across gain-loss regimes (CM's Table 1). As will be seen, both generalizations can be very important empirically.

Proposition 3's characterization (6) also plays a central role in Proposition 4's and 5's conditions for a rationalization with modelable reference points and constant sensitivity. With constant sensitivity a consumer's induced preferences over $\boldsymbol{q}$ and her/his optimal choice of $\boldsymbol{q}$ are independent of $\boldsymbol{r}$ within a gain-loss regime, but the maximized value of $u(\boldsymbol{q}, \boldsymbol{r})$ varies with $\boldsymbol{r}$ within a regime. (6)'s terms in $v_+^k(r^k)$ and $v_-^k(r^k)$ ensure continuity of $u(\boldsymbol{q}, \boldsymbol{r})$ despite such variation, by subtracting a regime-by-regime "loss cost". Because the loss costs depend on $\boldsymbol{r}$, not $\boldsymbol{q}$, a consumer faces a menu of fixed, exogenous regime charges, which influence her/his incentive to "defect" from an observation's consumption bundle to bundles in other regimes. This incentive constraint figures in Proposition 4's and 5's conditions for a rationalization.

6 Nonparametric Implications of Reference-Dependent Preferences with Modelable Reference Points, Constant Sensitivity, and Continuity

This section uses Proposition 3's characterization of reference-dependent utility functions that satisfy constant sensitivity and continuity to derive nonparametric conditions for a reference-dependent rationalization with modelable reference points, constant sensitivity, and continuity.

With modelable reference points and constant sensitivity, observations' consumption bundles can be objectively sorted into gain-loss regimes. By Afriat's Theorem (Section 3), GARP for each regime's observations is required for the existence of preferences that preclude defections from an observation's bundle to affordable bundles within the same regime, hence necessary for a rationalization. However, it is not sufficient, for two reasons. First, the gain-loss regime utility functions that rationalize the consumer's choices within each regime must satisfy Proposition 3's restrictions that their component utility functions must be the same across all regimes, and GARP regime-by-regime does not ensure that that is possible. Second, the rationalizing regime utility functions must also prevent defections from an observation's consumption bundle to affordable bundles in other regimes, in which the consumer's preferences may differ. This involves Section 5's loss costs, which are determined by the rationalizing regime utility functions.

Another difficulty in deriving conditions for a rationalization is that there is normally a range of rationalizing gain-loss regime utility functions, as in the neoclassical case (Varian 1982, Fact 4 and Figure 3). Choosing among them involves complex trade-offs, because a choice that lowers the gain from defecting *from* bundles in a regime raises the gain from defecting *to* them.

Propositions 4 and 5 approach these difficulties in two steps. Proposition 4 derives benchmark necessary and sufficient conditions for a rationalization, conditional on the choice of rationalizing gain-loss regime utility functions. Because those conditions are conditional, they are not directly applicable. Proposition 5 then derives directly applicable sufficient conditions based on rationalizing regime utility functions like Definition 3's Afriat utility functions. Because there are usually other rationalizing regime utility functions, Proposition 5's sufficient conditions are not necessary; but with rich enough data they should be asymptotically necessary, as explained below.

Let $\Gamma(g;\boldsymbol{r})\}$ be the set of $\boldsymbol{q}$ in regime g for $\boldsymbol{r}$. Let $\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g) \equiv \{t \in \{1,\dots,T\} \,|\, \boldsymbol{q}_t \in \Gamma(g;\boldsymbol{r}_t)\}$ be the set of t with $\boldsymbol{q}_t$ in regime g for $\boldsymbol{r}_t$.

PROPOSITION 4: [Rationalization with modelable reference points via preferences and utility functions with constant sensitivity.] Suppose that reference-dependent preferences and an associated utility function are defined over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function satisfies Proposition 3's (6). Consider data $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points. Then the statements [A] and [B] are equivalent:*
[A] There exists a continuous reference-dependent utility function $u(\boldsymbol{q},\boldsymbol{r})$ *that satisfies constant sensitivity; is strictly increasing in* $\boldsymbol{q}$ *and strictly decreasing in* $\boldsymbol{r}$*; and that rationalizes the data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$.
[B] Each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over consumption bundles, with continuous, strictly increasing consumption utility function $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$*, such that, for any regime* g *and any pair of observations* $\sigma,\tau \in \Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *again doing the work of* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

(10) $$U(\boldsymbol{q}_\sigma) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\sigma,\boldsymbol{r}_\tau)v_+^k(q_\sigma^k) + G_-^k(\boldsymbol{q}_\sigma,\boldsymbol{r}_\tau)v_-^k(q_\sigma^k)]$$

$$\leq \mathrm{U}(\boldsymbol{q}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)] \ + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau)$$

and for each observation $\{\boldsymbol{p}_\tau, \boldsymbol{q}_\tau, \boldsymbol{r}_\tau\}_{t=1,\dots,T}$ *with* $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ *and each* $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ *with* $g' \neq g$ *for which* $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \ \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$,

(11) $$U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_-^k(q^k) - v_-^k(r_\tau^k)\}]$$
$$\leq U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].$$

Proof: That [B] implies [A] is immediate. To prove that [A] implies [B], take the rationalizing regime preferences represented by $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$, which satisfy (10). Use Proposition 3 to write the condition preventing defections from the bundle of observation $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ in regime g to a bundle $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ in regime $g' \neq g$ for $\boldsymbol{r}_\tau$ with $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \ \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$:

$$u(\boldsymbol{q}, \boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau) \equiv U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} +$$
$$G_-^k(\boldsymbol{q}, \boldsymbol{r}_{\tau_\tau})\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\boldsymbol{r}_\tau)$$
(12) $$\equiv \{U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_+^k(q^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_-^k(q^k)]\} - \{U(\boldsymbol{r}_\tau) +$$
$$\textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_-^k(r_\tau^k)]\}$$
$$\leq \{U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)]\} - \{U(\boldsymbol{r}_\tau) +$$
$$\textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(r_\tau^k)]\}$$
$$\equiv U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) -$$
$$v_-^k(r_\tau^k)\}] \equiv u(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau).$$

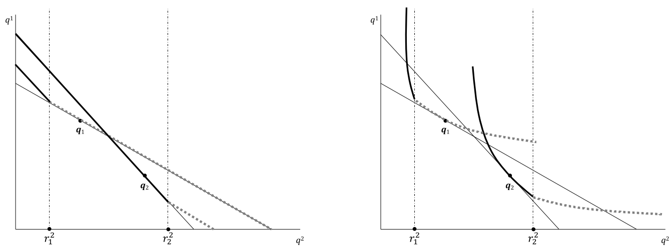
(12)'s central inequality can then be rearranged to yield (11). ■

Proving Proposition 4 requires linking Section 4's loss costs to things that can be estimated from the data, not only at given points but as functions of $\boldsymbol{r}$. The proof shows that this can be done, as in (12).

Figures 4 and 5 illustrate Proposition 4. In each case the entire dataset violates GARP, with observation 1's consumption bundle chosen in 1's budget set over observation 2's bundle, and vice versa. In each case the observations' reference points put their bundles in different gain-loss regimes, so constant sensitivity allows different preferences for each observation. And in each case each regime's single observation trivially satisfies GARP within its regime.

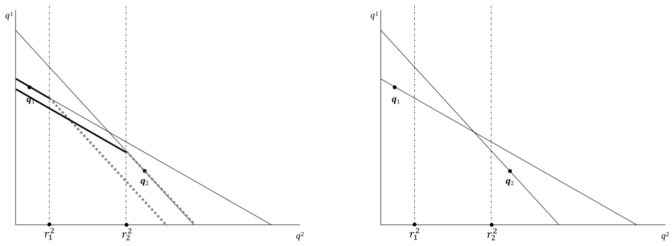
Figures 4a-b depict Afriat and non-Afriat rationalizing regime preferences. In each case condition (11) is satisfied, so that a rationalization is possible.

Figure 4. Rationalizing data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for loss maps, dashed lines for gains maps)



(a) (b)

Figure 5. Failing to rationalize data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for (a)'s loss maps, dashed lines for (a)'s gains maps)



(a) (b)

By contrast, in Figure 5a Afriat rationalizing regime preferences do not satisfy (11) and Figure 5b shows that there can be no choice of rationalizing regime preferences (Afriat or not) for which (11) is satisfied, so that a rationalization is impossible. A rationalization in Figure 5b would require regime preferences that connect a loss-regime indifference curve through observation 1's bundle to a gain-regime curve that cuts into observation 2's budget set and stays outside observation 1's budget set, thus passing northeast of 2's bundle; and also loss- and gain-regime indifference curves satisfying the analogous conditions interchanging observations 1 and 2. Such curves are inconsistent with optimality of each observation's consumption bundle.

The difference between Figure 4's and Figure 5's examples can be understood in terms of loss aversion (Definition 2). The change in rationalizing Afriat preferences across the gain-loss regimes in Figure 4a is consistent with loss aversion, but the analogous change in Figure 5a is not.

A Corollary shows that if the rationalizing regime preferences satisfy loss aversion, Proposition 4's no-cross-regime-defections constraints (11) must be satisfied, so that its conditions (10) are then sufficient for a rationalization. Thus loss aversion, which is empirically well-established but not usually seen as essential to modelling reference-dependent demand, plays a substantial role in the sufficient conditions for a reference-dependent rationalization.

Recall that the gain-loss indicator functions $G_+^k(\boldsymbol{q},\boldsymbol{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\boldsymbol{q},\boldsymbol{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise; and that $\Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g) \equiv \{t \in \{1,\ldots,T\} \,|\, \boldsymbol{q}_t \in \Gamma(g; \boldsymbol{r}_t)\}$ is the set of observation indicators t for which $\boldsymbol{q}_t$ is in regime g for $\boldsymbol{r}_t$.

COROLLARY*: [Rationalization with modelable reference points via preferences and utility functions with constant sensitivity that satisfy a condition weaker than loss aversion.] Suppose that reference-dependent preferences and an associated utility function are defined over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function therefore satisfies Proposition 3's (6). Consider data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$ *with modelable reference points. If each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over* *consumption bundles, with continuous, strictly increasing consumption utility function* $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$*, such that, for any regime* g *and any pair of observations* $\sigma, \tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ *for which* $\boldsymbol{p}_\tau \cdot \boldsymbol{q}_\sigma \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *doing the work of a regime indicator function* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

(13)
$$U(\boldsymbol{q}_\sigma) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_+^k(q_\sigma^k) + G_-^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_-^k(q_\sigma^k)]$$
$$\leq \mathrm{U}(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)] + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau),$$

and there are no observations for which $\boldsymbol{q}_t$ *is* not *on the boundary of the convex hull of* $\boldsymbol{q}_t$*'s upper contour set for the associated candidate global preference ordering for* $\boldsymbol{r}_t$*, then the consumption utility function* $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$ *rationalize the data.*

Proof: As in Proposition 4, by Afriat's Theorem, the hypothesized combination of preferences over bundles with consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$ prevent defections from any observation's consumption bundle to any affordable bundle in the same own gain-loss regime. If the hypothesized preferences are such that there are no observations *t* for which $\boldsymbol{q}_t$ is *not* on the boundary of the convex hull of the better-than-$\boldsymbol{q}_t$ set for the candidate global preference ordering given $\boldsymbol{r}_t$, then we can assume that they satisfy loss aversion without loss of generality. For, the candidate global ordering can then be replaced by a convexified ordering whose better-than-$\boldsymbol{q}_t$ sets are the convex hulls of the candidate global ordering, without changing any observation's optimal bundle. Definition 2 then implies that $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$ also prevent defections from any observation's bundle to any affordable bundle in a different regime. Alternatively, consider a defection from $\boldsymbol{q}_\tau \in \Gamma(g; \boldsymbol{r}_\tau)$ to some $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ with $g' \neq g$ and $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$. If $\boldsymbol{q}$ were in regime g, we would have, by Afriat's Theorem,

(14) $U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_-^k(q^k) - v_-^k(r_\tau^k)\}]$

$$\leq U(\boldsymbol{q}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].$$

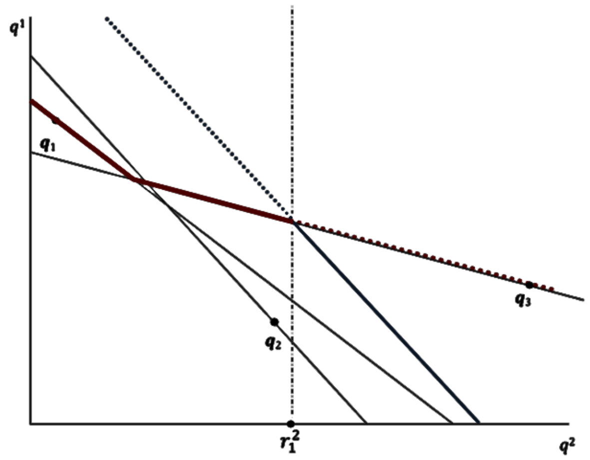
Given that $\boldsymbol{q}$ is actually in regime g', the interpretation of loss aversion in terms of marginal rates of substitution implies that the left-hand side of (14) is lower or at least no higher than if $\boldsymbol{q}$ were in regime g. (14) thus prevents defections from $\boldsymbol{q}_\tau$ to affordable bundles in different regimes. ■

Loss aversion is an empirically well-supported assumption known to have important implications, but to our knowledge it has not previously been linked to the *existence* of a reference-dependent rationalization. As the Corollary's proof suggests, loss aversion's testability is limited for the same reason that the convexity of neoclassical preferences is not nonparametrically testable.

The Corollary's final "no observations for which $\boldsymbol{q}_t$ is *not* on the boundary" condition rules out bunching of consumption bundles in regions of commodity space where the rationalizing regime preferences violate loss aversion and is vacuously satisfied for preferences that satisfy loss aversion. Such restrictions on bunching are unusual in a nonparametric analysis.

In Figure 6 the entire dataset violates GARP, the Afriat gain-loss regime preferences violate loss aversion, but the data satisfy the Corollary's final conditions, thus allowing a rationalization. Only reference point $\boldsymbol{r}_1$ is shown and observation 1 is in the good-2 loss regime. Assume that $\boldsymbol{r}_2 = [0, 0]$, so that observation 2's budget set is entirely in the good-2 gain regime; and that $\boldsymbol{r}_3 = [0, m]$, where *m* is large enough that observation 3's budget set is entirely in the good-2 loss regime. The Afriat regime preferences yield a candidate for global preferences that make all three observations' consumption bundles optimal: Observations 2's and 3's budget sets are entirely in their regimes (good-2 gain and good-2 loss, respectively), so their bundles' optimality in their regimes suffices for global optimality. Observation 1's bundle is optimal for its good-2 loss regime preferences and Corollary 1 ensures that its bundle's optimality extends to its entire budget set.

Figure 6. Rationalizing data that violate GARP when preferences violate loss aversion but satisfy the Corollary's sufficient conditions for a rationalization (solid curves for active parts of indifference maps, dashed for inactive parts)



As already noted, Proposition 4's necessary and sufficient conditions for a rationalization are not directly applicable because they are conditional on the choice of rationalizing gain-loss regime utility functions. Proposition 5 derives directly applicable sufficient conditions by specifying rationalizing regime utility functions in the style of the regime's Afriat utility functions (Definition 5). Those conditions include inequalities like (1) in Afriat's Theorem or Proposition 4's (10), which prevent defections from an observation's consumption bundle to affordable bundles in the same gain-loss regime, while enforcing Proposition 3's restrictions (6) on the component gain-loss utility functions. The conditions also include inequalities like Proposition 4's (11), which prevent defections to affordable bundles in other regimes.

PROPOSITION 5: [Sufficient conditions for rationalization with modelable reference points, via reference-dependent preferences and utility function with constant sensitivity and continuity.] The following conditions are sufficient for *the existence of continuous reference-dependent preferences and utility function with constant sensitivity* $u(\boldsymbol{q},\boldsymbol{r})$ *that rationalize data with modelable reference points* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$*: There exist numbers* U_t, v^k_{t+}, v^k_{t-}, *and* $\lambda_t > 0$ *for each* $k = 1,\dots,K$ *and* $t = 1,\dots,T$ *such that:*

[A] For any gain-loss regime g *and any pair of observations* $\sigma,\tau \in \Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ *(with the indicator functions* $G^k_+(\cdot,\cdot)$ *and* $G^k_-(\cdot,\cdot)$ *again doing the work of* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

(15) $$U_\sigma + \textstyle\sum_k [G^k_+(\boldsymbol{q}_\sigma,\boldsymbol{r}_\tau)v^k_{\sigma+} + G^k_-(\boldsymbol{q}_\sigma,\boldsymbol{r}_\tau)v^k_{\sigma-}] \leq U_\tau + \textstyle\sum_k [G^k_+(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v^k_{\tau+} + G^k_-(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v^k_{\tau-}] + \lambda_\tau \boldsymbol{p}_\tau\cdot(\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau).$$

[B] For observations σ,τ, $q^k_\sigma \geq q^k_\tau$ *for k = 1,..., K,* $U_\sigma \geq U_\tau$*; and for observations* σ,τ *and any k = 1,..., K,* $q^k_\sigma \geq q^k_\tau$, $v^k_{\sigma+} \geq v^k_{\tau+}$, *and* $v^k_{\sigma-} \geq v^k_{\tau-}$.

[C] For any pair of regimes g *and* $g' \neq g$, *observation* $\tau \in \Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}; g)$, *and bundle* $\boldsymbol{q} \in \Gamma(g';\boldsymbol{r}_\tau)$ *for which* $\boldsymbol{p}_\tau\cdot\boldsymbol{q} \leq \boldsymbol{p}_\tau\cdot\boldsymbol{q}_\tau$,

(16) $$\begin{aligned} &min_{\rho\in\Theta(\{q_t,r_t\}_{t=1,\dots,T};g')}\{U_\rho + \textstyle\sum_k[G^k_+(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho+} + G^k_-(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho-}] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}-\boldsymbol{q}_\rho)\} \\ &-min_{\rho\in\Theta(\{q_t,r_t\}_{t=1,\dots,T};g')}\{U_\rho + \textstyle\sum_k[G^k_+(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho+} + G^k_-(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho-}] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}_\tau-\boldsymbol{q}_\rho)\} \\ &\leq min_{\rho\in\Theta(\{q_t,r_t\}_{t=1,\dots,T};g)}\{U_\rho + \textstyle\sum_k[G^k_+(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho+} + G^k_-(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho-}] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}_\tau-\boldsymbol{q}_\rho)\} \\ &-min_{\rho\in\Theta(\{q_t,r_t\}_{t=1,\dots,T};g)}\{U_\rho + \textstyle\sum_k[G^k_+(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho+} + G^k_-(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v^k_{\rho-}] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}_\tau-\boldsymbol{q}_\rho)\}. \end{aligned}$$

Proof: Given choices of U_t, v^k_{t+}, v^k_{t-}, and λ_t, *t* = 1,…, *T*, that satisfy [A] and [B], let $u^g(\boldsymbol{q},\boldsymbol{r})$ denote the rationalizing Afriat regime utility function for regime g, including (6)'s loss costs, which exists by Afriat's Theorem. For $\boldsymbol{q} \in \Gamma(g;\boldsymbol{r})$, using (10) as in the proof of Afriat's Theorem:

$$\begin{aligned} u^g(\boldsymbol{q},\boldsymbol{r}) - U(\boldsymbol{r}) &\equiv U(\boldsymbol{q}) + \sum_k [G^k_+(\boldsymbol{q},\boldsymbol{r})\{v^k_+(q^k) - v^k_+(r^k)\} + G^k_-(\boldsymbol{q},\boldsymbol{r})\{v^k_-(q^k) - v^k_-(r^k)\}] - U(\boldsymbol{r}) \\ &\equiv \left\{U(\boldsymbol{q}) + \sum_k [G^k_+(\boldsymbol{q},\boldsymbol{r})v^k_+(q^k) + G^k_-(\boldsymbol{q},\boldsymbol{r})v^k_-(q^k)]\right\} \\ &\quad - \left\{U(\boldsymbol{r}) + \sum_k [G^k_+(\boldsymbol{q},\boldsymbol{r})\{v^k_+(r^k) + G^k_-(\boldsymbol{q},\boldsymbol{r})v^k_-(r^k)]\right\} \end{aligned}$$

(17)

$$\equiv min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\left\{U_\rho+\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho+}^k+G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho-}^k]+\lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}-\boldsymbol{q}_\rho)\right\}$$

$$-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho+\textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho+}^k+G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho-}^k]+\lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}-\boldsymbol{q}_\rho)\},$$

where the last identities follow as in Afriat's Theorem in the neoclassical case. The rationalizing reference-dependent utility function, including loss costs, is then $u(\boldsymbol{q},\boldsymbol{r})\equiv U(\boldsymbol{q})+\sum_g I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r})\,u^g(\boldsymbol{q},\boldsymbol{r})$. By construction, $u(\boldsymbol{q},\boldsymbol{r})$ is continuous, strictly increasing in $\boldsymbol{q}$, and strictly decreasing in $\boldsymbol{r}$.

For observations $\sigma,\tau\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)$ in the same gain-loss regime g, with $\boldsymbol{p}_\tau\cdot\boldsymbol{q}_\sigma\leq\ \boldsymbol{p}_\tau\cdot\boldsymbol{q}_\tau$, loss costs cancel out and (16) reduces to the usual Afriat inequalities (with its utilities expressed not as single numbers but as sums of consumption plus gain-loss utilities). Thus by Afriat's Theorem, [A] prevents defections to affordable bundles in the same regime.

For gain-loss regimes g and $g'\neq g$, observation $\tau\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)$, and bundle $\boldsymbol{q}\in\Gamma(g';\boldsymbol{r}_\tau)$ with $\boldsymbol{p}_\tau\cdot\boldsymbol{q}\leq\ \boldsymbol{p}_\tau\cdot\boldsymbol{q}_t$,

$$u(\boldsymbol{q},\boldsymbol{r}_\tau)-U(\boldsymbol{r}_\tau)$$

$$\equiv U(\boldsymbol{q})+\sum_k[G_+^k(\boldsymbol{q},\boldsymbol{r}_\tau)\{v_+^k(q^k)-v_+^k(r_\tau^k)\}+G_-^k(\boldsymbol{q},\boldsymbol{r}_{\tau_\tau})\{v_-^k(q^k)-v_-^k(r_\tau^k)\}]-U(\boldsymbol{r}_\tau)$$

$$\equiv\left\{U(\boldsymbol{q})+\sum_k[G_+^k(\boldsymbol{q},\boldsymbol{r}_{\boldsymbol{\tau}})v_+^k(q^k)+G_-^k(\boldsymbol{q},\boldsymbol{r}_{\boldsymbol{\tau}})v_-^k(q^k)]\right\}$$

$$-\left\{\mathrm{U}(\mathbf{r}_\tau)+\sum_{\mathrm{k}}[\mathrm{G}_+^{\mathrm{k}}(\mathbf{q},\mathbf{r}_{\boldsymbol{\tau}})\{\mathrm{v}_+^{\mathrm{k}}(\mathrm{r}_\tau^{\mathrm{k}})+\mathrm{G}_-^{\mathrm{k}}(\mathbf{q},\mathbf{r}_{\boldsymbol{\tau}})\mathrm{v}_-^{\mathrm{k}}(\mathrm{r}_\tau^{\mathrm{k}})]\right\}$$

$$\equiv min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho+\textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k+G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k]+\lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}-\boldsymbol{q}_\rho)\}$$

$$-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho+\textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k+G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k]+\lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}_\tau-\boldsymbol{q}_\rho)\}$$

(18)

$$\leq min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho+\textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k+G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k]+\lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}_\tau-\boldsymbol{q}_\rho)\}$$

$$-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho+\textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k+G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k]+\lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}_\tau-\boldsymbol{q}_\rho)\}$$

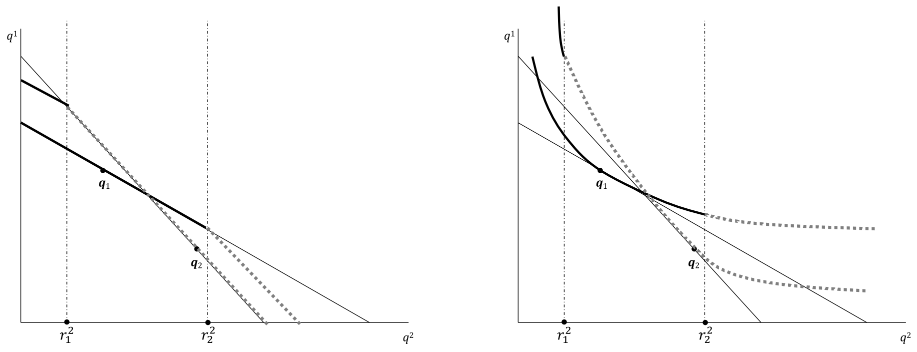
$$\equiv\left\{U(\boldsymbol{q}_\tau)+\sum_k[G_+^k(\boldsymbol{q}_\tau,\boldsymbol{r}_{\boldsymbol{\tau}})v_+^k(q_\tau^k)+G_-^k(\boldsymbol{q}_\tau,\boldsymbol{r}_{\boldsymbol{\tau}})v_-^k(q_\tau^k)]\right\}$$

$$-\left\{U(\boldsymbol{r}_\tau)+\sum_k[G_+^k(\boldsymbol{q}_\tau,\boldsymbol{r}_{\boldsymbol{\tau}})\{v_+^k(r_\tau^k)+G_-^k(\boldsymbol{q}_\tau,\boldsymbol{r}_{\boldsymbol{\tau}})v_-^k(r_\tau^k)]\right\}$$

$$\equiv u(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau))-U(\boldsymbol{r}_\tau),$$

which prevents defections across regimes. ■

Figure 7. A rationalization may require non-Afriat rationalizing regime preferences (solid lines for the loss map, dashed for the gain map)



(a) (b)

Proposition 5 depends on the choice of Afriat rationalizing regime utility functions.[14] As other choices might also suffice, its sufficient conditions are not necessary. For example, the Afriat regime preferences in Figure 7a do not yield a rationalization but the non-Afriat regime preferences in Figure 7b do.

In the neoclassical case, Mas-Colell (1978) and Forges and Minelli (2009) results study the limit of the rationalizing preferences as the data become "rich" in the sense that as $T \to \infty$ they include budget sets as close as desired to any possible budget set. They show that the range of convexified rationalizing preferences then collapses on Definition 5's Afriat preferences.[15] In the reference-dependent case with constant sensitivity, this result cannot be immediately applied gain-loss regime by regime, because of Proposition 3's constraint that the gain-loss utility functions must be the same in all regimes. But we conjecture that in the limit, as the data become rich in the sense of including {reference point × budget set} combinations as close as desired to

[14] Varian's (1982, Fact 4) bounds for the neoclassical case don't imply that all rationalizing preferences are convex, but examples show that requiring such convexity involves a loss of generality for some rationalizing regime preferences in Proposition 4. Proposition 5 avoids that difficulty by using the Afriat regime preferences, which are convex by construction.

[15] Also requiring richness of consumption bundles would rule out non-convex preferences.

any possible combination, if the Afriat regime preferences do not yield a reference-dependent rationalization, neither can any other regime preferences, so that Proposition 5's sufficient conditions are asymptotically necessary.

Propositions 4 and 5 somewhat refocus the view of reference-dependent consumer demand from structural models. Constant sensitivity, usually seen as a convenient simplification, is essential for reference-dependent models to have any nonparametrically refutable implications, as explained in footnote 4. And loss aversion, usually seen as empirically well-established but not as essential to the existence of a reference-dependent rationalization, now plays a role in the sufficient conditions for existence.

7 Empirical Illustration

Proposition 5's sufficient conditions for a reference-dependent rationalization with modelable reference points and constant sensitivity suggest methods for recovering rationalizing preferences when they exist. Although Proposition 3's characterization of reference-dependent preferences in that case would be well suited to a structural econometric analysis, here we illustrate our methods by reconsidering Farber's and CM's econometric analyses nonparametrically, using sample proxies like CM's for the targets. Our choice is motivated by curiosity regarding the robustness of Farber's and CM's structural analyses and by the computational difficulty of nonparametric estimation using more recent datasets, which are much larger than Farber's.[16]

We consider preferences over levels of and changes in earnings and leisure. With two goods, GARP (Definition 4) reduces to the Weak Axiom of Revealed Preference ("WARP"). WARP is then necessary and sufficient for a neoclassical rationalization. In this section we use "WARP" for "GARP".

[16] We are just as curious about Camerer et al.'s data, but they are no longer fully available.

DEFINITION 6: [Weak Axiom of Revealed Preference ("WARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *and* $\boldsymbol{q}_s \neq \boldsymbol{q}_t$ *implies not* $\boldsymbol{q_t} R \boldsymbol{q_s}$*, where R indicates that there is some sequence of observations* $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \dots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i$, $\boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \dots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

We relax Camerer et al.'s (1997), Farber's (2005, 2008), and CM's assumption that drivers have homogeneous preferences, instead allowing unrestricted heterogeneity of preferences. (Our theory covers both cases, distinguished only by whether the data are pooled across drivers.)

To guide future work, we compare several models of reference points, including expectations-based and recent-experience-based alternatives to CM's sample proxies. Like CM, but unlike Camerer et al. and Farber, we allow three different forms of reference-dependence: in earnings alone, in hours alone, or in both earnings and hours.

Section 7.1 reviews Farber's data. Section 7.2 outlines the models of reference-dependent preferences we compare. Section 7.3 discusses Selten and Krischker's (1983), Selten's (1991), and Beatty and Crawford's (2011) nonparametric notions of predictive success. Section 7.4 describes our estimation procedure. Section 7.5 reports our results.

7.1 Data

Like CM, we use Farber's (2005, 2008) data.[17] Farber collected 593 trip sheets for 13461 trips by 21 drivers between June 1999 and May 2001. Each sheet records the driver's name, hack number, date, each fare's start time and location, each fare's end time and location, and the fare paid. Nine sheets duplicate the day and driver, so there are only 584 shifts. Because our methods

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[17] The datasets are posted at https://www.aeaweb.org/aer/data/june08/20030605_data.zip, and https://www.aeaweb.org/aer/data/aug2011/20080780_data.zip. The CPI data are posted at https://data.bls.gov/timeseries/CUURS12ASA0, under the years 1999-2001.

make some allowance for sample size, in addition to the 15 drivers Farber and CM studied we include the 6 with samples of 10 or fewer shifts they excluded.

The Supplemental Appendix's Table A.1 reports descriptive statistics driver by driver. The values are the same as those in Farber's (2005) Table B1, except for the hourly wage variable and the Afriat efficiencies in the last two columns. Our earnings and wage variables differ from Farber's and CM's in two ways, which affect the Afriat efficiencies. First, we use the NY/NJ urban CPI to control for price level changes in the sample period. Second, Farber's and CM's wage variable is income per hour spent working, with working time defined as the sum of time driving with a passenger and time waiting for the next passenger. However, as waiting time varies randomly from shift to shift with weather, the flow of customers, etc., and is not directly linked to earnings, it appears largely exogenous. Accordingly, we define the wage as earnings per hour driving, treating waiting time as an exogenous fixed cost. This seems a natural choice in a model where income targeting is day by day.

Our redefinition of the wage matters more in our nonparametric analysis than it would in Farber's and CM's structural analyses. Drivers' waiting times range from about 25-40% of their shifts. If we included waiting in driving time, shift-to-shift wage variation would make a driver's observations' budget lines pivot around their common zero-hours end, they would never cross, he would trivially satisfy WARP, and a nonparametric analysis would give only a meaningless recapitulation of his data. By contrast, treating waiting times as a fixed cost allows a driver's budget lines to cross. The Appendix's Figure A.1 shows that with our wage definition drivers' budget lines cross frequently, making WARP a meaningful restriction and allowing a nonparametric analysis to provide a meaningful interpretation of the data.

Table A.1's last column reports each driver's Afriat efficiency index. The index is 1 for a driver whose data satisfy WARP but is otherwise less than 1. Only 7 of Farber's 21 drivers satisfy WARP. Except for drivers 12 (sample size 13), 14 (sample size 17), and 17 (sample size 10), the drivers with exact neoclassical fits (2, 3, 6, 9, 11, 13, and 15) are the ones with the smallest sample sizes among the 21 drivers. Except for drivers 2 (sample size 14), 9 (sample size 19), and 17 (sample size 10), those drivers are the same as the six (3, 6, 11, 13, 15, and 17) that Farber and CM excluded due to sample sizes ≤ 10. Small samples make it easier to satisfy WARP by chance, and those drivers' data may simply be too under-powered to reject the neoclassical model. We return to the issue of correcting for power to reject in Section 7.3.

7.2 Alternative models of reference-dependent preferences

Our reference-dependent models vary in three dimensions.[18] The first distinguishes models based on proxied rational expectations from those based on recent experience. Our expectations-based models are sample averages of a driver's choices, excluding the current shift. Our experience-based models are one-shift lags. For each kind of model we consider both unconditional models and models that condition on Farber's and CM's variables that shift demand and influence waiting time: weather (rain, snow, or dry) and time of day (day or night). This yields 18 different *kinds* of reference-point model.

The second dimension distinguishes three *forms* of reference-dependence: with respect to hours, earnings, or both hours and earnings.

The third dimension distinguishes reference-dependent or neoclassical models that do or do not impose additive separability across goods.

7.3 Nonparametric notions of predictive success

The simplest possible measure of a model's predictive success is its pass rate. A model's pass rate for driver *i*, denoted $r^i \in [0,1]$, is defined as the maximal proportion of the driver's observations that are consistent with the model. A closely related measure replaces r^i with a model's proximity π^i,

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[18] We explicitly compare only static models of reference points, but our theory covers cases where reference points are dynamic, as long as they are modelable.

defined as one minus the Euclidean distance, rescaled as a proportion of the maximum possible distance, between i's set of observations and the set of sets of observations that fit the model exactly (Beatty and Crawford 2011, pp. 2786-87). Like r^i, $\pi^i \in [0,1]$, with higher values for more successful models.

However, neither measure is adequate for comparing models of varying flexibility. Reference-dependent models are more flexible than neoclassical models and must have pass rates and proximities at least as high. This accounts for much of the profession's skepticism about their parsimony. Even neoclassical models can be highly restrictive or without nonparametric content depending on the number of observations and whether budget lines cross.

To control for flexibility, Selten and Krischker (1983) and Selten (1991) penalize a model's pass rate for flexibility using what they call the model's "area", $a^i \in [0,1]$. The area is the size of the set of all model-consistent sets of observations for driver i, relative to the size of the set of all feasible sets of observations of the same size, or equivalently the probability that uniformly random data are consistent with the model. Noting that successful models have small values of a^i and large values of r^i, Selten and Krischker define a measure of predictive success, $m(r^i, a^i) \equiv r^i - a^i \in [-1,1]$.[19] As $m \to 1$ a model's restrictions grow tighter but behavior satisfies them: a highly successful model. As $m \to -1$ a model's restrictions become looser but behavior fails to satisfy them: a pathologically bad model. As $m \to 0$ a model's compliance approaches random: a harmless but useless model.

Selten and Krischker's all-or-nothing pass rate r^i is not discriminating enough for our application, in which drivers with more than a few trips have little chance of satisfying even a reference-dependent model exactly. Accordingly, we replace their pass rate r^i with Beatty and Crawford's

[19] Selten (1991) shows that three axioms, *monotonicity* $m(1,0) > m(0,1)$; *equivalence of trivial theories* $m(1,1) = m(0,0)$; and *aggregability* $m(\lambda r_1 + (1-\lambda) r_2, \lambda a_1 + (1-\lambda) a_2) = \lambda m(r_1, a_1) + (1-\lambda) m(r_1, a_1)$ for $\lambda \in [0,1]$, characterize the measure $m(r^i, a^i) \equiv r^i - a^i$.

proximity measure π^i, following Beatty and Crawford in continuing to penalize it via Selten and Krischker's area. Thus our proposed measure is $n(\pi^i, a^i) \equiv \pi^i - a^i \in [-1,1]$. Like Selten and Krischker's measure, $n(\pi^i, a^i)$ levels the playing field between more- and less-flexible models in a well-defined, objective way. Both measures are similar in spirit to the adjusted R^2 or the Akaike Information Criteria in structural econometrics, which penalize model fit and likelihood for a model's number of free parameters. From now on we use "Selten measure" loosely for Beatty and Crawford's proximity-based measure of predictive success.

7.4 Estimation procedure

We estimate driver by driver and model by model.[20] For each model we fix whether preferences are additively separable across goods and the form and kind of reference-dependence.[21] The details are in our replication materials.

Proposition 5 immediately suggests an estimation procedure:[22]

(i) Use the observations' modeled reference points to sort their consumption bundles into gain-loss regimes.

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[20] Rather than nesting and estimating the form and kind of reference-dependence we condition on them and compare the resulting models. Nesting and estimating would be computationally complex, in part because the Afriat regime preferences are not invariant to merging regimes.

[21] With regard to additive separability across goods, Debreu's (1960) necessary and sufficient "double cancellation" condition shows that with two goods the Afriat rationalizing regime preferences preclude it in gain-loss regimes with more than one observation. We therefore use Varian's (1983, Theorem 6) linear program, specializing inequalities like those in condition [B] of Afriat's Theorem, and a version of condition (14) modified to require such separability. For proximities and Selten measures, we design and implement a computationally efficient search algorithm using the fact that that the proximity for a separable model cannot exceed that for its non-separable counterpart. Details and code are in our Replication files.

[22] This description ignores the choice of rationalizing regime preferences for drivers who are reference-dependent on less than *K* dimensions, or who are neoclassical. But Propositions 3-5 continue to hold, mutatis mutandis, for such preferences and our arguments extend to them.

(ii) Pooling the data from all regimes, use linear programming to find Afriat numbers U_t, v_{t+}^k, v_{t-}^k, and $\lambda_t > 0$ for each $k = 1, \ldots, K$ and $t = 1, \ldots, T$ that satisfy [A]'s Afriat inequalities (15).

(iii) Use the fact that for each observation in a regime, (15) can hold with equality for another observation in the regime, to choose numbers so that for observation *t* in regime *g*, the rationalizing Afriat utilities are given as in (17) in the proof of Proposition 5:

(iv) $U_t = u^g(\boldsymbol{q}_t, \boldsymbol{r}_t) \equiv min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)}\{U_\rho + \sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_t - \boldsymbol{q}_\rho)\}$.

(v) Use (ii)'s Afriat numbers U_t, v_{t+}^k, and v_{t-}^k to check that [B]'s monotonicity restrictions are satisfied.

(vi) Use (iii)'s rationalizing Afriat utilities to check, regime by regime and observation by observation, that [C]'s conditions (16) are satisfied by scanning along the budget surface.

Proposition 5's conditions (15) involve linear inequalities in a finite number of variables; and its conditions (16) involve nonlinear inequalities in a continuum of $\boldsymbol{q}$ values. Both sets of inequalities are finitely parameterized by the U_t, v_{t+}^k, v_{t-}^k, and λ_t that satisfy [A]'s (15).

Thus, as with Diewert's (1973) and Varian's (1982) methods for the neoclassical case, our procedure rests on inequality restrictions that can be checked directly in the data, without estimating econometric models of unobservable objects such as demand or labor supply curves. It also largely avoids the need for the auxiliary statistical assumptions that structural econometric approaches require for consistent estimation. Measurement error is an exception, but it too can be handled nonparametrically (Varian 1985).

For computational efficiency, instead of using the estimation procedure directly suggested by Proposition 5, or analogues of Varian's (1982, Appendix I) methods for finding solutions to (15), we use linear programming methods analogous to Diewert's (1973, Section 3) methods for the neoclassical case.

We estimate Selten and Krischker's area a^i by checking the conditions for a rationalization repeatedly for random data, as in Beatty and Crawford.[23] For a neoclassical model we use WARP or, for models that impose additive separability across goods, Varian's (1983, Theorem 6) conditions.[24] For a reference-dependent model we use Proposition 5's conditions [A]-[C], with Varian's (footnote 21) modifications for additive separability across goods.

When a model of either type does not fit exactly for driver i, we define its proximity π^i as the Euclidean distance, rescaled as a proportion of the maximum possible distance, between driver i's set of observations and the set of sets of observations that fit exactly, with the latter estimated in the process of estimating the area a^i (Beatty and Crawford 2011, pp. 2786-87). However, the conditions for fitting a model exactly greatly increase in stringency with the number of observations, and for the drivers with the seven largest sample sizes of the 21 (1, 4, 10, 16, 18, 20, and 21; sample sizes 39 to 70), repeated sampling (up to 20,000 times) yielded no passes. For such drivers we set $\pi^i = 0$, as if we found passing observations only at the maximum possible distance.

Given Propositions 5's gap between the sufficient and necessary conditions for a reference-dependent rationalization, which precludes precise estimation of proximities and Selten measures for reference-dependent models, we bound them as follows.[25] Imposing Proposition 5's within-regime conditions [A] ((15)) and monotonicity conditions [B], but not its cross-regime conditions [C]

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[23] We calculate the area by numerical (Monte Carlo) integration over the budget sets. New sets of choices that satisfy the budget constraints are repeatedly drawn and the conditions of interest are tested for each draw. The area is the proportion of those draws that satisfy the conditions. The area estimate converges as the square root of the number of draws. We draw until the uncertainty of the estimate is confined to the fifth decimal place.

[24] Because we only need WARP, not GARP, this is easily implemented for the non-additively separable model using R's igraph package. Details and code are in our Replication files.

[25] Such bounds are unnecessary for a neoclassical model because GARP is necessary and sufficient for a rationalization without regard to Varian's (1982, Fact 4).

((16)), yields an approximate upper bound on the proximity—"approximate" because conditions [A] assume the Afriat regime utilities and so are sufficient but not necessary; so the true proximity could be higher than the upper bound. Imposing [A]-[C] yields an approximate lower bound on the proximity, which could also be higher than the lower bound. The approximate lower and upper bounds on a reference-dependent model's Selten measure follow similarly. In each case a one-sided approximate lower bound suffices for our purposes.

7.5 Results

We now summarize our estimation results. To create a comprehensible path through the large number of models considered, we proceed sequentially. We first compare neoclassical and reference-dependent models that impose or relax additive separability across goods. This comparison so strongly favors relaxing separability that that can be seen by looking at aggregate summaries.

Next, relaxing additive separability across goods, we compare reference-dependent models that differ in the kind and form of reference-dependence. Although reference-dependent models differ significantly from neoclassical models for many drivers, the kind and form of reference-dependence make little difference, as we show again via aggregate summaries.

Finally, continuing to relax additive separability across goods, we compare neoclassical and reference-dependent models more comprehensively, first via aggregate summaries and then driver by driver.

7.5a *Additive separability across goods*

Additive separability across goods has been assumed in all previous theoretical and empirical work on this topic, but it lacks theoretical or empirical justification, and Proposition 3's characterization of reference-dependent preferences with constant sensitivity shows that it is unnecessary.

Figures 8-11 give the empirical cumulative distribution functions ("CDFs") of proximities and Selten measures for neoclassical and reference-dependent models that impose or relax additive separability across goods.[26] Each CDF pools over all 21 drivers. For reference-dependent models each CDF also pools over all 18 kinds and three forms of reference-dependent model.

Figure 8: Empirical CDFs of Proximities for Neoclassical Models Imposing and Relaxing Additive Separability Across Goods

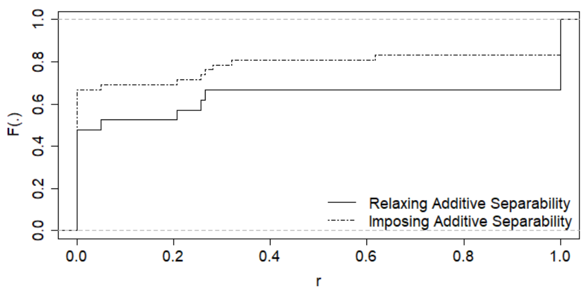
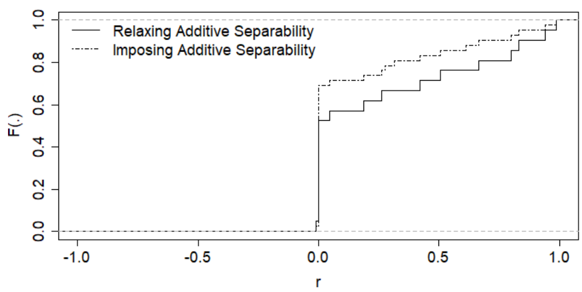


Figure 9: Empirical CDFs of Selten Measures for Neoclassical Models Imposing and Relaxing Additive Separability Across Goods



Figures 8-9 show that neoclassical models that relax additive separability across goods have higher proximities and Selten measures than models that impose it. These aggregate summaries don't show for how many drivers

[26] These comparisons also relax KR's constant-sensitivity constraints on how marginal rates of substitution vary across gain-loss regimes. A reference-dependent model must have at least as high a proximity as its neoclassical counterpart, but its Selten measure could be higher or lower. There is a minor exception for experience-based reference-point models, in which we lose one observation (two for models that condition on something) due to the construction of the lag. This can yield a slightly higher upper proximity bound for the neoclassical model.

relaxing additive separability improves a neoclassical model's fit enough to justify the added flexibility but the gap in Selten measures is large enough to confirm that relaxing separability is preferable for neoclassical models.

Figures 10-11 show that reference-dependent models that relax additive separability across goods also have significantly higher proximities and Selten measures than models that impose it. Figure 11's gap in Selten measures is large enough to confirm that relaxing separability is also preferable for reference-dependent models. From now on, we set aside models that impose additive separability across goods and focus on models that relax it.

Figure 10: Empirical CDFs of Proximities for Reference-dependent Models Imposing and Relaxing Additive Separability Across Goods

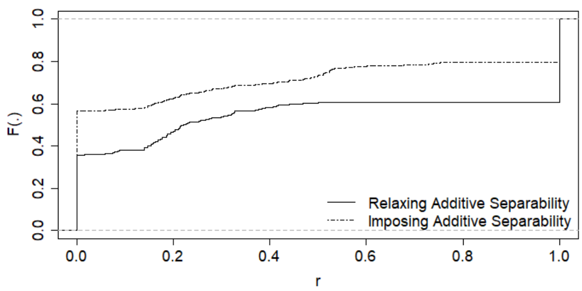
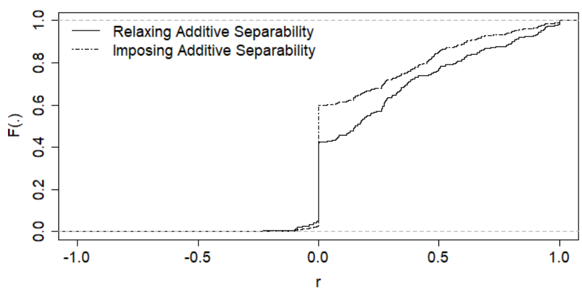


Figure 11: Empirical CDFs of Selten Measures for Reference-dependent Models Imposing and Relaxing Additive Separability Across Goods



7.5b. *Reference-point models*

Figures 12-15 give the empirical CDFs of proximities and Selten measures for the unconditional reference-dependent models we consider, again relaxing additive separability across goods. Figures 12 and 13 compare the CDFs for our 18 different kinds of reference-point model, again pooling over all 21 drivers. Figures 14 and 15 compare the CDFs for our three forms of reference-dependence, also pooling over all 21 drivers.

Figure 12: Empirical CDFs of Proximities
for Different Kinds of Reference-dependent Model

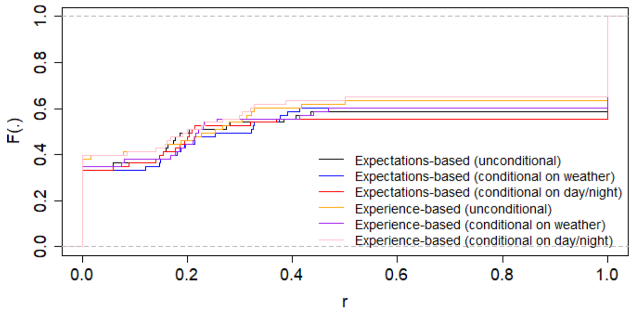


Figure 13: Empirical CDFs of Selten Measures
for Different Kinds of Reference-dependent Model

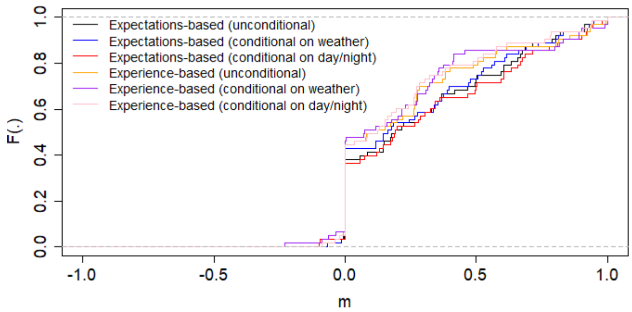


Figure 14: Empirical CDFs of Proximities

for Different Forms of Reference-dependence

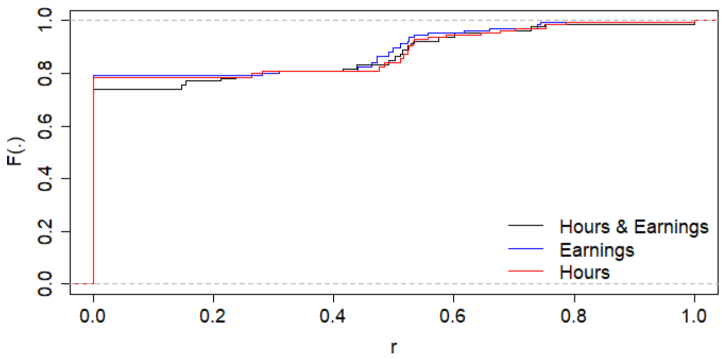
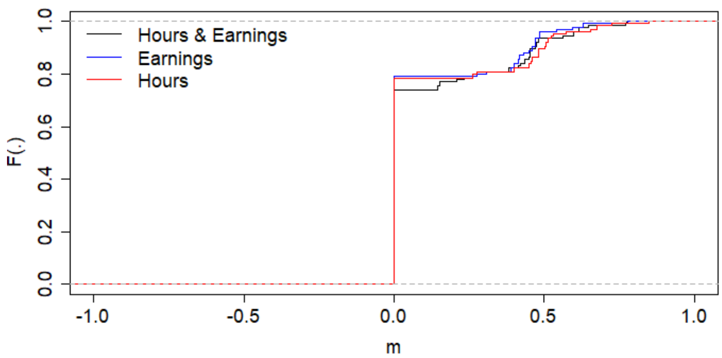


Figure 15: Empirical CDFs of Selten Measures

for Different Forms of Reference-dependence



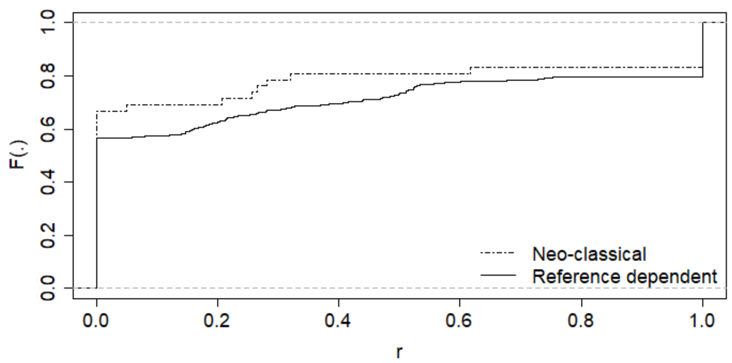
Figures 13's and 15's Selten measures show that in these data there are comparatively small differences among models' kinds and forms of reference-dependence. Expectations-based models usually have higher Selten measures than experience-based models, and unconditioned expectations-based models have measures almost as high as conditioned ones, though expectations-based models that are conditioned on day/night usually have even higher Selten measures. Expectations-based models with hours- and earnings-targeting have measures approximately as high as such models with only hours-targeting and somewhat higher measures than such models with only earnings-targeting.[27]

Accordingly, from now on we focus on unconditioned expectations-based models (also relaxing additive separability across goods), but we also report results for unconditioned experience-based reference-point models, in each case considering all three forms of reference-dependence.

7.5c *Comparing neoclassical and reference-dependent models*

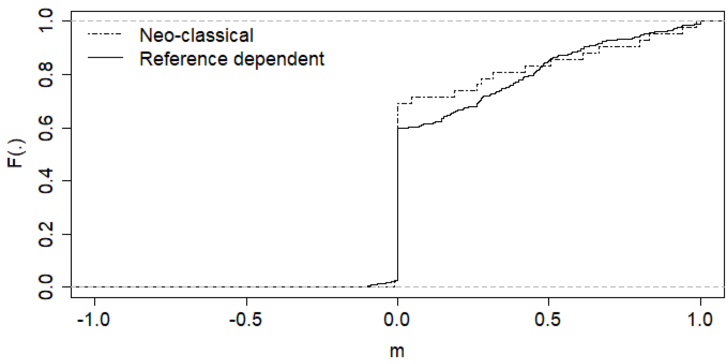
Figures 16 and 17 give the empirical CDFs of proximities and Selten measures for neoclassical versus expectations-based reference-dependent models, pooling over drivers and kinds and forms of reference-dependent model. In these aggregate summaries, neoclassical models have higher Selten measures than reference-dependent models for measure values from 0 to 0.5, but slightly lower Selten measures for values from 0.5 to 1.0, so the comparison is inconclusive, we believe due mainly to driver heterogeneity.

Figure 16: Empirical CDFs of Proximities
for Neoclassical and Reference-dependent Models



[27] Figure 15's demonstration that expectations-based models with only hours-targeting perform better than those with only earnings-targeting is surprising, given the exclusive focus in most previous empirical work on earnings-targeting. We stress that our analysis uses modelable targets to identify and distinguish the influences of hours through consumption versus via gain-loss utility (see (12) and (15)), so this is not a "neoclassical" effect.

Figure 17: Empirical CDFs of Selten Measures
for Neoclassical and Reference-dependent Models



Figures 18-21 give driver-by-driver plots for neoclassical and expectations-based and experience-based reference-dependent models' proximities and Selten measures. (The Appendix's Tables A.2-A.5 give the precise numerical values behind the plots.) Each figure has separate plots for different forms of reference-dependence, with a separate "spoke" for each driver. Figures 14's and 16's proximity plots are centered at -0.25, for clarity a tick below the lowest possible value of 0; with outer rims at the highest possible value of 1. The solid lines trace proximities for the neoclassical model. The shaded areas depict Section VII.D's approximate bounds on the proximities for the reference-dependent models. Figures 19's and 21's Selten measure plots are centered at the lowest possible value of -1, with outer rims at the highest possible value of 1. The solid lines trace measures for the neoclassical model.

Overall, the qualitative model comparisons differ only slightly across forms of reference-dependence, so we focus on models with reference-dependence in both hours and earnings, whose plots are in the left-most panels.

Neither model has uniformly higher Selten measures. In Figure 19, the expectations-based reference-dependent model has the same bounded Selten measure as the neoclassical model (thus possibly higher, Section 7.3) for seven of 21 drivers (1, 4, 10, 16, 18, 20, and 21); an unambiguously higher measure for six (5, 7, 8, 12, 17, and 19); and an unambiguously lower measure for eight (2, 3, 6, 9, 11, 13, 14, and 15). Similarly, in Figure 21, the experience-based reference-dependent model has the same (possibly higher) bounded Selten measure as the neoclassical model for six drivers: 1, 10, 16, 18, 20, and 21; a higher measure for four: 4, 8, 17, and 19; a lower measure for nine: 2, 3, 6, 9, 11, 12, 13, 14, and 15; and ambiguous bounds for two: 5 and 7.

However, not all drivers' comparisons are equally informative. Consider first the expectations-based model with reference-dependence in both hours and earnings. With our CPI adjustment, all but one of the six drivers Farber and CM excluded due to small (≤ 10) sample sizes (3, 6, 11, 13, 15, and 17) has an exact neoclassical fit, and the neoclassical model has a higher Selten measure than its more flexible reference-dependent counterpart. This is good news for the neoclassical model, but might only reflect overfitting. For seven other drivers (1, 4, 10, 16, 18, 20, and 21) the sample sizes were too large for us to estimate the set of sets of observations that fit exactly. So for them the proximities are set to 0 for both models and the neoclassical model again has a higher Selten measure; but that does not truly favor the neoclassical over the reference-dependent model. For the eight remaining drivers (2, 5, 7, 8, 9, 12, 14, and 19), the expectations-based model with reference-dependence in hours and earnings has a higher Selten measure for five (5, 7, 8, 12, and 19) and the neoclassical model has a higher Selten measure for three (2, 9, and 14).

Similarly, the experience-based model with reference-dependence in hours and earnings has a higher Selten measure for four drivers (7, 8, 14, and 19) and the neoclassical model has a higher measure for four (2, 5, 9, and 12).

Thus, for many of Farber's drivers who violate rationality for a neoclassical model a reference-dependent model gives a coherent rationality-based account of their choices. Judging by Selten measures, for many of these drivers the reference-dependent model is more parsimonious despite its greater flexibility.

Figure 18. Proximities for neoclassical and unconditional expectations-based reference-dependent models

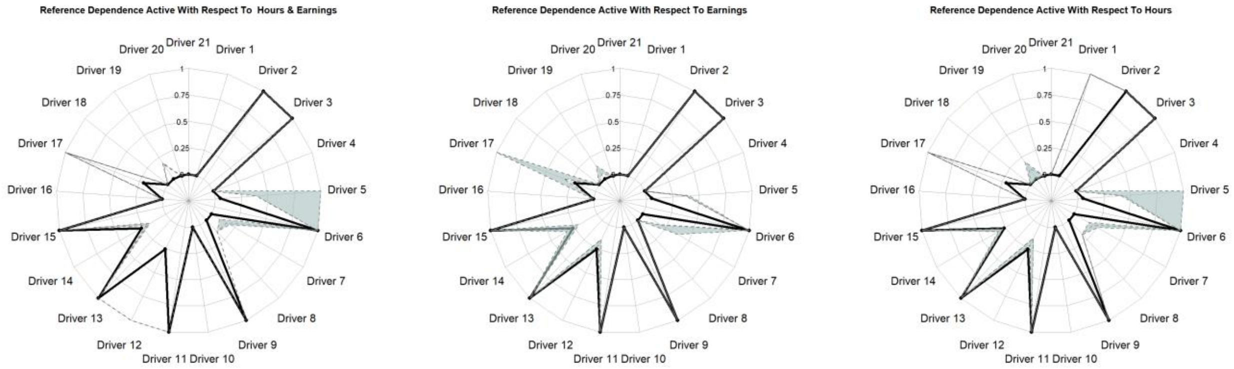


Figure 19. Selten measures for neoclassical and unconditional expectations-based reference-dependent models

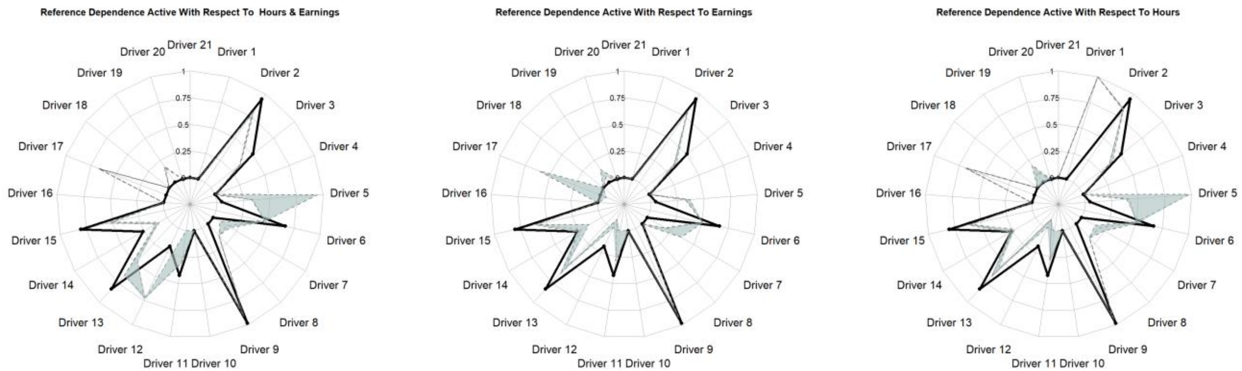


Figure 20. Proximities for neoclassical and unconditional experience-based reference-dependent models

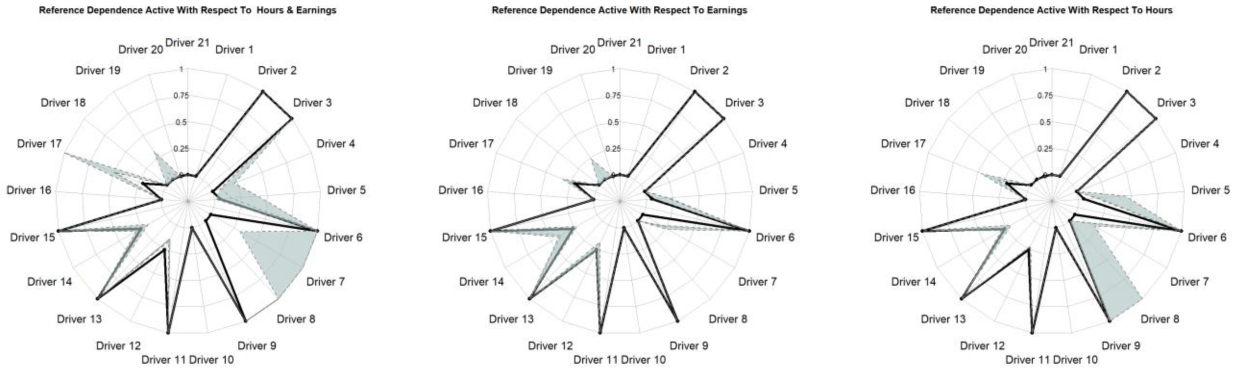
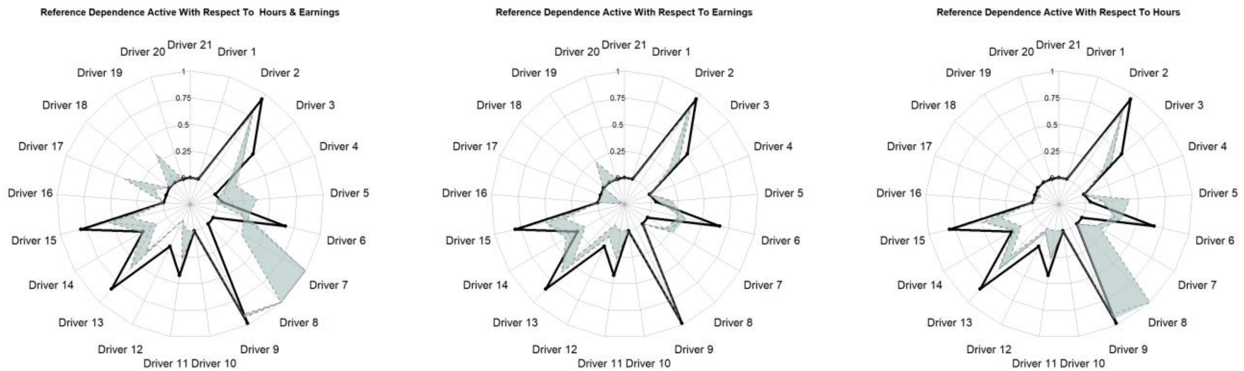


Figure 21. Selten measures for neoclassical and unconditional experience-based reference-dependent models



8 Conclusion

This paper presents a nonparametric analysis of the theory of consumer demand or, equivalently, labor supply with reference-dependent preferences. Our nonparametric model closely follows KR's structural model, maintaining their and others' assumption that preferences are additively separable across components of consumption and gain-loss utility but relaxing their and others' unnecessarily restrictive assumptions on functional structure and form.

Propositions 1 and 2 show that unless reference points are precisely modelable or observable and sensitivity is constant, reference-dependent models of consumer demand are flexible enough to fit any data, with a minor exception when sensitivity is variable.

Assuming modelable reference points, Proposition 3 characterizes preferences that satisfy constant sensitivity and are continuous, paving the way for positive results. It identifies the most general class of reference-dependent utility functions that have nonparametrically refutable implications for consumer demand, relaxing the unnecessarily restrictive assumptions in all previous theoretical and empirical work, that the sum of consumption and gain-loss utility that determines consumer demand is additively separable across goods and that its marginal rates of substitution vary across gain-loss regimes in a particular way.

Proposition 3 directly suggests methods for structural estimation of reference-dependent consumer demand, with conventional assumptions on the now-separate functional forms of consumption and gain-loss utility and using sample proxies like Camerer et al.'s and CM's for the targets.

In this paper, however, we continue our nonparametric theoretical analysis in Propositions 4 and 5, which use Proposition 3's characterization to derive sufficient and, with rich enough data, asymptotically necessary conditions for a reference-dependent rationalization, relaxing the unnecessarily restrictive assumptions maintained in all previous theoretical and empirical work. Like the classic nonparametric analyses of neoclassical consumer demand, our conditions make essential use of rationality, but now within the daily bracket and over a preference domain expanded in the disciplined way suggested by reference-dependence. Our analysis requires substantial generalizations of the neoclassical analyses, because levels of and changes in a good's consumption are bundled and priced together, and a reference-dependent consumer can, in effect, change her/his preferences by buying a bundle in a different gain-loss regime.

We illustrate our results by re-analyzing Farber's (2005, 2008) data. Although the GARP condition for a neoclassical rationalization is violated for most of Farber's drivers, our methods yield coherent reference-dependent rationalizations for almost all of most drivers' choices. For these drivers, models that relax the restrictive assumptions have significantly higher Selten measures of predictive success than their counterparts imposing additive separability, showing that our relaxations of the assumptions in previous work are empirically important.

For many of Farber's drivers an expectations-based reference-dependent model has at least as high or higher Selten measure than a neoclassical model, This suggests that reference-dependent models of consumer demand are a useful addition to the neoclassical consumer demand toolkit, which might allow parsimonious, rationality-based explanations of otherwise puzzling behaver in labor supply, consumer demand, housing, and finance.

Acknowledgements

We are especially grateful to Ian Crawford, whose contributions to the current version were extremely valuable. We are also grateful for valuable advice to Abi Adams, Richard Blundell, Alec Brandon, Colin Camerer, David Cooper, Stefano DellaVigna, Ernst Fehr, David Freeman, Georgios Gerasimou, Botond Kőszegi, John List, Juanjuan Meng, Robert Moffit, Adam Sanjurjo, Ricardo Serrano-Padial, Matthew Rabin, Joel Sobel, and Richard Thaler.

Funding

Blow's work was supported by the European Research Council under grant ERC-2009-StG-240910-ROMETA and the Economic and Social Research Council's Centre for Microeconomic Analysis of Public Policy at the Institute for Fiscal Studies under grant RES-544-28-5001. Crawford's work was supported by All Souls College, the University of Oxford, and the European Research Council under grant ERC-2013-AdG 339179-BESTDECISION. The contents reflect only our views and not those of the E.R.C. or the European Commission, and the European Union is not liable for any use that may be made of the information contained herein.

Conflict of interest

The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

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Supplemental Appendix

A.1 *Data*

Like CM, we use Farber's (2005, 2008) data.[28] As explained in the text, Farber collected 593 trip sheets for 13461 trips by 21 drivers between June 1999 and May 2001. Each sheet records the driver's name, hack number, date, each fare's start time and location, each fare's end time and location, and the fare paid. Nine sheets duplicate the day and driver, so there are only 584 shifts. Because our methods make some allowance for sample size, in addition to the 15 drivers Farber and CM studied we include the 6 with samples of 10 or fewer shifts they excluded.

Table A.1 reports descriptive statistics driver by driver. The values are the same as those in Farber's (2005) Table B1, except for the hourly wage variable and the Afriat efficiencies in the last two columns. Our earnings and wage variables differ from Farber's and CM's in two ways, which affect the Afriat efficiencies. First, we use the NY/NJ urban CPI to control for price level changes in the sample period. Second, Farber's and CM's wage variable is income per hour spent working, with working time defined as the sum of time driving with a passenger and time waiting for the next passenger. As explained in the text, we treat waiting time as an exogenous fixed cost and define the wage as earnings per hour driving.

Table A.1: Descriptive statistics, driver by driver

	T	Working Hours	Driving Hours	Waiting Hours	Break Hours	Earnings ($/CPI)	Wage ($/hr)	Afriat Efficiency
Driver 1	39	6.85	4.32	2.53	0.90	153.01	36.41	0.9952
Driver 2	14	3.89	2.78	1.11	2.41	95.98	34.68	1
Driver 3	6	6.66	4.61	2.05	0.74	160.07	36.19	1
Driver 4	40	6.28	4.52	1.76	0.39	145.89	33.02	0.9978
Driver 5	23	6.46	3.98	2.48	2.11	144.00	38.12	0.9971
Driver 6	6	8.62	6.48	2.14	2.42	202.71	33.49	1
Driver 7	24	6.47	4.42	2.05	0.74	159.50	36.69	0.9991
Driver 8	37	7.78	5.13	2.64	0.86	170.33	34.23	0.9897
Driver 9	19	7.17	5.47	1.70	0.54	158.82	30.61	1
Driver 10	45	6.35	3.90	2.45	1.65	129.68	33.83	0.9954
Driver 11	6	7.15	5.22	1.93	0.71	182.40	35.50	1
Driver 12	13	6.15	4.03	2.13	0.55	155.57	39.44	0.9972
Driver 13	10	7.03	4.72	2.31	0.53	153.99	33.26	1
Driver 14	17	7.06	4.49	2.57	0.64	157.15	37.37	0.9930
Driver 15	8	10.82	7.64	3.17	0.19	217.29	29.92	1
Driver 16	70	6.84	4.56	2.28	0.93	163.56	37.72	0.9936
Driver 17	10	5.88	3.71	2.17	0.54	137.28	39.10	0.9946
Driver 18	72	8.53	5.84	2.69	0.60	194.88	35.07	0.9849
Driver 19	33	6.91	4.63	2.29	0.97	155.65	36.01	0.9870
Driver 20	46	7.10	4.80	2.30	0.67	148.76	32.73	0.9842
Driver 21	46	5.32	3.66	1.66	0.24	123.57	35.62	0.9915

[28] The datasets are posted at https://www.aeaweb.org/aer/data/june08/20030605_data.zip, and https://www.aeaweb.org/aer/data/aug2011/20080780_data.zip. The CPI data are posted at https://data.bls.gov/timeseries/CUURS12ASA0, under the years 1999-2001.

Table A.1's last column reports each driver's Afriat efficiency index. The index is 1 for a driver whose data satisfy WARP; otherwise less than 1. Only seven of Farber's 21 drivers satisfy WARP. Except for drivers 12 (sample size 13), 14 (sample size 17), and 17 (sample size 10), the drivers with exact neoclassical fits (2, 3, 6, 9, 11, 13, and 15) are the ones with the smallest sample sizes among the 21 drivers. Except for drivers 2 (sample size 14), 9 (sample size 19), and 17 (sample size 10), those drivers are the same as the six (3, 6, 11, 13, 15, and 17) Farber and CM excluded due to small (≤ 10) sample sizes. Small samples make it easier to satisfy WARP by chance, and those drivers' data may simply be too under-powered to reject the neoclassical model. Discuss the issue of correcting for power to reject in Section 7.3.

Figure A.1 shows that with our wage definition drivers' budget lines cross frequently, making WARP a meaningful restriction and allowing a nonparametric analysis to provide a meaningful interpretation of the data.

Tables A.2-A.5 give the precise numerical values behind the driver-by-driver plots for neoclassical and expectations-based and experience-based reference-dependent models' proximities and Selten measures in Figures 18-21. The Selten measures are Section 7.4's lower bounds, the estimates imposing Proposition 5's full conditions [A]-[C].

Figure A.1: Farber's (2005, 2008) drivers' hours and earnings choices

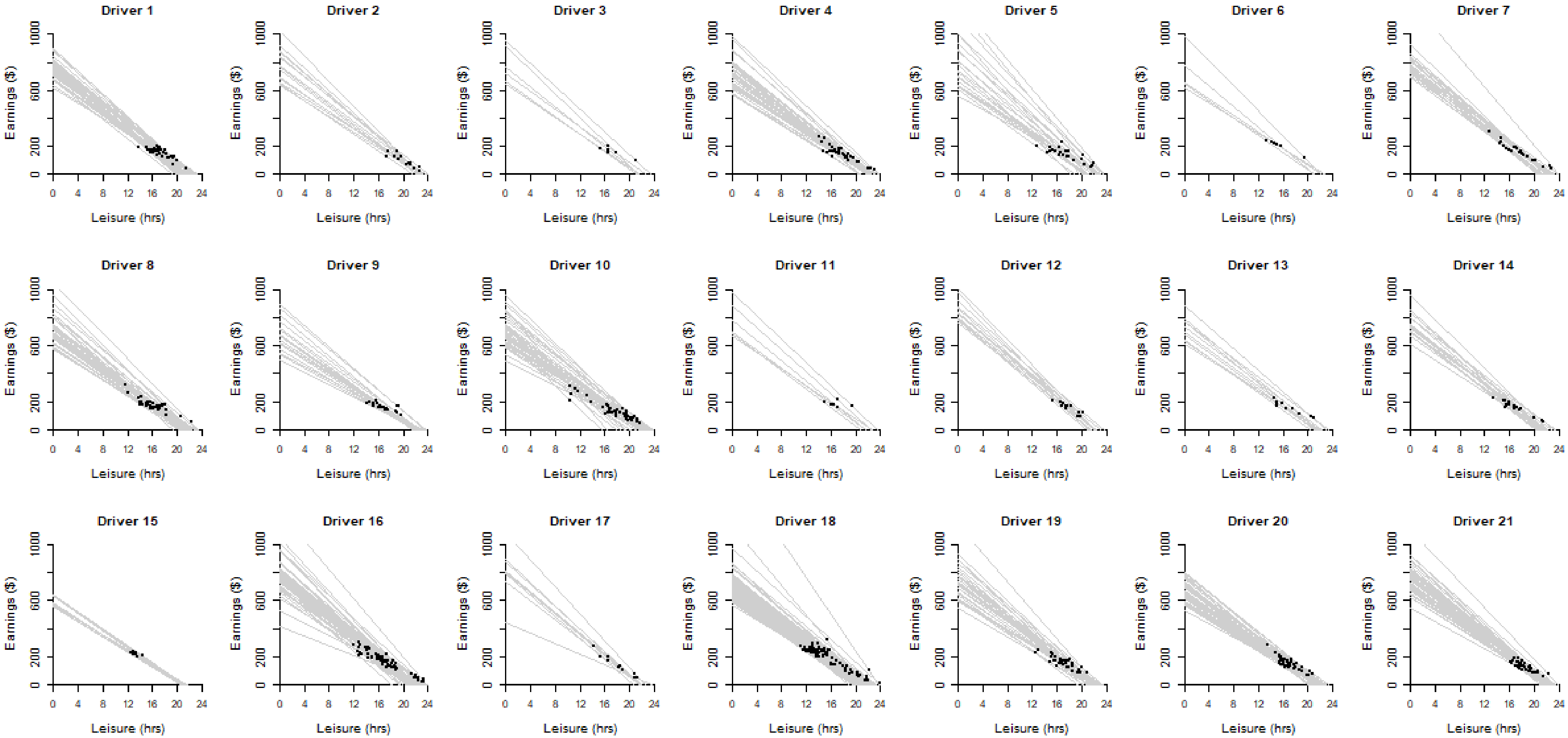

Table A.2: Proximities for neoclassical and unconditional expectations-based reference-dependent models

	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	1
Driver 2	14	1	1	1	1
Driver 3	6	1	1	1	1
Driver 4	40	0	0	0	0
Driver 5	23	0.048492	0.384315	0.406994	0.43469
Driver 6	6	1	1	1	1
Driver 7	24	0	0.162525	0.203046	0.15848
Driver 8	37	0	0.146842	0	0.17589
Driver 9	19	1	1	1	1
Driver 10	45	0	0	0	0
Driver 11	6	1	1	1	1
Driver 12	13	0.256557	1	0.15315	0.13958
Driver 13	10	1	1	1	1
Driver 14	17	0.265625	0.172794	0.280798	0.27437
Driver 15	8	1	1	1	1
Driver 16	70	0	0	0	0
Driver 17	10	0.207382	1	1	1
Driver 18	72	0	0	0	0
Driver 19	33	0	0.185454	0.057727	0
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0

Table A.3: Selten measures for neoclassical and unconditional expectations-based reference-dependent models

	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	1
Driver 2	14	0.942	0.788	0.803	0.82
Driver 3	6	0.505	0.343	0.344	0.361
Driver 4	40	0	0	0	0
Driver 5	23	0.047492	0.328315	0.367994	0.416699
Driver 6	6	0.665	0.466	0.499	0.497
Driver 7	24	0	0.148525	0.193046	0.145481
Driver 8	37	0	0.144842	0	0.174891
Driver 9	19	0.987	0.912	0.913	0.941
Driver 10	45	0	0	0	0
Driver 11	6	0.421	0.303	0.287	0.279
Driver 12	13	0.187557	0.714	-0.09685	-0.09742
Driver 13	10	0.832	0.659	0.632	0.669
Driver 14	17	0.261625	0.084794	0.217798	0.222375
Driver 15	8	0.8	0.504	0.604	0.594
Driver 16	70	0	0	0	0
Driver 17	10	-0.01062	0.677	0.604	0.685
Driver 18	72	0	0	0	0
Driver 19	33	0	0.178454	0.056727	-0.002
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0

Table A.4: Proximities for neoclassical and unconditional experience-based reference-dependent models

	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	0
Driver 2	14	1	1	1	1
Driver 3	6	1	1	1	1
Driver 4	40	0	0.078039	0	0
Driver 5	23	0.048492	0.016492	0.212637	0.187298
Driver 6	6	1	1	1	1
Driver 7	24	0	1	0.278643	0.157611
Driver 8	37	0	1	0	0
Driver 9	19	1	1	1	1
Driver 10	45	0	0	0	0
Driver 11	6	1	1	1	1
Driver 12	13	0.256557	0.13873	0.265933	0.226169
Driver 13	10	1	1	1	1
Driver 14	17	0.265625	0.313394	0.416434	0.251828
Driver 15	8	1	1	1	1
Driver 16	70	0	0	0	0
Driver 17	10	0.207382	0.500883	0.326962	0.304039
Driver 18	72	0	0	0	0
Driver 19	33	0	0.322158	0	0
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0

Table A.5: Selten measures for neoclassical and unconditional experience-based reference-dependent models

	T	Neoclassical	Hours and Earnings	Earnings alone	Hours alone
Driver 1	39	0	0	0	0
Driver 2	14	0.942	0.804	0.856	0.814
Driver 3	6	0.505	0.373	0.401	0.381
Driver 4	40	0	0.077039	0	0
Driver 5	23	0.047492	-0.00751	0.197637	0.175298
Driver 6	6	0.665	0.267	0.274	0.285
Driver 7	24	0	0.994	0.276643	0.153611
Driver 8	37	0	1	0	0
Driver 9	19	0.987	0.924	0.935	0.938
Driver 10	45	0	0	0	0
Driver 11	6	0.421	0.271	0.261	0.261
Driver 12	13	0.187557	-0.08827	0.081933	0.036169
Driver 13	10	0.832	0.575	0.616	0.577
Driver 14	17	0.261625	0.270394	0.381434	0.219828
Driver 15	8	0.8	0.527	0.505	0.49
Driver 16	70	0	0	0	0
Driver 17	10	-0.01062	0.124883	-0.02104	-0.03996
Driver 18	72	0	0	0	0
Driver 19	33	0	0.321158	0	0
Driver 20	46	0	0	0	0
Driver 21	46	0	0	0	0