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LEARNING DYNAMICS, LOCK-IN, AND EQUILIBRIUM SELECTION IN EXPERIMENTAL COORDINATION GAMES

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Abstract

This paper compares the leading theoretical approaches to equilibrium selection, both traditional and adaptive, in the light of recent experiments by Van Huyck, Battalio, and Beil (henceforth "VHBB") in which subjects repeatedly played coordination games, uncertain only about each other's strategy choices. The large strategy spaces of VHBB's designs and the variety of interaction patterns they considered yielded rich dynamics, with systematic differences in limiting outcomes across treatments. Explaining these differences promises to shed considerable light on equilibrium selection and coordination more generally, in the field as well as the laboratory. Following earlier analyses by Bruno Broseta and myself, I propose a model that gives a flexible characterization of individual behavior and allows for strategic uncertainty, in the form of idiosyncratic random shocks to players' adjustments. The model includes representatives of the leading approaches to equilibrium selection, which are distinguished by different values of behavioral parameters, including variances that represent the level of strategic uncertainty. The model provides a framework within which to estimate the parameters econometrically, using data from the experiments. The estimates suggest that VHBB's treatments evoked high initial levels of strategic uncertainty, declining steadily to zero as subjects learned to predict each other's responses. The resulting model has nonstationary transition probabilities and history-dependent dynamics that lock in on an equilibrium of the stage game, whose prior probability distribution is nondegenerate due to persistent effects of strategic uncertainty. The analysis shows that even when strategic uncertainty is eventually eliminated by learning, it imparts a drift to the learning dynamics, whose magnitude and direction depend on the environment and the behavioral parameters. This drift makes the distribution of the limiting outcome vary across treatments much as its empirical frequency distribution varied in the experiments. In this sense, taking the persistent effects of strategic uncertainty into account allows a simple, unified explanation of VHBB's results.

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I. Introduction

Coordination and equilibrium selection are central to many questions in economics, from the determination of bargaining outcomes to the design of incentive schemes, the efficacy of implicit contracts, the influence of expectations in macroeconomics, and the nature of competition in markets.[1] Such questions are usually modeled as noncooperative games with multiple *Nash equilibria*--combinations of strategies such that each player's strategy maximizes his expected payoff, given the others' strategies. The analysis of such games must address the issue of *equilibrium selection*, the determination of which, if any, equilibrium should be taken to represent the model's implications. This issue is particularly acute in games with multiple *strict* equilibria--those in which players strictly prefer playing their equilibrium strategies to deviating. In applications involving such games, equilibrium selection is often done by introspection, goodness of fit, custom, or convenience. Yet this begs the questions of how players come to expect a particular equilibrium and what they do when their expectations are less sharply focused, limiting the insight the analysis can give into how the environment influences coordination outcomes.

Theorists have now begun to study equilibrium selection more systematically. Several leading approaches can now be distinguished: traditional equilibrium analyses and refinements, including Harsanyi and Selten's (1988) general theory of equilibrium selection and its underlying

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[1] Coordination failure models have considerable unrealized potential in macroeconomics because they yield Keynesian conclusions about the importance of expectations and the nonneutrality of policy without relying on irrationality, uncertainty, or frictions; see for example Bryant (1994) and Hahn and Solow (1995, pp. 138-139 and 150-152).

notions of risk- and payoff-dominance; equilibrium analyses of perturbed games; rational learning models that extend equilibrium analysis to the repeated game that describes the learning process; deterministic evolutionary dynamics; analyses of the long-run equilibria of stochastic evolutionary dynamics; and adaptive learning models.

These approaches sharpen the predictions of equilibrium analysis in different ways, emphasizing different aspects of the process by which players choose their strategies or form the expectations or *beliefs* about others' strategies on which their choices are based. They differ in the amount of *strategic sophistication* they attribute to players--the extent to which their beliefs and behavior reflect their analysis of the environment as a game rather than a decision problem, taking its structure and other players' incentives into account--and the amount of *strategic uncertainty* (players' uncertainty about each other's strategy choices) they allow. Traditional equilibrium analyses and rational learning models take a deductive view of behavior, attributing perfect rationality and a very high degree of sophistication to players, in effect assuming that they have complete models of each other's decisions and thereby ruling out all strategic uncertainty. Models of evolutionary or adaptive dynamics, by contrast, take an inductive view of behavior that attributes far less sophistication to players and usually allows unlimited strategic uncertainty.

The leading approaches also differ in the extent to which they follow the game-theoretic custom of striving to predict behavior as much as possible by theory, without recourse to empirical information. The general theory of equilibrium selection and the analysis of long-run equilibria are the most ambitious in this regard, determining behavior entirely by theory even in games with multiple strict equilibria. Equilibrium refinements and deterministic evolutionary dynamics do not discriminate a priori among strict equilibria, and implicitly admit some empirical information by taking players' initial beliefs as determined partly or wholly outside the model. Adaptive learning models have exogenous behavioral parameters that also create a role for empirical information.

Recent theoretical analyses have the potential to make game theory more useful in applications, but the persistence of approaches with such diverse behavioral assumptions and conclusions suggests that the issue of equilibrium selection will not be resolved by theory alone. Confronting theory with empirical evidence provides useful discipline by highlighting unrealistic behavioral assumptions, and should help to fill in the gaps in our understanding. Experiments are a particularly useful source of such evidence, because they have the tight control of preferences and information needed to test game-theoretic predictions and they make it possible to observe the entire coordination process. Crawford (1997) surveys a number of studies whose subjects repeatedly played games with multiple equilibria, uncertain only about each other's strategies. The typical result was convergence to equilibrium, often with a systematic pattern of equilibrium selection in the limit. Explaining such patterns and the dynamics that led to them promises to shed considerable light on coordination and equilibrium selection, in the field as well as the laboratory.

In this paper I assess the leading approaches to equilibrium selection in the light of some of the clearest and most intriguing experimental evidence of which I am aware, that reported in Van Huyck, Battalio, and Beil (henceforth "VHBB")(1990, 1991, 1993), following the analyses in Crawford (1995), Broseta (1993ab), and Crawford and Broseta (1997). VHBB's (1990, 1991) designs are of unusual economic interest, with subjects repeatedly playing symmetric coordination games in which their payoffs each period were determined by their own strategies, called "efforts," and an order statistic of their own and other subjects' efforts. VHBB's (1993) subjects repeatedly played one such game preceded by an auction in which a larger group of subjects bid for the right to play it. In each case, with minor exceptions, subjects were uncertain only about each other's strategies, and usually converged to an equilibrium of the *stage game* that was repeated each period--VHBB's (1990, 1991) coordination games or VHBB's (1993) auction-cum-coordination game. However, the large strategy spaces and variety of interaction patterns yielded rich dynamics, with large, systematic differences in equilibrium selection for different values of the *treatment variables* that define the environment, the order statistic and the number of players in the coordination game and the auction.

Crawford's and Broseta's models are based on a flexible characterization of individual behavior, which allows for strategic uncertainty in the form of idiosyncratic random shocks to players' initial responses and their adjustments to new observations. They include representatives of the leading theoretical approaches, distinguished by different values of empirical behavioral parameters that represent certain aspects of players' adjustments, including variances that represent the level of strategic uncertainty as it varies over time as players learn to predict each other's responses. These variances play a crucial role in equilibrium selection, but cannot be reliably explained by theory alone because they reflect differences in the beliefs of players who have identical roles, preferences, and information. The need for empirical parameters will come as no surprise to anyone who recalls Schelling's (1960) comparison of purely theoretical analyses of coordination to attempts to "prove, by purely formal deduction, that a particular joke is bound to be funny."

Crawford's and Broseta's models provide a framework within which to estimate the behavioral parameters econometrically using data from the experiments, and permit an informative analysis of the dynamics, which shows with considerable generality how the parameters interact with the treatment variables to determine the outcome. Both the estimates and the dynamics they imply discriminate sharply among the leading approaches, favoring an adaptive learning model with strategic uncertainty declining gradually to zero.

The resulting model is a Markov process with nonstationary transition probabilities and history-dependent dynamics, which lock in on behavior consistent with equilibrium in the stage game in the limit. The model's implications for equilibrium selection can be summarized by the prior probability distribution of the limiting outcome, which is normally nondegenerate due to persistent effects of strategic uncertainty. The analysis shows that strategic uncertainty imparts a drift to the learning dynamics, whose magnitude and direction are determined by the environment and the behavioral parameters, and whose effects persist long after strategic uncertainty has been eliminated by learning. This drift makes the distribution of the limiting outcome vary across treatments much as its empirical frequency distribution varied in the experiments. Thus, studying how strategic uncertainty interacts with the learning dynamics allows a simple, unified explanation of VHBB's results. The mechanism of equilibrium selection in VHBB's experiments seems typical of other experiments of this type, and presumably also of field environments.

The paper is organized as follows. Section II describes VHBB's (1990, 1991, 1993) experimental designs and results. Section III describes a class of environments that generalizes VHBB's designs and introduces a portmanteau model of behavior that includes representatives of the leading approaches to equilibrium selection. The next three sections compare VHBB's results with the implications of the leading approaches. Section IV considers traditional equilibrium analyses and refinements, Harsanyi and Selten's general theory of equilibrium selection, equilibrium analyses of perturbed games, and rational learning models. Section V considers deterministic and stochastic evolutionary dynamics. Section VI considers adaptive learning models. Section VII is the conclusion.

II. VHBB's Experimental Designs and Results

In VHBB's (1990, 1991) experiments, populations of indistinguishable subjects played symmetric coordination games in which they chose among seven strategies called "efforts," {1,...,7}, with payoffs determined by their own efforts and an order statistic of their own and other subjects' efforts. Effort had a commonly understood scale, which makes it meaningful to say that a subject chose the same effort in different periods or that different subjects chose the same effort, and which made it possible to define the order statistic. Subjects played these coordination games repeatedly, usually for 10 periods. Explicit communication was prohibited throughout; the relevant order statistic was publicly announced after each play; and with minor exceptions the structure was publicly announced at the start, so subjects were uncertain only about each other's efforts. Subjects seemed to understand the rules and were paid enough to induce the desired preferences.

I introduce the structures of VHBB's games by considering a simplified version, in which each of n players choose between two efforts, 1 and 2, with the minimum of their efforts determining total output, which they share equally. Effort is costly, but is productive enough that if all n players choose the same effort their output shares more than repay their costs. If anyone shirks, however, the balance of the others' efforts is wasted.

This game has a long history in economics, which can be traced to the Stag Hunt example Rousseau (1973 [1755], p. 78) used to discuss the origins of the social contract. To see the connection, imagine (adding some game-theoretic detail to Rousseau's discussion) that each of a number of hunters must independently decide whether to join in a stag hunt (effort 2) or hunt rabbits by himself (effort 1). Hunting a stag yields each hunter a payoff of 2 when successful, but success requires the cooperation of every hunter and failure yields 0. Hunting rabbits yields each hunter a payoff of 1 with or without cooperation, and thereby determines the opportunity cost of effort devoted to the stag hunt.

		Minimum Effort	
		2	**1**
Player's	**2**	**2**	**0**
Effort	**1**	**1**	**1**

Figure 1. Stag Hunt

Figure 1 gives the payoffs when output per capita is twice the minimum effort and the unit cost of effort is 1. For any *n*, Stag Hunt has two pure-strategy Nash equilibria, "all-2" and "all-1." All-2 is the best feasible outcome for all players, better for all than all-1. This rationale for playing effort 2 does not depend on game-theoretic subtleties; it is clearly the "correct" coordinating principle. However, effort 2's higher payoff when all players choose it must be traded off against its risk of a lower payoff when someone does not.

For a player to prefer effort 2, treating the influence of his choice on future developments as negligible, he must believe that the correctness of this choice is sufficiently clear that it is more likely than not that all of the other players will believe that its correctness is sufficiently clear to all. People are often uncertain about whether other people will believe this, and most choose effort 2 in small groups but not in large groups.[2]

In VHBB's (1990, 1991) experiments there were five leading treatments with seven efforts per subject, which varied the order statistic and the size of the groups playing the game. In VHBB's

[2] Note that these beliefs are self-confirming. They are plausible because if players choose independently, with probabilities independent of the number of players, the clarity of the principle is less likely to be sufficient the larger the group. Because this intuition concerns a choice between strict equilibria, however, it is not captured by traditional refinements like trembling-hand perfectness or strategic stability.

(1990) "minimum" treatments, populations of 14-16 subjects repeatedly played games like Stag Hunt, first in large groups with the population minimum effort determining payoffs, and then in random pairs (with new partners each period) in which each subject's payoff was determined by his current pair's minimum. Denoting subjects' efforts and the relevant minimum at time t by $x_{1t},...,x_{nt}$ and N_t respectively, subject *i*'s period *t* payoff in dollars was $0.2N_t - 0.1x_{it} + 0.6.$ In VHBB's (1991) "median" treatments, populations of nine subjects repeatedly played games like Stag Hunt in large groups, with the population median effort determining payoffs, and variations in the payoff function across three treatments. In the leading median treatment, *G*, subject *i*'s period *t* payoff in dollars, denoting the median by M_t, was $0.1M_t - 0.05(M_t - x_{it})^2 + 0.6.$ There were two other median treatments, *O* and F. *O* retained *G*'*s* premium for equilibria with higher efforts but replaced its quadratic penalty for efforts away from the median with a uniform payoff, and F eliminated the premium but retained the quadratic penalty.

Each treatment's strategic structure was qualitatively similar to Stag Hunt. Subjects' payoffs were highest, other things equal, when their efforts equaled the relevant order statistic. Because a subject's effort can influence the minimum only by lowering it, which is never advantageous, and no subject's effort can influence the median when all other players are choosing the same effort, any symmetric configuration of efforts, with $x_{it} = N_t$ or M_t for all *i*, is an equilibrium. These seven symmetric equilibria are the only pure-strategy equilibria. These equilibria are strict and Pareto-ranked, with equilibria with higher efforts better for all subjects than those with lower efforts. The efficient equilibrium is plainly the "correct" coordinating principle, but it is best for a subject to play his part of that equilibrium only if he thinks it likely that enough other subjects will do so. This creates a tension between the higher payoff of the efficient equilibrium and its fragility due to strategic uncertainty. (These features extend to the random-pairing minimum treatment, with the stage game describing the interactions of the entire population each period, and payoffs evaluated taking into account uncertainty about how they interact. It is shown in Crawford (1995, fn. 10, p. 110) that players' best responses in this treatment are given by an order statistic of the population effort distribution, the median for VHBB's payoffs.)

VHBB's (1990, 1991) designs are among the simplest models of the emergence of conventions to solve coordination problems, with a range of possible outcomes and a natural measure of efficiency. They pose central questions about how the difficulty of coordination depends on the number of players and the robustness of the efficient equilibrium, which is greater the farther the order statistic is from the minimum. Their structures are mirrored in important economic models, from the Stag Hunt to Keynes's beauty contest analogy and the more prosaic macroeconomic models surveyed in Cooper and John (1988) and Bryant (1994). To bring them closer to home, they have the structure of a faculty meeting that cannot start until a given quorum is achieved (100% in the large-group minimum game, 50% in the large-group median games). All would prefer that the meeting start on time, but waiting is costly, so each member prefers to arrive just when he expects the quorum to be achieved. Thus there is a range of Pareto-ranked equilibrium, one in which all members come on time, one in which all are one minute late, and so on.

In VHBB's (1993) experiment subjects played the nine-person median coordination game of VHBB's (1991) treatment for 10-15 periods, with the right to play auctioned each period in a population of 18 subjects. The auction was a multiple-unit ascending-bid English clock auction, as in McCabe, Rassenti, and Smith (1990), in which subjects indicated their willingness to pay the current asking price by holding up bid cards, with subjects who dropped out not allowed to re-enter the bidding. The market-clearing price was determined as follows. If the lowest price at which nine or fewer subjects remained in the auction left exactly nine subjects, all nine were awarded the right to play at that price. If that price left fewer than nine subjects, they were all awarded the right to play, with the remainder of the nine slots filled randomly from those who dropped out at the last increase, and all nine subjects paying the price before the last increase.[3] As in VHBB's (1990, 1991) experiments, explicit communication was prohibited; the median was publicly announced after each play; the structure was publicly announced at the start; and the market-clearing price was publicly announced after each auction, before the median game was played.

VHBB's (1993) auctions are an interesting form of preplay communication, in which subjects' willingnesses to pay may signal how they expect to play, and thereby alleviate the tension due to strategic uncertainty.[4] They also capture important aspects of "general equilibrium" analogs of VHBB's (1990, 1991) environments, in which players choose among coordination games with the market-clearing price analogous to opportunity costs determined by their best alternatives. The models in Cooper and John (1988) and Bryant (1994), for instance, view the entire economy as a single coordination game, but it may be more realistic to view it as composed of sectors, regions, or firms, each of which is a coordination game. Because participants must often choose among these, such an economy may be closer to VHBB's (1993) design than to their 1990 and 1991 designs.

The strategic structures of VHBB's (1990, 1991, 1993) designs and their tight control of preferences and information make them of considerable game-theoretic interest, and the experiments yielded striking results, which allow powerful tests of alternative approaches to

[3] Thus subjects never pay more than they have indicated they are willing to, and they are never excluded involuntarily unless they have indicated approximate indifference.

[4] The auction is an unusual form of preplay communication in that players' messages can directly influence their payoffs, hence are not "cheap talk"; and they are communicated only through an aggregate, the market-clearing price.

equilibrium selection.[5] Their treatments elicited roughly similar initial effort distributions, with high to moderate variances and inefficiently low efforts. With minor exceptions subjects quickly converged to behavior consistent with equilibrium in the stage game. However, the large strategy spaces and variety of interaction patterns yielded rich dynamics, in which subjects' subsequent behavior differed strikingly and systematically across treatments, with persistent differences in equilibrium selection.

In the large-group minimum treatment subjects' choices gravitated toward the lowest effort, even though this led to the least efficient equilibrium. In the random-pairing minimum treatment, by contrast, subjects' efforts converged very slowly with little or no trend. And in the median treatments subjects invariably converged to the equilibrium determined by the initial median, even though it varied across runs in each treatment and was usually inefficient. Thus, the dynamics were sensitive to the size of the groups playing the coordination game, with very different drifts, rates of convergence, and limiting equilibria in the large-group and random-pairing minimum treatments. They were also sensitive to the order statistic that determined payoffs, with strong drift and no history-dependence in the large-group minimum treatment but no drift and strong history-dependence in the median treatments.

Auctioning the right to play the median game in VHBB's (1993) experiment yielded even more striking results. When that game was played without auctions in VHBB's (1991) experiment, most subjects initially chose inefficiently low efforts, and six out of six subject groups converged to inefficient equilibria. Auctions might be expected to yield more efficient outcomes simply because subjects have diverse beliefs about each other's efforts, auctions select the most optimistic subjects to play, and optimism favors efficiency in the median game. But VHBB's (1993) subjects did much

[5] VHBB's results are replicable, with similar results reported for many related designs.

better than this argument suggests: In eight out of eight groups, they bid the market-clearing price to a level recoverable only in the efficient equilibrium and then converged to that equilibrium within 3-5 periods. Their limiting behavior was consistent with subgame-perfect equilibrium in the stage game, and their beliefs appeared to be focused as in the intuition for forward induction refinements, in which players infer from their partners' willingnesses to pay to play a game that they expect their payoffs to repay their costs, and intend to play accordingly (Ben-Porath and Dekel (1992)). The efficiency-enhancing effect of auctions suggests a novel and potentially important way in which competition might promote efficiency.

III. A Portmanteau Model

Although VHBB's results convey a powerful impression by themselves, they raise more questions than they answer. Were the outcomes VHBB observed inevitable in their environments? How would they vary with changes in the environment? Would they extend to environments beyond those that directly generalize VHBB's designs? What guidance can they offer about equilibrium selection? Only an analysis that identifies the mechanism behind VHBB's results can provide a firm basis for generalization beyond the environments they studied, and realize the full power of their experiments to inform analysis.

Because learning models easily explain the rapid convergence to equilibrium in the stage game observed in almost all treatments, the main difficulty the analysis must address is explaining the differences in equilibrium selection across VHBB's treatments. The similarity across treatments of subjects' initial responses suggests that the differences in their limiting behavior cannot be understood without analyzing the dynamics. I begin by describing a class of environments that generalizes VHBB's designs. I then introduce a portmanteau model of behavior that nests representatives of the leading approaches to equilibrium selection, both static and dynamic, so that the analysis can use the experimental evidence to distinguish among them. I apply all but rational learning to the stage game rather than the repeated game, which is less plausible, and does no better. In its most general form, the model is an adaptive learning model, based on a flexible characterization of individual behavior that allows for strategic uncertainty in the form of idiosyncratic shocks to players' initial responses and their adjustments to new observations. The leading approaches are distinguished by different values of behavioral parameters, including variances that represent the level of strategic uncertainty as it varies over time.

I assume complete information because the structures of VHBB's designs were publicly announced. For simplicity I focus on VHBB's (1990, 1991) experiments, and consider VHBB's (1993) extensive-form auction environment only when it yields additional insight; and I take effort to be continuously variable.[6]

A. Strategic environments

I begin by describing a class of environments that generalizes VHBB's designs by allowing arbitrary values of their treatment variables and more general payoff structures.

A finite population of *n* indistinguishable players play an *n*-person coordination game each period. The coordination game has symmetric player roles and one-dimensional strategy spaces, with strategies called "efforts." Effort has a commonly understood scale, the same for all players.[7]

[6] Crawford and Broseta (1997) models VHBB's (1993) experiment formally. Like Crawford (1995) and Broseta (1993ab), it allows for the discrete efforts of VHBB's (1990, 1991, 1993) experiments by viewing players' continuous efforts as the latent variables in an ordered probit model of discrete choice, in which players' efforts are determined by rounding the latent variables to the nearest feasible integer.

[7] The scale concerns the labeling of strategies, and would thus be assumed irrelevant in a traditional analysis. Here it plays an important role in defining the order statistic and players' learning rules, as the language in which they interpret their experience.

Any symmetric effort combination is an equilibrium; such combinations are the only pure-strategy equilibria; and, by symmetry, those equilibria are Pareto-ranked. Each player's best responses are given by an order statistic of all players' efforts whenever it is unaffected by his own effort. I write this order statistic, y_t, as a function of the x_{it}, players' continuous efforts at time t:

(1) $$y_t \equiv f(x_{it},\ldots,x_{nt}),$$

where $f(\cdot)$ is continuous, and for any $x_{1t},\ldots x_{nt}$ and constants a and $b \geq 0$,

(2) $$f(a+bx_{1t},\ldots,a+bx_{nt}) \equiv a+bf(x_{1t},\ldots,x_{nt}).$$

These assumptions are satisfied for the coordination games in VHBB's (1990, 1991, 1993) environments, including the random-pairing minimum treatment when players' expected payoffs are evaluated before the uncertainty of pairing is resolved (the relevant order statistic is then the population median effort; see Crawford (1995, fn. 10, p. 110)).

The structure is made public before play begins. Players then choose their efforts, and the resulting value of y_t is publicly announced.[8] Players then choose new efforts, and so on. Throughout this process, players face uncertainty only about each other's decisions, and the effects of those decisions on their payoffs are filtered through the order statistic.

B. Behavior

The model of behavior must be flexible enough to include representatives of the leading approaches to equilibrium selection, so that the empirical analysis can identify the essential features of an explanation of VHBB's results. It must, in particular, capture the idea that even if players form their beliefs and choose their efforts sensibly, they may differ in unpredictable ways. It must

[8] Although a subject was told only his pair minimum in the random-pairing minimum treatment, this can be viewed as a noisy estimate of the population median that determined his best response, which causes an increase in the dispersion of beliefs.

also describe the dynamics of beliefs and efforts realistically, in terms of observable variables, in a way that permits an informative analysis.

With these desiderata in mind, I assume that players are rational in the standard sense that their decisions maximize expected payoffs given their beliefs. I also assume that players treat their individual influences on y_t as negligible, which is plausible given the large subject populations in VHBB's experiments. This implies that players' optimal efforts each period are determined by their current payoff implications, and thus by their beliefs about the current y_t. I further imagine that in forming their beliefs, players focus on the stage game, beginning with prior beliefs about the process that generates y_t, using standard statistical procedures to revise their beliefs each period in response to new observations, and then choosing the efforts that maximize expected payoffs. Players whose priors differ may then have different beliefs even if they have always observed the same history and used the same procedures to interpret it.

I now describe a simplified version of the model in Crawford (1995), which exploits the "evolutionary" structure of VHBB's designs to give a simple statistical characterization of the dynamics in the style of the adaptive control literature, in a way that allows a more informative analysis of the effects of strategic uncertainty than now seems possible for games in general.[9] The key insight of the control literature is that describing how beliefs respond to new information does not require representing them as probability distributions or their moments. It is enough to model the dynamics of the optimal decisions they imply, which are the only aspects of beliefs that directly affect the outcome.

[9] See Nevel'son and Has'minskii (1973) or Woodford (1990, Section 2).

I represent players' beliefs by their optimal efforts, x_{it}, which when continuously variable preserve enough information to realistically describe beliefs.[10] The x_{it} adjust toward the value suggested by the latest observation of y_t according to simple linear rules:

(3) $$x_{i0} = \alpha_0 + \zeta_{i0}$$

and

(4) $$x_{it} = \alpha_t + \beta_t y_{t-1} + (1-\beta_t) x_{it-1} + \zeta_{it},$$

where the α_t and β_t are exogenous behavioral parameters, with $0 < \beta_t \leq 1$ and $\alpha_t \rightarrow 0$ as $t \rightarrow \infty$. These assumptions imply that players quickly and reliably learn to predict y_t if it converges and choose the x_{it} that are optimal given their predictions, and that the long-run steady states of the dynamics coincide with the pure-strategy equilibria of the stage game. The ζ_{it} are independently and identically distributed (henceforth, "i.i.d.") random shocks with zero means and given variances, $\sigma^2_{\zeta t}$, which represent the level of strategic uncertainty. In effect each player has his own theory of others' behavior, which gives his initial beliefs and his interpretations of new observations an unpredictable component. The independence of the ζ_{it} means that any correlation in players' beliefs or efforts that emerges, does so in response to their common observations of y_t rather than as an artifact of the distributional assumptions.

Although the form of (4) suggests partial adjustment, it is best thought of as full adjustment to players' current estimates of their optimal efforts, which respond less than fully to new observations because they are only part of players' information about the process.

10 The model with discrete efforts, by contrast, describes beliefs and efforts separately.

Suppose, for instance, that player *i*'s decisions are certainty-equivalent, so that his optimal effort equals his current estimate of the mean of y_t. Then if he thinks that the y_t are independent draws from a fixed distribution, and puts as much weight on his prior as he would on such draws, he will set $a_t = 0$ and $b_t = 1/(t + t + 1)$, as in "fictitious play." If he believes instead that the y_t are generated by a driftless random walk, he will set $a_t = 0$ and $b_t = 1$, as in a "best-response" process. Thus (4) includes as special cases two of the learning rules most often studied in the game theory literature, while allowing a wide class of values for parameters that represent the initial levels, trends, and inertia in beliefs. Learning rules of this kind have been shown in the control literature to be a robust approach to the estimation problems faced by agents who understand their environments but are unwilling to make the specific assumptions about it or unable to store and process the large amounts of information an explicitly Bayesian approach would require.

In general (4) is not fully "rational" in the game-theoretic sense, because players' priors need not be correct and linearity may be inconsistent with the rules that are optimal given their priors. However, (4) also includes representatives of traditional equilibrium analysis and the other leading approaches to equilibrium selection, as explained below.

(4) differs from the "reinforcement learning" model of Roth and Erev (1995), which is its leading competitor in the adaptive learning literature, in that in the Roth-Erev model players adjust the probabilities with which they play their strategies directly in response to their realized payoffs, without reference to their best responses. The models are otherwise similar; both allow the variances of players' responses to decline over time with experience. Although reinforcement learning describes observed behavior in experiments with matrix games quite well, as explained in Crawford (1997, Section 6.3) VHBB's subjects seemed to understand the best-response structures of the games they played, and taking this into account yields a better description of their behavior.

Roth (1995, Figure 1.2, p. 39), for instance, found that the Roth-Erev model tracks the dynamics in VHBB's large-group minimum treatment much better if it is modified to allow "common learning," in which players adjust as if they had played the most successful strategy in the population. When the population is diverse, this approximates the situation when players' learning rules incorporate the best-response structure, as in (4). In these environments, reinforcement learning also yields adjustments an order of magnitude too slow; this discrepancy cannot be corrected simply by adjusting the behavioral parameters because this disturbs the balance between the stochastic and deterministic parts of the learning dynamics.

IV. Traditional Equilibrium Analyses

In traditional equilibrium analyses, players are assumed to be rational in the decision-theoretic sense that their strategies are best responses to their beliefs, and this assumption is supplemented with the hypothesis that players share common beliefs, so that there is no strategic uncertainty (Aumann and Brandenburger (1995)). In Section III's portmanteau model, equilibrium analyses are distinguished by the restrictions $z_{it} \equiv s^2_{zt} \equiv 0$ for all i and t. When they are applied to the stage game, it is natural also to require $a_t = 0$ for all $t = 1, . . .$ because there is no reason to predict different equilibria in different periods. Different approaches are distinguished by different assumptions about a_0. When equilibrium analyses are applied to the repeated game, as in rational learning models, may vary over time. (Since $x_{it} = y_t$ when $z_{it} \equiv s^2_{zt} \equiv 0$, $ß_t$ is irrelevant in each case.)

Even though in most of VHBB's (1990, 1991, 1993) treatments subjects converged to an equilibrium in the stage game, equilibrium analyses stop well short of what is needed to understand the observed patterns of equilibrium selection. It is nevertheless instructive to consider their implications in VHBB's environments. I now discuss the implications of traditional refinements; the general theory of equilibrium selection, including risk- and payoff-dominance; equilibrium analyses of perturbed games; and rational learning models.

A. Traditional refinements

Traditional refinements such as trembling-hand perfectness and strategic stability refine equilibrium analyses by requiring that a given equilibrium is "self-enforcing" in some stronger sense. Because they take beliefs as given, they do not discriminate among the strict equilibria in any of VHBB's (1990, 1991) treatments. Traditional refinements simply do not address the strategic issues raised by these games, which all pose essentially the same question in the absence of strategic uncertainty.

The situation is a bit different in VHBB's (1993) stage game, where extensive-form refinements such as subgame-perfectness and forward induction do address some of the issues it raises. In this experiment subjects' bids and efforts were almost always consistent with the intuition for forward induction, in that they seldom bid more than their efforts made it possible to recoup, and never after the first few periods; and their limiting behavior was consistent with subgame-perfect equilibrium in the stage game (VHBB (1993, Tables V-VI)). However, both refinements are consistent with any of the seven symmetric pure-strategy equilibria of the coordination game, with players bidding their equilibrium payoffs. They are therefore too unrestrictive to help in explaining VHBB's (1993) results.

B. The general theory of equilibrium selection

Harsanyi and Selten's (1988) general theory of equilibrium selection is the most ambitious attempt to define a comprehensive theory of equilibrium selection to date. Its main building blocks are their notions of risk- and payoff-dominance, which discriminate among strict equilibria and thereby (along with ad hoc devices that come into play in special cases) yield unique predictions in a wide class of games, including VHBB's.

Payoff-dominance is defined as Pareto-superiority within the set of equilibria, with certain qualifications involving dominated strategies and symmetry. *Risk-dominance* is defined via a mental tâtonnement called the "tracing procedure," in which players begin with naive priors and then mentally simulate each other's best responses and adjust their priors accordingly, until they converge to identical (and therefore equilibrium) beliefs. These simulations are independent across players but the same for all. Roughly speaking, risk-dominance selects an equilibrium whose *basin of attraction*--the set of beliefs that make its efforts best responses--is larger than those of other equilibria, in a certain sense.

Harsanyi and Selten's theory gives priority to payoff-dominance, so that where one equilibrium is Pareto-superior to all others, as in all but one of VHBB's treatments, risk-dominance has no influence. However, their theory is coherent without this priority; and risk-dominance is of particular interest here because it embodies most of their ideas about the effects of strategic uncertainty, and it captures some of the variations in behavior across VHBB's (1990, 1991) treatments. I therefore consider the implications of Harsanyi and Selten's theory both with and without the priority they give payoff-dominance.

The logic of Harsanyi and Selten's theory suggests comparing its predictions with subjects' initial efforts in each stage game. Payoff-dominance selects the all-7 equilibrium in all but VHBB's median treatment *?* , where payoff-dominance is neutral and the theory selects all-4 in response to symmetry. This yields success rates of 31% and 37% in the large-group and random-pairing minimum treatments and 15%, 52%, and 41% in median treatments *G, O, and F* (VHBB (1991, Table II)). Without priority to payoff-dominance, Harsanyi and Selten's theory selects all-1 in the large-group minimum treatment in response to risk-dominance; all-4 in the random-pairing minimum treatment, where risk-dominance is neutral and the theory applies the tracing procedure to a uniform prior over undominated strategies; all-7 in median treatments *G* and *O* in response to risk-dominance; and all-4 in median treatment *F* in response to risk-dominance, after imposing symmetry. This lowers the success rates to 2% and 17% in the large-group and random-pairing minimum treatments and leaves it unchanged in the median treatments.

Although it is impressive that any purely theoretical principle can pick up this much of the variation in subjects' responses to VHBB's treatments, these success rates--though mostly better than random, which with seven efforts would imply a success rate of 14%--fall far short of the perfect prediction the logic of the theory requires. And this cannot be attributed to the dispersion of subjects' responses: The theory predicts the modal outcome in three out of five or two out of five treatments respectively with or without payoff-dominance, missing a large part of the central tendency of initial responses.

The theory's overall success rates are no higher for subjects' limiting behavior. This comparison leaves them unchanged for median treatments; roughly unchanged for the random-pairing minimum treatment; and significantly lower or higher, respectively with and without payoff-dominance, for the large-group minimum treatment. Without priority to payoff-dominance, Harsanyi and Selten's theory essentially reduces to the tracing procedure, so the latter comparison amounts to using the tracing procedure as a model of the learning process. Section VI's analysis shows that there were significant interactions between strategic uncertainty and the learning dynamics. Perhaps it is unsurprising that a model in which players mentally simulate each other's responses does not adequately capture these, so that learning from actual observations of behavior is different from mental tâtonnements. However, describing the central tendency of initial responses is also important in predicting equilibrium selection via history-dependent learning dynamics, which may create an important role for theories like Harsanyi and Selten's.

C. An aside on equilibrium analyses of perturbed games

I now discuss equilibrium analyses of randomly perturbed versions of VHBB's stage games that do not fit neatly into the portmanteau model, but are potentially relevant.

Anderson, Goeree, and Holt (1996) adapt McKelvey and Palfrey's (1995) notion of *quantal response equilibrium* to analyze a continuous-effort version of the large-group minimum game. A quantal response equilibrium is an equilibrium in a perturbed game in which players' payoffs are subject to privately observed, i.i.d. mean-zero shocks with a commonly known distribution, whose variance is treated as a behavioral parameter. Thus it roughly resembles a snapshot of behavior at some point (depending on the variance) during the tracing procedure. Anderson, Goeree, and Holt show that for a given variance their minimum game has a unique, symmetric quantal response equilibrium. Its effort distributions stochastically decrease with the number of players and the cost of effort, capturing some of the sensitivity to changes in treatment variables suggested by VHBB's results.[11] However, the mechanism that yields this sensitivity is very different from the one suggested by the analysis below, where all random variation is eliminated in the limit and the learning dynamics can lock in on equilibria far from the quantal response equilibrium.

Carlsson and van Damme (1993) study a class of *n*-person, two-effort games against the field that includes Section II's Stag Hunt example and other order-statistic games, in which the payoff of high effort is a nondecreasing function of the number of players who play it. However, there is incomplete information, with each player's payoff for low effort perturbed by a privately

[11] They do not consider changes in the order statistic, and their uniqueness result appears not to hold for all order statistics other than the minimum.

observed, i.i.d. mean-zero shock with a commonly known, uniform distribution. They show that when this distribution is sufficiently concentrated around zero, the only strategies that survive iterative elimination of strictly dominated strategies are those in which each player plays low effort: Inferences about these arbitrarily small payoff perturbations turn the unperturbed game (which has two strict equilibria) into a dominance-solvable game with a unique equilibrium. For more dispersed distributions, low efforts are more likely the more players there are and the closer the order statistic is to the minimum, and the equilibrium displays a sensitivity to treatment variables qualitatively similar to VHBB's results. Then, however, each player plays both efforts with nonnegligible probabilities, so that there is a significant probability of disequilibrium even in the limit. Thus, the model can capture either the sensitivity of equilibrium selection to VHBB's treatment variables or their subjects' tendency to lock in on an equilibrium, but not both at once. In any case, the inferences required to identify the equilibrium by iterated dominance go far beyond the degree of strategic sophistication usually exhibited by experimental subjects, and are presumably beyond the sophistication of players in the field.

D. Rational learning

Most rational learning models extend equilibrium analyses to the repeated game that describes players' entire interaction. They are dynamic in the sense that they allow nontrivial interactions between periods; but like equilibrium analyses of the stage game they rule out all strategic uncertainty. Just as any configuration in which all players choose the same effort is an equilibrium in any of the stage games of VHBB's (1990, 1991) experiments, any deterministic pattern of jumping among such configurations over time is an equilibrium in their repeated games. Rational learning models therefore add little to the power of equilibrium analyses to explain behavior in these environments.

V. Evolutionary Dynamics

Evolutionary analyses study environments in which games are played repeatedly in populations, characterizing the dynamics of the population strategy frequencies under simple assumptions about how they respond to current expected payoffs. In the simplest evolutionary models, a large population of indistinguishable players repeatedly play a symmetric stage game with undistinguished roles. Players' stage-game strategies are one-dimensional and have a fixed common scale, as in VHBB's (1990, 1991) experiments. Individual players play only pure strategies, with payoffs determined by their own strategies and the population frequencies of others' strategies, often by a simple summary statistic such as the mean, minimum, or median.

Most discussions of evolutionary games treat them as synonymous with random pairing to play a two-person game, as in VHBB's random-pairing minimum treatment. However, many applications are better modeled by assuming that the entire population simultaneously plays a single symmetric *n*-person game, known in biology as a *game against the field*.[12] The above description allows both interaction patterns, and it is often convenient to view random pairing as a special case of games against the field, with payoffs evaluated before the uncertainty of pairing is resolved. "Stage game" refers to the game that describes the simultaneous interaction of the entire population, rather than the two-person game played by matched pairs.

Evolutionary models attribute little or no rationality or sophistication to players and allow unlimited strategic uncertainty, and thus occupy the opposite end of the behavioral spectrum from

[12] The founding analysis of evolutionary game theory, Fisher's (1930) explanation of the tendency of the ration of male to female births to remain near 1,is a game against the field (Maynard Smith (1982, pp. 23-27)). This problem--one of the most beautiful in science--requires a game-theoretic explanation because a ratio of 1 equalizes the fitnesses of having male and female offspring, and does not maximize the growth rate of the entire population. The model is a game against the field because the fitnesses depend (nonlinearly) on the population frequencies of male and female offspring.

equilibrium analyses. Instead behavior is summarized by the law of motion of the population strategy frequencies, which in biology is derived (usually in a form known as the "replicator dynamics") from the assumption that players inherit their actions from their parents, who reproduce at rates proportional to their payoffs; and in economics from plausible assumptions about individual adjustment (Schelling (1978)).

In each case the goal is to identify the locally stable steady states of the dynamics. A remarkable conclusion emerges: If the dynamics converge, they converge to a steady state in which strategies that persist are optimal in the stage game, given the limiting strategy frequencies. Thus the limiting strategy frequencies (viewed as a mixed strategy) are in Nash equilibrium in the stage game. Although strategies are not rationally chosen (in biology, not even chosen) the population collectively "learns" the equilibrium as its frequencies evolve, with selection doing the work of rationality and strategic sophistication.

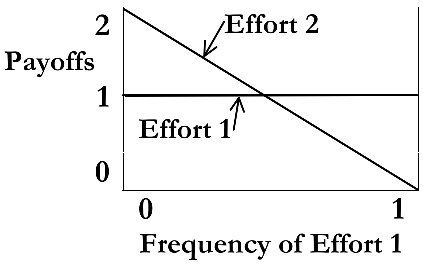
Although evolution, taken literally, has little direct influence on behavior in experiments, most experimental designs are similar in structure to evolutionary models. Crawford (1991) showed that VHBB's (1990, 1991) designs satisfy the structural assumptions of evolutionary game theory, in most cases as games against the field. This fact and an imperfect but informative analogy between evolutionary and learning dynamics make an "evolutionary" analysis helpful in understanding VHBB' results. Here I discuss two leading evolutionary approaches, deterministic evolutionary dynamics and analyses of the long-run equilibria of stochastic evolutionary dynamics.

A. Deterministic evolutionary dynamics

In the portmanteau model, deterministic evolutionary dynamics are represented by the parameter restrictions $\sigma_{\zeta 0}^2 > 0$ but $\zeta_{it} \equiv \sigma_{\zeta t}^2 \equiv \alpha_t \equiv 0$ for all *i* and *t* = 1, . . . , which allow initial heterogeneity but rule out subsequent differences in players' adjustment rules. Crawford (1991) considered the extent to which VHBB's (1990, 1991) results could be explained by finite-population variants of deterministic evolutionary dynamics or (equivalently here) static refinements such as evolutionary stability. The flavor of the analysis and the relationship between random pairing and games against the field are well illustrated by an evolutionary analysis of Section II's two-effort Stag Hunt example, whose conclusions extend immediately to VHBB's seven-effort games.

Recall that for any number of players, Stag Hunt has two symmetric pure-strategy equilibria, all-2 and all-1, both of which correspond to steady states of the dynamics. The same conclusions hold when n players are randomly paired to play two-person versions of Stag Hunt. In each case there is also a symmetric mixed-strategy equilibrium, which also corresponds to a steady state.

Figure 2 (Crawford (1991, Figure 1)) graphs the expected payoffs of efforts 1 and 2 against the population frequency of effort 1 for Stag Hunt with random pairing and against the field. Effort 1's payoff is constant at 1 in each case. With random pairing effort 2's payoff is a population frequency-weighted average of its payoffs when the other player chooses efforts 1 and 2, hence linear in those frequencies; while against the field, effort 2's payoff drops discontinuously from 2 to 0 as the frequency of effort 1 rises above 0. The mixed-strategy equilibrium frequency is always unstable in this game. With random pairing both all-2 and all-1 are locally stable, and their basins of attraction are equally large. Against the field only all-1 is locally stable (even though all-2 is strict in the stage game, as explained in Crawford (1991)) and its basin of attraction is almost the entire state space. Moving the order statistic above the minimum moves the discontinuity in effort 2's payoff to the right, making all-2 locally stable, but with small basins of attraction for order statistics near the minimum. These conclusions extend to finite populations.



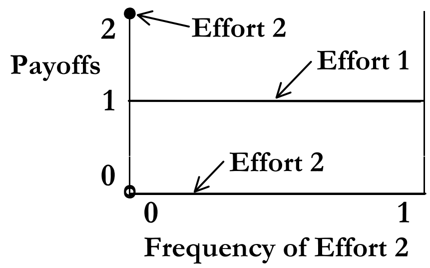


Figure 2a. Random pairing Stag Hunt **Figure 2b. Stag Hunt against the field**

Similarly, in VHBB's large-group minimum game with seven efforts only the all-1 equilibrium is locally stable. But in VHBB's median treatments, or in any such game with an order statistic other than the minimum (including the random-pairing minimum treatment, which is equivalent to a large-group median treatment as explained in Crawford (1995), all seven symmetric pure-strategy equilibria are locally stable.

Deterministic evolutionary dynamics have two advantages over equilibrium analyses for the purpose of explaining results like VHBB's. Together with the dispersion of initial responses, the effect of the order statistic on the sizes of the basins of attraction begins to capture the interaction between strategic uncertainty and learning dynamics. And the dynamics give a rudimentary account of history-dependent equilibrium selection, in which the population always converges to the equilibrium whose basin of attraction includes its initial state. I now record a result (stated informally in Crawford (1995)) that gives a more general account of this history-dependence, and thereby shows how to use the model as a kind of accounting system for keeping track of the probabilities of possible changes in y_t.

Proposition 1: Suppose $a_t = 0$ and $b_t \in (0,1]$ for all t = 1, . . . , and that there exists an integer T = 1 such that $z_{it} \equiv s^2_{zt} \equiv 0$ for all $t = T$, ... Then for all i, $x_{it} \to y_{T-1}$ monotonically, without overshooting, and $y_t = y_T$ for all $t = T$, . . . , independent of the number of players n and the order statistic $f(\cdot)$.

Proof: (4) with $a_t = 0$ and $b_t \in (0,1]$ for all $t = 1, \ldots,$ and $\zeta_{it} = 0$ for $t = T, \ldots,$ implies that $x_{iT} - y_{T-1}$ always has the same sign as $x_{iT-1} - y_{T-1}$, with x_{iT} closer to y_{T-1} than x_{iT-1} was. Because $f(\cdot)$ is an order statistic, it follows that $y_T = y_{T-1}$, and so on.

The proof exploits the fact that, by the definition of an order statistic, y_t can change only if more players overshoot it in one direction than in the other. When players' adjustments follow (4), on average they do not overshoot, because $b_t \in (0,1]$. Thus, unless their responses to new observations continue to differ, their efforts collapse mechanically on the current y_t, and continued change in y_t depends on *persistent* strategic uncertainty. In particular, if players differ in their initial responses but interpret new observations in the same way, y_t remains forever at y_0, independent of the treatment variables.

This perfect history-dependence is consistent with the results for VHBB's median treatments, but not for their large-group minimum treatment. There, the average subject with effort above the minimum adjusted only part of the way toward the minimum, and if subjects did not differ in their responses to new observations, the minimum could never have fallen. In fact there was enough variation in subjects' responses to make the minimum fall in 9 out of the 13 instances in which it was not already at the lowest possible level. The model of adaptive learning dynamics discussed below captures this variation by allowing moderate values of $s_{\zeta 0}^2$, with $s_{\zeta t}^2 \to 0$ steadily over time as one would expect as subjects learn from their common observations of y_t. This yields enough overshooting to explain the dynamics of y_t in the large-group minimum treatment, while also reproducing the failure of y_t to change in the median treatments, where the median smoothes the effects of individual dispersion.

Overall, deterministic evolutionary dynamics hint at the patterns of equilibrium selection in VHBB's (1990, 1991) experiments, but are too unrestrictive to explain them. They have two main drawbacks: (i) they take the initial state of the population as a datum, even though it is the principal determinant of the limiting outcome; and (ii) they give little guidance on how the sizes of basins of attractions affect equilibrium selection, beyond the intuition that the initial state is more likely to fall into a larger basin. Section VI's adaptive learning model responds to these drawbacks by estimating the distribution of subjects' initial responses and using an explicit, stochastic model of learning at the individual level, which allows "tunneling" across basins of attraction with positive probability and the more subtle forms of history-dependence found in the experiments.

B. Stochastic evolutionary dynamics and long-run equilibria

Analyses of long-run equilibria of stochastic evolutionary dynamics (Kandori, Mailath, and Rob (1993) and Young (1993)) assume population interaction patterns like those in simple evolutionary game theory. The state of the population is characterized by its current mix of strategies, and players' adjustments are assumed to move their strategies to or toward their best responses to the current state. The main difference from deterministic evolutionary dynamics is that players' adjustments are subject to random "mutations," whose probability is constant over time and independent of the state.

In the portmanteau model, the required assumptions correspond to the parameter restrictions $s_{zt}^2 \equiv e > 0$ (or $s_{zt}^2 \to e > 0$) and $a_t \equiv 0$ and $b_t \equiv b$ for all t = 1, ... The resulting dynamics are a Markov process with (at least eventually) stationary transition probabilities. The analysis allows strategic uncertainty, in that the population need not be in equilibrium from the start, and assumes little or no strategic sophistication.

When $e = 0$ the dynamics may have many steady states, which normally include the game's symmetric pure-strategy equilibria, and in games like VHBB's coincide with them. Which one, if any, the population converges to depends on the initial state, as in deterministic evolutionary models, and is difficult to predict. But when $e > 0$ the limiting outcome paradoxically becomes more predictable. The dynamics are then ergodic, so that they converge to a distribution over states that is independent of history. In the long run the process cycles perpetually among those states, with their prior probabilities at any given time determined by the ergodic distribution. This distribution depends on *e*, and is difficult to characterize in general. But when $e \to 0$ the ergodic distribution is concentrated around the steady states of the dynamics without mutations (in VHBB's games, the symmetric pure-strategy equilibria) and approaches a limit that can be characterized by estimating the relative likelihoods of entering and exiting the steady states from the number of simultaneous mutations such changes require. The *long-run equilibrium* is defined as the support of the ergodic distribution as $e \to 0$. This distribution usually puts probability one on a single steady state, which in sufficiently simple random-pairing environments is determined by risk-dominance, and more generally by the relative difficulty of entering and leaving the various steady states.[13]

Although analyses of long-run equilibria usually assume random pairing, VHBB's games against the field are no harder to analyze this way. Transitions between equilibria occur if and only

[13] Bergin and Lipman (1996) have shown that these predictions have an element of arbitrariness, in that if one allows state-dependent mutation probabilities and defines long-run equilibria using the limit of the ergodic distribution as they approach zero together, remaining in fixed proportions, the long-run equilibria vary with the proportions, and for some proportions put probability one on any given steady state.

if more players cross the order statistic from below than above, or vice versa. Consider Figure 2b's Stag Hunt game. When the order statistic is below the median, the discontinuous drop in effort 2's expected payoff occurs in the left half of the horizontal axis, so that the basin of attraction of the low-effort equilibrium at the right end of the horizontal axis is larger than the basin of attraction of the high-effort equilibrium at the left end, and fewer simultaneous mutations are required to go from the high-effort equilibrium to the edge of the basin of attraction of the low-effort equilibrium than from the low-effort equilibrium to the edge of the basin of attraction of the high-effort equilibrium. A noninfinitesimal mutation probability therefore makes the probability of tunneling leftward across the boundary lower than the probability of tunneling rightward, so that the ergodic distribution assigns higher probability to the low-effort equilibrium. As the mutation probability approaches zero, the ratio of the two tunneling probabilities approaches infinity and the probability the ergodic distribution assigns to the low-effort equilibrium approaches one. Robles (1997) gives a complete characterization of long-run equilibria in these environments. Here I give the result for the portmanteau model, whose proof formalizes the intuitive argument just given.

Proposition 2: In VHBB's (1990, 1991) games, the long-run equilibrium assigns probability one to the equilibrium with lowest (highest) effort whenever the order statistic is below (above) the median, and positive probability to every equilibrium when the order statistic is the median. In each case the long-run equilibrium is independent of the number of players and the order statistic, as long as it remains below (or above) the median.

Proposition 2 shows that analyses of long-run equilibria discriminate among strict equilibria and obtain unique predictions in most of VHBB's (1990, 1991) environments. These predictions are appealing in their simplicity. They are obtained without modeling players' initial responses or using empirical information about behavior, by studying ergodic dynamics and passing to the limit as the mutation probability approaches zero. However, although they distinguish between VHBB's large-group minimum treatment and their median and random-pairing minimum treatments in a way that is qualitatively consistent with the variations in observed outcomes, they are otherwise undiscriminating. By limiting the effects of history, such analyses eliminate much of the information about the effects of changes in treatment variables an analysis of VHBB's results can provide.

An analysis of long-run equilibria also seems possible in VHBB's (1993) environment, as in Kim's (1996) analysis of stochastic evolutionary dynamics. This would likely reproduce Kim's conclusion that limiting outcomes assure efficient coordination for any numbers of players in the auction and the coordination game, and any order statistic. Although this was the case in VHBB's experiment, Cachon and Camerer's (1996) and Weber's (1994) closely related experiments suggest that efficiency is not inevitable for all such treatments. Once again, the long-run equilibrium appears to lose some information that is important in understanding the full implications of VHBB's results.

VI. Adaptive Learning

In the portmanteau model, adaptive learning models are distinguished by the parameter restrictions $s^2_{z0} > 0$, $s^2_{zt} \to 0$, and $a_t \to 0$ (or $a_t \equiv 0$ for all $t = 1, \ldots$). The crucial difference between adaptive learning as defined here and stochastic evolutionary dynamics is that $s^2_{zt} \to 0$ gradually, rather than $s^2_{zt} \equiv e > 0$ or $s^2_{zt} \to e > 0$, so that learning dynamics are inherently nonstationary. This reflects the belief that players in an environment that approaches stationarity sufficiently rapidly will learn to predict each other's behavior accurately enough to lock in on a particular equilibrium. Only if there is new uncertainty each period, or if players continually enter and leave the population, will the process have the kind of perpetual randomness that allows an analysis of ergodic dynamics.

Although this may seem like a fine point, adaptive learning and stochastic evolutionary dynamics have quite different implications. The choice is still a subtle one, because predictions based on long-run equilibria are qualitatively consistent with the patterns of equilibrium selection in VHBB's experiments, and no amount of real data can refute a long-run prediction. The real issue is whether a model in which $s_{zt}^2 \rightarrow 0$ gradually, but quickly enough to allow the dynamics to lock in on a particular equilibrium, is a more useful description of behavior. As I will show, such a model makes patterns of equilibrium selection like those VHBB observed a long-run phenomenon, and is a better language in which to express and generalize the lessons to be learned from their experiments.

A. Convergence

Once the distributions of the ζ_{it} are specified, (3) and (4) define a Markov process with time-varying transition probabilities and state vector $\boldsymbol{x_t}$, in which players' beliefs and strategy choices are identically distributed ex ante (but not in general otherwise). The model's dynamics are driven by strategic uncertainty, as represented by the ζ_{it} or the s_{zt}^2. Its recursive structure and the conditional independence of players' deviations from the average learning rule capture the requirement that players must form their beliefs and choose their strategies independently which is the essence of the coordination problem.

Proposition 3 shows that, unless the level of strategic uncertainty declines to 0 very slowly, the learning dynamics converge, with probability 1, to one of the symmetric equilibria of the coordination game.[14] In this proposition, for technical reasons, I bound players' strategies by increasing x_{it} to its lower bound, denoted $\underline{x}$, or reducing it to its upper bound, denoted $\bar{x}$, whenever it would otherwise fall outside the interval $[\underline{x}, \bar{x}]$.

Proposition 3: Assume that the distributions of the ζ_{it} are truncated so that the x remain in the interval $[\underline{x}, \bar{x}]$. Then if $0 < ß = 1$ and $\sum_{s=0}^{\infty} s_{zs}^2$ is finite, y_t and the x_{it} converge, with probability 1, to a common, finite limit, which is an equilibrium of the stage game.

The proof follows the martingale convergence arguments of Nevel'son and Has'minskii (1973, Theorem 2.7.3), using the stochastic Lyapunov function $V_t \equiv \sum_{i,j} (x_{it} - x_{jt})^2$. Clearly $V_t \geq 0$, with $V_t = 0$ if and only if $x_{it} = x_{jt}$ for all *i* and *j*. It can be shown that the expected motion of the Lyapunov function is downward out of equilibrium. With the stated variance conditions, which are what would be needed to establish the strong law of large numbers in the absence of interactions between the stochastic and deterministic parts of the dynamics, this assures convergence with probability 1.

B. Closed-form solution

Given Proposition 3, the model's implications for equilibrium selection can be summarized by the prior probability distribution of the limiting equilibrium, which is normally nondegenerate due to the persistent effects of strategic uncertainty by Proposition 4 below. The probabilistic

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[14] For random-pairing environments, Robles (1995) shows that if $s_{zt}^2 \to 0$ there are only two possibilities: (i) the model is ergodic, and the result is the same as the long-run equilibrium when $s_{zt}^2 \equiv e \to 0;$ or (ii) the dynamics lock in on a particular equilibrium as in Proposition 3. (Robles's result continues to hold with state-dependent mutation probabilities as in Bergin and Lipman (1996), and ergodic dynamics yield their generalized long-run equilibrium.) Thus with random pairing at least, the key issue is whether $s_{zt}^2 \to 0$ fast enough to make the dynamics nonergodic.

nature of the model's predictions follows naturally from its statistical characterization of players' beliefs, and is fully consistent with the variation across runs within treatments that are commonly observed in experiments.

The distribution of the limiting equilibrium is influenced by interactions between strategic uncertainty and the learning dynamics. To build intuition about the interactions, reconsider Section II's analogy between VHBB's (1990) large-group minimum game and a faculty meeting with a 100% quorum. Members who can perfectly predict when others will arrive might coordinate on an inefficient equilibrium, but with perfect prediction there is no reason for them to favor such an equilibrium. On the contrary, one would expect such sagacious beings to coordinate on the efficient equilibrium in which everyone arrives on time. There is then also no reason to expect the outcome to vary with the quorum or the size of the faculty. But as anyone who has attended a series of such meetings can testify, such perfect prediction is unlikely, and the resulting dispersion of arrival times creates incentives for members to arrive later and later as they try to avoid having to wait for their colleagues. The meetings begin progressively later until the members gain enough experience to predict each other's arrival times, but by then they have converged to a common arrival time later than all would have preferred. Further, it is intuitively clear that this problem is less severe the smaller the faculty, or the more moderate the quorum.

I now show how the portmanteau model, with $s_{zt}^2 \to 0$ gradually, captures this intuition. Note first that with no restrictions on the a_t, the portmanteau model is formally consistent with any history of the y_t for any n and $f(\cdot)$, with $z_{it} \equiv s_{zt}^2 \equiv 0$ for all i and t and a_t varying over time so that $\sum_0^t a_s = y_t = x_{it}$. Such solutions, in which players jump from one stage-game equilibrium to the next following some commonly understood pattern, are not inconsistent with game-theoretic ideas of rationality. But they are empirically bizarre because the ad hoc variation in a_t they require violates the hypothesis that past behavior in an environment is directly indicative of current behavior, which most people take for granted, and which is the foundation of adaptive learning models. I therefore rule out such ad hoc variation by imposing the restrictions $a_t = 0$ and $b_t \equiv b$ for all $t = 1, \ldots$. In most of the analysis I leave s_{zt}^2 free to vary, although the model's dynamics are closest to those observed in the experiments when s_{zt}^2 declines steadily toward zero as players learn to predict each other's responses, and it is sometimes useful to impose intertemporal restrictions of the form $s_{zt}^2 = s_{z1}^2 / t^l$ for t = 1, . . . , where ? = 0.

The resulting model is nonlinear, with nonstationary transition probabilities and nonergodic, history-dependent dynamics that would normally be difficult to analyze. The key to the analysis is the evolutionary structure of VHBB's designs and the scaling properties of order statistics noted in (2), which Proposition 4 exploits to obtain a closed-form solution for y_t and the x_{it} as functions of the behavioral parameters, the treatment variables, and the shocks that represent strategic uncertainty. This allows an informative analysis of the dynamics, whether or not $s_{zt}^2 \rightarrow 0$. In what follows, sums with no terms (like $\sum_{s=0}^{t-1} b_{s+1} f_s$ for $t = 0$) equal 0 and products with no terms equal 1.

***Proposition* 4**: The unique solution of (3) and (4) is given, for all *i* and *t*, by

(5)
$$x_{it} = a_0 + \sum_{s=0}^{t-1} bf_s + z_{it}$$

and

(6)
$$y_t = \sum_{s=0}^{t-1} bf_s + f_t,$$

where

(7) $$z_{it} \equiv \sum_{s=0}^{t} (1-\beta)^{t-s} \zeta_{is} \text{ and } f_t \equiv f(z_{1t},\ldots,z_{nt}).$$

Proof: As in Crawford (1995, Proposition 1), the proof is immediate by induction on *t* once the solution has been found. The solution is constructed by using (2) to pass the common elements of the x_{it} through $f(\cdot)$. Combining (1) and (4) and using (2) to simplify,

$$y_t - y_{t-1} = f(x_{1t},\ldots,x_{nt}) - f(x_{1t-1},\ldots,x_{nt-1})$$

(8) $$= f(\beta y_{t-1} + (1-\beta)x_{1t-1} + \zeta_{1t},\ldots,\beta y_{t-1} + (1-\beta)x_{nt-1} + \zeta_{nt}) - f(x_{1t-1},\ldots,x_{nt-1})$$

$$= \beta f(x_{1t-1},\ldots,x_{nt-1}) + f((1-\beta)x_{1t-1} + \zeta_{1t},\ldots,(1-\beta)x_{nt-1} + \zeta_{nt}) - f(x_{1t-1},\ldots,x_{nt-1})$$

$$= f((1-\beta)x_{1t-1} + \zeta_{1t},\ldots,(1-\beta)x_{nt-1} + \zeta_{nt}) - f((1-\beta)x_{1t-1},\ldots,(1-\beta)x_{nt-1}).$$

Using (2) to remove the common elements of the x_{it-1} from both parts of the last expression, noting that $z_{it} = (1\text{-}\beta)z_{it-1} + \zeta_{it}$, and using (2) and (7) yields

(9) $$y_t - y_{t-1} = f((1-\beta)z_{1t-1} + \zeta_{1t},\ldots,(1-\beta)z_{nt-1} + \zeta_{nt}) - f((1-\beta)z_{1t-1},\ldots,(1-\beta)z_{nt-1})$$

$$= f_t - (1-\beta)f_{t-1}.$$

Given y_0, this yields (6) by successive substitution. To derive (5) from (6), note that

(10) $$x_{it} - y_t \equiv x_{it} - f(x_{1t},\ldots,x_{nt})$$

$$\equiv \beta y_{t-1} + (1-\beta)x_{it-1} + \zeta_{it} - f(\beta y_{t-1} + (1-\beta)x_{1t-1} + \zeta_{1t},\ldots,\beta y_{t-1} + (1-\beta)x_{nt-1} + \zeta_{nt})$$

$$\equiv (1-\beta)x_{it-1} + \zeta_{it} - f((1-\beta)x_{1t-1} + \zeta_{1t},\ldots,(1-\beta)x_{nt-1} + \zeta_{nt})$$

$$\equiv (1-\beta)z_{it-1} + \zeta_{it} - f((1-\beta)z_{1t-1} + \zeta_{1t},\ldots,(1-\beta)z_{nt-1} + \zeta_{nt}) \equiv z_{it} - f_t,$$

which immediately yields (5).

Proposition 4 shows how the outcome is built up period by period from the shocks that represent strategic uncertainty, each of whose effects persist indefinitely. The persistence of the shock terms makes the learning process resemble a random walk in the aggregate, but with declining variances and nonzero drift. Thus, equilibrium selection via adaptive learning is an inherently dynamic phenomenon: The limiting equilibrium is the cumulative result of interactions between strategic uncertainty and the learning process, and depends on the entire history; a shock that affects y_t in some period will persist in the limiting outcome. This persistence (and the fact that the extent of strategic uncertainty cannot be explained by theory alone) makes the analysis inherently partly empirical.

C. Comparative dynamics

The form of Proposition 4's solution indicates that unless the behavioral parameters vary sharply with changes in the treatment variables--which econometric estimates suggest is unlikely--outcomes will vary across treatments in stable, predictable ways. Changes in treatment variables have a direct effect, holding the behavioral parameters constant, and an indirect effect via induced changes in behavioral parameters. As I will show, theory has a lot to say about the direct effect, which shows up in the drift of the process. It is important to note, however, that theory by itself has little to say about the indirect effect: If sunspots that do not affect payoffs can focus players' expectations, then so can changes in treatment variables, however small. It is not irrational for players to expect all to choose effort 7 when n is even and effort 1 when n is odd, in which case their expectations will be confirmed; it is just inconsistent with sound empirical judgment.

The point is that any analysis of the effects of changes in treatment variables will inevitably be based in part on empirical knowledge about induced changes in behavioral parameters. This reliance on empirical knowledge is unusual in game theory, though accepted without question elsewhere in economics. What is needed is a theory that indicates what empirical knowledge is needed to predict the outcome, and that provides a framework within which to gather it. Making strong assumptions that yield conclusions independent of behavioral parameters can give a misleading view of important strategic phenomena. This problem is dealt with in Crawford (1995), Broseta (1993ab), and Crawford and Broseta (1997) by conducting as much of the theoretical analysis as possible contingent on behavioral parameters, closing the model when necessary by using it to estimate the parameters, separately for each treatment, from the experimental data, taking the discreteness of efforts into account. The estimated parameters generally satisfy the restrictions suggested by theory, but differ significantly from the values needed to justify an equilibrium analysis or an analysis of long-run equilibria. Instead they show the characteristic pattern of adaptive learning analyses: large initial levels of strategic uncertainty, declining to zero gradually, though with one exception quickly enough to assure lock-in on an equilibrium of the stage game. The estimated model gives an adequate statistical summary of individual subjects' behavior, and the model generates dynamics and limiting outcomes whose prior probability distributions (estimated by repeated simulation) closely resemble their empirical frequency distributions in the experiments.

Proposition 5 uses the closed-form solution of Proposition 4 to characterize the direct effect of changes in the treatment variables, expressing the mean outcome as a function of behavioral parameters, statistical parameters, and treatment variables. Let s_{zt}^2 denote the common variance of the z_{it}. (7) implies that $s_{zt}^2 \equiv \sum_{s=0}^{t} [(1-b)^{t-s}]^2 s_{zs}^2$. Define $m_t \equiv Ef(z_{1t}/s_{zt},\ldots,z_{nt}/s_{zt})$. Because the random variables z_{it}/s_{zt} are standardized, with common mean 0 and common variance 1, m_t is completely determined by n, $f(\cdot)$, and the joint distribution of the z_{it}/s_{zt}. m_t is subscripted only because the distribution of the z_{it}/s_{zt}. is generally time-dependent; its dependence on n and $f(\cdot)$ is suppressed for clarity.

Proposition 5: The ex ante means of y_t and the x_{it} are given, for all i and t, by

(11) $$Ex_{it} = a_0 + b\sum_{s=0}^{t-1} s_{zs} m_s$$

and

(12) $$Ey_t = a_0 + b\sum_{s=0}^{t-1} s_{zs} m_s + s_{zt} m_t .$$

Proof: The proof uses the fact that although the x_{it} become correlated as players respond to their common observations of y_t, adopting an ex ante point of view makes it possible to express their means as simple functions of the behavioral parameters, statistical parameters, and treatment variables. The shock terms in (5) and (6) are known functions of the z_{it}, which are ex ante i.i.d. across *i* with zero means for any given *t*. Taking expectations in (5) and (6), using (7), and noting that

(13) $$Ef(z_{1s},...,z_{ns}) \equiv E[s_{zs} f(z_{1s}/s_{zs},...,z_{ns}/s_{zs})] \equiv s_{zs} m_s$$

immediately yields (11) and (12).

Proposition 5 expresses the mean coordination outcome as the sum of the initial mean level of players' beliefs and the cumulative drift of the process. (The remaining term, $s_{zt} m_t$ in (12), is subsumed in the sum after the period in which it first appears.) The drift in period *s* is the product of the behavioral parameter b, which measures the extent to which players' beliefs respond to new observations of y_t, on average; the behavioral parameter s_{zs} (determined by b and the s^2_{zt} as indicated above), which measures the cumulative dispersion of players' beliefs; and the statistical parameter m_s, which is completely determined by *n*, $f(\cdot)$, and the distribution of the z_{is}.

Using the properties of order statistics, m_s, is easily shown to be decreasing in *n* and increasing in $f(\cdot)$, as required for qualitative consistency with the variations across VHBB's treatments. Crawford (1995) and Broseta (1993ab) conduct more detailed qualitative comparative dynamics analyses, making precise the common intuitions that coordination tends to be less efficient, the less robust desirable equilibria are to disruption by subgroups (here, the closer the order statistic is to the minimum) and less efficient in larger groups because it requires coherence among a larger number of independent decisions.

To say more than this, values must be assigned to a_0, b, the s_{zs}, and the m_s. The behavioral parameters a_0, b, and the s_{zs} can be estimated from the experimental data, as described above. The statistical parameters m_s. are difficult to evaluate due to the complexity of their dependence on the distribution of the z_{is}. This problem can be sidestepped by estimating the probability distributions the model implies directly by repeated simulation, as in Crawford (1995, Section 7); but it is informative to approximate the outcome distributions analytically. The approximations are based on the simplifying assumption that the z_{is} are jointly normally distributed for any given *s*. Normality is a reasonable approximation because the z_{is} are weighted sums of the z_{it}, which are weakly dependent and likely to be approximately conditionally normal, for familiar reasons. This makes the common distribution of the z_{is}/s_{zs}.independent of *s*, so that $m_s \equiv m$. Given that the z_{is}/s_{zs}.are uncorrelated for any given *s*, the parameter m is tabulated in Teichroew (1956) for any order statistic of the normal distribution and any $n \leq 20$.

An explanation for the dynamics in VHBB's experiments can now be discerned. Suppose, for simplicity, that $\sum_{s=0}^{t} s_{zs} \to S$ as $t \to \infty$. Propositions 3 and 5 then imply that Ey_t and Ex_{it} approach the approximate common limit $a_0 + mb\mathrm{S}$. This formula shows how the mean coordination outcome is determined by the behavioral parameters a_0 and b; the number of players and the order statistic, via m; and the initial dispersion of beliefs and the rate at which it is eliminated by learning, via S.

By symmetry, $m = 0$ for VHBB's median treatments and their random-pairing minimum treatment (viewed as a median treatment, as in Crawford (1995)), so there is no drift in those treatments and the approximate common limit of Ey_t and Ex_{it} is a_0. The estimates of a_0 were 4.30 in the random-pairing minimum treatment; 4.71 and 4.75 in median treatments F and G; and 6.26 in median treatment O, whose structure made the all-7 equilibrium more prominent.[15] m is negative (positive) for order statistics below (above) the median and, as suggested by the faculty meeting example above, strongly negative for the large-group minimum treatment, where, setting $n = 15$ for simplicity, $m = -1.74$ (Teichroew (1956, Table I). There the estimates of a_0 and b were 5.45 and 0.25 respectively, and S (which is difficult to estimate for this treatment) appeared highly unlikely to be less than 10. Thus, the approximate common limit of Ey_t and Ex_{it} in the large-group minimum treatment, $a_0 + mb$S, is at most 1.10.

Comparing these approximations with VHBB's results shows that for large initial levels of strategic uncertainty, declining gradually to zero over time, differences in drift across treatments make the prior probability distribution of the limiting outcome vary with the number of players and the order statistic in a way that yields patterns of equilibrium selection like those in the experiments. The most important changes across treatments were between the random-pairing and

[15] The estimated model also makes it possible to allocate the variance of the limiting outcome among periods. This suggests that in VHBB's median treatments G, F, and O, the initial median y_0 "explains" 46%, 58%, and 81% of the respective variances of the limiting medians, and that the contributions of subsequent periods decline very rapidly over time. In VHBB's twelve trials with treatments G, F, and O, subjects always converged to the initial median. Thus, the model suggests that VHBB's sample somewhat overstated the history-dependence of the dynamics.

large-group minimum treatments, and between the median treatments and the large-group minimum treatment. Viewing the random-pairing minimum treatment as a median treatment, the model treats the differences between these treatments primarily as changes in the order statistic (even though the former difference is "really" a change in group size and the latter also involves a change in group size, from 9 to 14-16). The above estimates suggest that each of these changes altered the drift of the process much more than the changes in behavioral parameters they induced.

D. Extension to VHBB's (1993) auction environment

Crawford and Broseta (1997) generalized the analysis just summarized to the extensive-form stage game of VHBB's (1993) auction environment, in which subjects played one of the nine-person median coordination games of VHBB's (1991) experiments with the right to play auctioned each period in a population of 18. Recall that when that game was played without auctions subjects always converged to inefficient equilibria, but that in eight out of eight trials with auctions subjects bid the market-clearing price to a level recoverable only in the efficient equilibrium and then converged to that equilibrium.

The model combines a learning rule like (4) with a bidding equation in the same style, and a stochastic structure that allows for the diversity of subjects' initial beliefs and inferences from their observations of the market-clearing price and the order statistic. If players treat their individual influences on the market-clearing price and the order statistic as negligible, their optimal bids and efforts each period are determined by their beliefs about the current value of the order statistic. This makes it possible to summarize their decisions throughout the extensive-form stage game by a single beliefs variable for each player, as in our earlier analyses. This and the fact that the market-clearing price is also an order statistic makes it possible to generalize Propositions 1 and 3-5 to VHBB's auction environments, including a closed-form solution for the dynamics as in Proposition

4. The solution shows that the coordination process resembles that in environments without auctions, and that interactions between strategic uncertainty and the learning dynamics generate an order statistic effect like the one that drives the dynamics without auctions, plus optimistic subjects and forward induction effects that are large enough together to explain the efficiency-enhancing effect of auctions in VHBB's experiment.

These effects can be approximated as functions of the treatment variables, behavioral parameters, and tabulated statistical parameters. The optimistic subjects and order statistic effects together have approximately the same magnitude in VHBB's environment (where the right to play a nine-person median game was auctioned in a group of 18) as the order statistic effect in an 18-person coordination game without auctions in which payoffs and best responses are determined by the fifth highest (the median of the nine highest) of all 18 players' efforts. In this respect the auctions transformed VHBB's median game, whose order statistic effect without auctions would contribute zero drift to the dynamics, into a 75th percentile game (0.75 = 13.5/18) whose order statistic effect contributes a large upward drift. Our estimates suggest that this drift is responsible for roughly half of the efficiency-enhancing effect of auctions in VHBB's environment, and that the other half is due to a strong forward induction effect. A qualitative comparative dynamics analysis generalizes to auction environments our earlier results that coordination tends to be less efficient, the less robust desirable equilibria are to disruption by subsets of the population, and less efficient in larger groups. It also establishes a new result for auction environments, that increased competition for the right to play favors efficiency because it tends to yield higher market-clearing prices and intensifies the optimistic-subjects effect.

VII. Conclusion

This paper outlines a model to explain the results of recent experiments by VHBB (1990, 1991, 1993), in which subjects repeatedly played coordination games, uncertain only about each other's strategy choices, with striking patterns of equilibrium selection across treatments. Theoretical and econometric analyses suggest that these outcomes were governed by history-dependent learning dynamics, which lock in on a particular equilibrium with high probability. The model's implications can be summarized by the prior probability distribution of this limiting equilibrium, which is normally nondegenerate due to the persistent effects of interactions between strategic uncertainty and the learning dynamics. The analysis shows that taking these effects into account yields a unified explanation of VHBB's results. The need for an analysis of history-dependent learning dynamics stands out especially clearly in VHBB's experiments because their structure permits a closed-form solution of the dynamics and an informative analysis. But the mechanism suggests that similar patterns of equilibrium selection will be found in a variety of related laboratory or field environments. This has already been confirmed by the results of other experiments, such as Brandts and Holt (1992, 1993) and Roth and Erev (1995).

In environments like VHBB's--as in many other environments--equilibrium selection is an inherently dynamic phenomenon: a shock that affects the outcome early on persists in the limiting outcome. This persistence and the fact that the extent of strategic uncertainty and other aspects of behavior cannot usefully be explained by theory alone make the analysis inherently partly empirical. Approaches like Harsanyi and Selten's general theory of equilibrium selection and analyses of long-run equilibria reach precise conclusions about equilibrium selection, independent of behavioral parameters, only by making unrealistic assumptions that rule out either strategic uncertainty or the history-dependence that allows it to affect the limiting outcome. Such approaches can capture the outcomes of history-dependent dynamics only by coincidence, and lose much of the information results like VHBB's can provide about how the structure of the environment affects coordination outcomes. The same criticism applies to equilibrium analyses of perturbed versions of VHBB's stage games, as in Carlsson and van Damme (1993) and Anderson, Goeree, and Holt (1996). Finally, it applies for different reasons to analyses of deterministic evolutionary dynamics as in Crawford (1991), which yield history-dependence too mechanical to describe VHBB's results.

Results like VHBB's, properly understood, have the power to change the way we think about models with multiple equilibria, even in field environments where one is confident that equilibrium has long since been reached. In Cooper and John (1988), for instance, all pure-strategy equilibria are treated as equal. VHBB's results show that this kind of agnosticism about equilibrium selection is no longer supportable. We should ask of any explanation based on a model that has an equilibrium--or a risk- or payoff-dominant equilibrium, or an evolutionarily stable or long-run equilibrium--that seems to fit the data, whether that equilibrium is likely to be reached by a realistic learning process.

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