MEANINGFUL THEOREMS: NONPARAMETRIC ANALYSIS OF REFERENCE-DEPENDENT PREFERENCES*

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Abstract: This paper derives nonparametric conditions for the existence of reference-dependent preferences that rationalize a price-taking consumer's demand behavior. Unless reference points are modelable and sensitivity is constant, reference-dependent models of consumer demand are flexible enough to fit virtually any data. Assuming modelable reference points and constant sensitivity, we characterize continuous reference-dependent preferences, relaxing some of Kőszegi and Rabin's (2006) strong functional-structure assumptions, which to our knowledge have been maintained without testing in all empirical studies of reference-dependent consumer demand. Our characterization enables more general structural econometric as well as nonparametric analyses. Preliminary analysis suggests that our relaxations are empirically important. (*JEL* C14, C23, D11, D12, J22)

Keywords: consumer theory, labor supply, reference-dependent preferences, revealed preference, nonparametric demand analysis, loss aversion

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I. Introduction

Kahneman and Tversky (1979) and Tversky and Kahneman (1991; "TK") introduce a model of individual decisions in which people have preferences over gains and losses relative to a reference point. Such reference-dependence alters the domain of preferences from levels of outcomes to changes in outcomes; but it remains consistent with a complete and transitive preference ordering over changes, thus not inherently irrational. Although Kahneman and Tversky and TK focus on changes alone, Kőszegi and Rabin (2006; "KR") and most recent analyses allow preferences over both levels and changes, and such reference-dependence is also not inherently irrational.

Building on TK's and KR's landmark theory papers, this paper studies the theory of the leading microeconomic application of reference-dependent preferences, to consumer demand. Reference-dependent consumer theory is the basis of many structural econometric applications, in finance, housing, and labor supply, as well as consumer demand itself. We derive nonparametric conditions for the existence of reference-dependent preferences that rationalize a price-taking consumer's demand behavior, precisely identifying the cases where reference-dependent modelling can be empirically useful.

For those cases we also characterize the functional structure of continuous preferences, relaxing some unnecessarily restrictive functional-structure assumptions that TK and KR imposed, which to our knowledge have been maintained without testing in all theoretical and empirical studies of reference-dependent consumer demand. We also relax the functional-form assumptions commonly maintained in structural econometric applications.

Our characterization makes it possible to conduct more general structural econometric or nonparametric analyses in applications, which we plan to pursue in companion papers. Preliminary analysis suggests that our relaxations of TK's and KR's functional-structure assumptions are empirically important.

Some history is helpful in describing our results. Econometric studies of reference-dependent consumer demand stem from Camerer et al.'s (1997) classic analysis of New York City cabdrivers' labor supply.[1] A standard model of labor supply is analogous to a model of consumer demand for leisure and earnings (black-boxing the goods earnings can buy), in this case taking expected earnings per hour as a proxy for the wage. In a neoclassical version of the model, with preferences over levels of earnings and leisure, the elasticity of hours with respect to the wage is positive unless there are large income effects. But Camerer et al. estimate a strongly negative wage elasticity.

To explain this anomaly, Camerer et al. propose a reference-dependent model in which drivers' behavior is rational but with preferences over a domain including not only the neoclassical model's levels of leisure and earnings but also changes in earnings measured relative to daily earnings targets, analogous to TK's and KR's reference points.[2] Camerer et al. also suggest that the targets can be proxied by the average daily level of earnings.

Experiments suggest that most people are loss-averse—more sensitive to changes below their targets (losses) than above them (gains). Loss aversion creates kinks in drivers' preferences that make their earnings tend to bunch around their earnings targets, thus possibly working less on days with higher wages. This allows Camerer et al.'s model to reconcile their estimated negative wage elasticity of hours with a positive neoclassical incentive effect of higher anticipated wages, even without large income effects.

Farber (2005, 2008) econometrically analyzes another dataset on New York City cabdrivers. In his data, as in Camerer et al.'s, drivers' hours worked and earnings per hour are negatively correlated. He finds that a model with daily

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[1] Cabdrivers are of particular interest in labor supply because many choose their own hours, unlike most workers in modern economies. Another important impetus to empirical applications of reference-dependent models is Kahneman, Knetsch, and Thaler's (1990) experimental analysis of the endowment effect, whereby people's reservation prices for goods they own often exceeds their willingness to pay to acquire them. More recent applications include Oettinger (1999), Genesove and Mayer (2001), Fehr and Goette (2007), Post et al. (2008), Pope and Schweitzer (2011), Lien and Zheng (2015), and Meng and Weng (2018).
[2] Some have argued that such income targeting is irrational per se because it leads a driver to sacrifice levels of earnings in exchange for something less tangible. But our use of the term "rational" refers only to the consistency of a driver's decisions in the specified larger domain.

earnings targets treated as latent variables fits better than a neoclassical model. However, his model yields unstable estimates of the targets, which he argues limits the usefulness of reference-dependence in modeling labor supply.[3]

In a paper inspired by Camerer et al.'s and Farber's analyses, KR propose a more general theory of reference-dependent preferences. KR assume that utility is additively separable across components of neoclassical consumption utility and reference-dependent "gain-loss" utility. Like TK, but unlike Camerer et al. and Farber, KR assume that reference-dependence is active for each good. KR also assume that gain-loss utility is determined by the good-by-good differences between realized and reference consumption utilities. Finally, as in Camerer et al.'s use of average daily earnings to proxy earnings targets, but unlike TK and Farber, who take no clear position on how reference points are determined, KR close their model by equating a consumer's reference points to her/his good-by-good rational expectations of future consumption.

Like Camerer et al.'s model, KR's model can reconcile a negative overall correlation between hours and earnings per hour with the neoclassical prediction that higher anticipated wages tend to increase labor supply: With perfectly anticipated changes in earnings or hours, gain-loss utility drops out of KR's model, which then replicates the neoclassical prediction. But with unanticipated changes, loss aversion makes daily earnings tend to bunch around its reference point, which may yield a negative overall correlation.

Crawford and Meng (2011; "CM") adapt KR's theory to reconsider Farber's (2005, 2008) econometric analyses, using Farber's data. Instead of limiting drivers to earnings targets, CM allow both hours and earnings targets. And instead of treating the targets as latent variables, CM model them via natural sample proxies, in the spirit of Camerer et al.'s daily earnings averages and KR's rational-expectations reference points. Modeling the targets avoids

[3] Farber (2015) studies a much larger dataset on New York City cabdrivers and finds evidence of reference-dependence, but he again concludes "…gain-loss utility and income reference-dependence is not an important factor in the daily labor supply decisions of taxi drivers."

the unstable estimates that made Farber doubt his reference-dependent models and appears to yield a useful reference-dependent model of drivers' behavior.[4]

Although reference-dependent models allow rationality-based explanations of some observed behavior that is anomalous from a neoclassical point of view, several factors have limited their appeal. Because they expand the domain of preferences, many researchers doubt that they yield any testable implications—Samuelson's (1947) "meaningful theorems". Such doubts are exacerbated if reference points are unmodelled or treated as latent variables.

Further, theoretical studies of reference-dependent consumer demand like TK's and KR's, and empirical studies like Camerer et al.'s (1997), Farber's (2005, 2008, 2015), CM's, and all others of which we are aware, make ancillary functional-structure assumptions that are not directly supported by theory or evidence. All assume—in our view naturally—that preferences are additively separable across separate components of consumption and gain-loss utility. All but occasionally KR also assume that preferences have "constant sensitivity" (TK's "sign-dependence"; KR's A3′). (As explained in Section II, a reference point divides commodity space into gain-loss regimes, such as "earnings loss, hours gain" in labor supply. With constant sensitivity, a consumer's preferences over consumption bundles must be the same throughout a given regime but may vary across regimes.) And all assume—again naturally—that the sum of consumption and gain-loss utility is continuous, even when preferences over bundles change across regimes.

[4] Thakral and Tô (2021) study the labor supply of New York City cabdrivers using a 2013 dataset comparable to Farber's (2015) dataset, replicating CM's main findings and adding an analysis of the dynamics of drivers' reference points. Anderson et al. (2023) study the effects of transitory incentives and income for vendors in an open-air market in India. In their words, their results suggest that "…reference dependence is moderated by experience with an incentive regime or by the time at which income is accumulated." Brandon et al. (2023) study the labor supply of Lyft drivers in four U.S. cities. In the observational part of their study, they replicate some of the main findings of CM's and other previous observational studies but raise questions about other findings. In their field experiment, random windfalls to drivers yield statistically insignificant effects on stopping decisions, suggesting that for most drivers the mental accounting of windfalls differs from that of income earned by driving.

All previous theoretical and empirical studies also assume—explicitly in KR's analysis, implicitly in some others—that gain-loss utility is determined by the good-by-good differences between realized and reference consumption utilities. In models with reference points for each good, like KR's, and with constant sensitivity, this implies—in our view unnaturally—that the sum of consumption and gain-loss utility that determines consumer demand is additively separable across goods, and that its marginal rates of substitution satisfy strong knife-edge restrictions on how they vary across gain-loss regimes (as summarized for the two-good case in CM's Table 1). The former restriction is prima facie unacceptable in neoclassical consumer demand, and neither restriction is well justified in reference-dependent consumer demand.

Finally, previous empirical studies all also rely on strong, though commonly maintained, functional form assumptions.[5]

McFadden (1985) remarks that using econometrics to flesh out the theory in this way "interposes an untidy veil between econometric analysis and the propositions of economic theory." Are the empirical successes of reference-dependent models due to reference-dependence per se, or are they artifacts of assumptions on functional structure or form—assumptions maintained, with neither theoretical justification nor testing, in all previous work in this area?

Our derivation of nonparametric conditions for the existence of reference-dependent preferences that rationalize a price-taking consumer's demand behavior begins to lift this veil.[6] The classic nonparametric analyses of neoclassical consumer demand make essential use of the rationality assumption, in that the consumer must have a complete and transitive preference ordering over levels of consumption. We adapt the neoclassical

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[5] Farber (2005, 2008, 2015), CM, Thakral and Tô (2021), Anderson et al. (2023), and Brandon et al. (2023) all reduce reliance on functional form assumptions in various ways, without eliminating it.
[6] There are other nonparametric theoretical analyses of reference-dependent models, including Gul and Pesendorfer (2006); Abdellaoui, Bleichrodt, and Paraschiv (2007); Ok, Ortoleva, and Riella (2015); and Freeman (2017, 2019). All but Gul and Pesendorfer (2006) and Freeman (2017), which are discussed in footnote 11, focus on different aspects of the problem.

analyses by extending rationality to allow preferences over a domain expanded in the disciplined way suggested by reference-dependence, to include changes in as well as levels of consumption.[7] Our adaptation requires more than a translation of the neoclassical analyses for two reasons: (i) levels of and changes in consumption are, by definition, bundled and priced together and (ii) a reference-dependent consumer might find it beneficial to change her/his preferences by altering how consumption relates to the reference point.

Section II introduces our general model of reference-dependent preferences. Following KR, we assume throughout that preferences are additively separable across component of consumption and gain-loss utility.

Section III reviews Afriat's (1967), Diewert's (1973), and Varian's (1982) classic nonparametric analyses of consumer demand in the neoclassical case.

Section IV begins to study the nonparametric implications of reference-dependent preferences. Propositions 1 and 2 show that unless reference points are modelable *and* sensitivity is constant, reference-dependent models of consumer demand are flexible enough to fit any data, with a minor qualification when constant sensitivity fails. These results precisely identify the grain of truth in the common belief that allowing reference-dependence destroys the parsimony of neoclassical consumer theory. Proposition 1 also shows that analyses that treat targets as latent variables may be as influenced by how they constrain estimating the targets as by reference-dependence.

Section V, assuming modelable reference points and constant sensitivity so that reference-dependent modeling can be empirically useful, characterizes continuous reference-dependent preferences. Proposition 3 show that constant sensitivity and continuity imply that gain-loss utility must be determined, additively separably across goods, by the good-by-good differences between

[7] By contrast, Farber (2008, p. 1070) takes the neoclassical view that preferences are only over levels of consumption and concludes that most of his drivers are irrational: "This [earnings-targeting] is clearly nonoptimal from a neoclassical perspective, since it implies quitting early on days when it is easy to make money and working longer on days when it is harder to make money. Utility would be higher by allocating time in precisely the opposite manner."

realized and reference utilities. This derives KR's assumption regarding the functional structure of gain-loss utility from continuity. Although KR simplified by extending that structure to consumption utility, Proposition 3 also shows that under KR's and our assumption that preferences are additively separable across consumption and gain-loss utility, continuity does *not* restrict consumption utility, which is constant across gain-loss regimes.

Thus, with nonparametric flexibility Proposition 3 allows us to relax KR's implied restriction that the sum of consumption and gain-loss utility that determines consumer demand is additively separable across goods and their restrictions on how that sum's marginal rates of substitution vary across gain-loss regimes. As we have said, neither restriction is well justified. Relaxing them is likely to be empirically important.

Proposition 3's characterization shows how to conduct a general structural econometric analysis of data like Farber's (2005, 2008, 2015), CM's, Thakral and Tô's (2021), Andersen et al.'s (2023), or Brandon et al.'s (2023), using sample proxies like CM's for KR's rational-expectations targets. Such an analysis should reveal the extent to which unnecessarily restrictive functional-structure assumptions bias the results of previous econometric analyses.

Continuing to assume modelable reference points and constant sensitivity, Section VI studies the nonparametric implications of reference-dependent preferences in more detail. Proposition 4 derives general necessary and sufficient conditions for a rationalization, which are not directly applicable because they are conditional on an unspecified choice of rationalizing gain-loss regime preferences. A Corollary shows that loss aversion significantly simplifies those conditions. Proposition 5 derives directly applicable sufficient and, with rich enough data, asymptotically necessary conditions for a rationalization, based on a specific choice of rationalizing regime preferences.

Our nonparametric analysis refocuses the view of reference-dependent consumer demand from structural models to some extent. Constant sensitivity, usually seen as merely a convenient simplification, is essential for reference-dependent models to have any nonparametrically refutable implications.[8] And loss aversion, usually seen as empirically well-established but not essential to modelling reference-dependent demand, now plays an important role in the sufficient conditions for the existence of a reference-dependent rationalization.

Proposition 5's sufficient conditions show how to conduct a nonparametric analysis of data like Farber's, CM's, Thakral and Tô's, Andersen et al.'s, or Brandon et al.'s, relaxing TK's and KR's unnecessarily restrictive functional-structure assumptions and the functional-form assumptions maintained in previous work. A preliminary analysis of Farber's (2005, 2008) data, reported in Blow and V.P. Crawford (2024, Section III), estimates neoclassical and reference-dependent models driver by driver, as in most nonparametric demand analyses. Controlling for the models' varying flexibility using Beatty and I. Crawford's (2011, pp. 2786-87) proximity-based variant of Selten and Krischker's (1983) and Selten's (1991) nonparametric measure of predictive success, which judges flexibility by the likelihood that random data would fit a model, Blow and V.P. Crawford's analysis strongly rejects KR's constant-sensitivity assumption of additive separability across goods and KR's restrictions on how marginal rates of substitution vary across gain-loss regimes. Relaxing those restrictions, for many drivers a reference-dependent model fits more than enough better than its neoclassical counterpart to justify its greater flexibility.

Section VII is the conclusion.

II. Reference-dependent Preferences

We consider reference-dependent preferences with a finite number of demand observations for a single consumer—or equivalently for a pooled group of consumers assumed to have homogeneous preferences—but we will

[8] Limiting the variability of sensitivity is important for testable implications because otherwise preferences could vary freely across observations. Structural analyses implicitly limit the variability of sensitivity via functional-form assumptions, but a nonparametric analysis can do so only via a strong assumption like constant sensitivity.

speak of a single consumer. We index goods k = 1,…, K and observations t = 1,…, T. We assume that the consumer is a price-taker, choosing a consumption bundle $\boldsymbol{q} \in \mathbb{R}_+^K$ with a linear budget constraints. Her/his preferences are represented by a family of utility functions $u(\boldsymbol{q}, \boldsymbol{r})$, parameterized by an exogenous reference point $\boldsymbol{r} \in \mathbb{R}_+^K$, conformable to a K–good consumption bundle as in TK and CM.[9] If reference points are unmodelable the data are prices and quantities $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$, with hypothetical reference points $\{\boldsymbol{r}_t\}_{t=1,\dots,T}$. If they are modelable the data are prices, quantities, and reference points $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$. The context will make the interpretation of $\boldsymbol{r}_t$ clear. Sometimes we denote goods by scalars with superscripts, so for k = 1,…, K, $\boldsymbol{q} \equiv (q^1, \dots, q^K)$ and for observation t = 1,…, T, $\boldsymbol{q_t} \equiv (q_t^1, \dots, q_t^K)$, with analogous notation for $\boldsymbol{p}$, $\boldsymbol{p_t}$, $\boldsymbol{r}$, and $\boldsymbol{r_t}$.

To describe preferences that respond positively to changes in consumption relative to the reference point, as well as to levels, we take the utility function $u(\boldsymbol{q}, \boldsymbol{r})$ to be strictly increasing in $\boldsymbol{q}$ and strictly decreasing in $\boldsymbol{r}$. Our specification is then at least as flexible as a general strictly increasing function of levels $\boldsymbol{q}$ and changes $\boldsymbol{q} - \boldsymbol{r}$. It nests the neoclassical case where preferences respond only to levels; Kahneman and Tversky's (1979) and TK's case where they respond only to changes; and cases like Camerer et al.'s (1997), Farber's (2005, 2008), KR's, and CM's where preferences respond to both. As in those papers, we take $u(\boldsymbol{q}, \boldsymbol{r})$ to be continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$; and we assume that preferences have separate consumption utility and gain-loss utility components that enter $u(\boldsymbol{q}, \boldsymbol{r})$ additively separably, with the consumption utility function the same for all gain-loss regimes and independent of the reference point.

We call the general case of preferences that can be represented by a utility function $u(\boldsymbol{q}, \boldsymbol{r})$ in the class just described "variable sensitivity". An important

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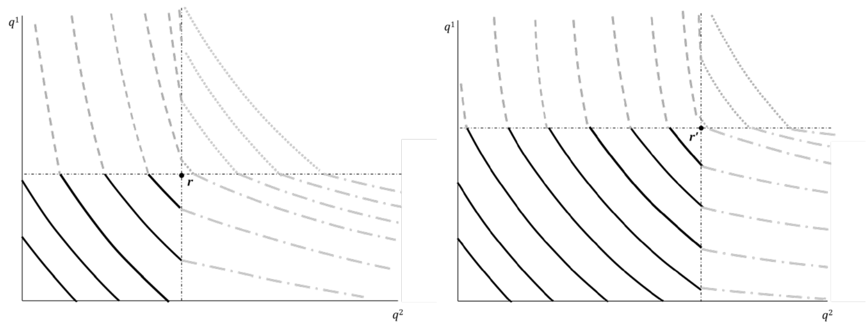
[9] In KR's theoretical model, which makes no allowance for errors, only probabilistic targets make possible the unanticipated changes in outcomes that allow expectations-based reference-dependence to have any effect. CM use the fact that sampling variation causes unanticipated changes to simplify KR's probabilistic targets to point expectations, as we do here.

special case is “constant sensitivity” (TK’s sign-dependence; KR’s assumption A3′). Let $\text{sign}(\boldsymbol{q}-\boldsymbol{r})$, the vector whose kth component is $\text{sign}(q^k - r^k)$, be the good-by-good sign pattern of gains and losses. A reference point divides commodity space into gain-loss regimes, such as “earnings loss, hours gain” in labor supply, throughout each of which $\text{sign}(\boldsymbol{q}-\boldsymbol{r})$ remains constant. With constant sensitivity a consumer’s preferences over $\boldsymbol{q}$ are the same, independent of $\boldsymbol{r}$, throughout a regime; but may vary freely across regimes.

DEFINITION 1: [Preferences and utility functions with constant sensitivity.] A reference-dependent utility function $u(\boldsymbol{q},\boldsymbol{r})$ *satisfies constant sensitivity if and only if, for any consumption bundles* $\boldsymbol{q}$ *and* $\boldsymbol{q}^*$ *and reference points* $\boldsymbol{r}$ *and* $\boldsymbol{r}^*$ *such that* $sign(\boldsymbol{q}-\boldsymbol{r}) = sign(\boldsymbol{q}^*-\boldsymbol{r}) = sign(\boldsymbol{q}-\boldsymbol{r}^*) = sign(\boldsymbol{q}^*-\boldsymbol{r}^*)$, $u(\boldsymbol{q},\boldsymbol{r}) \geq u(\boldsymbol{q}^*,\boldsymbol{r})$ *if and only if* $u(\boldsymbol{q},\boldsymbol{r}^*) \geq u(\boldsymbol{q}^*,\boldsymbol{r}^*)$.

Figure 1. A set of gain-loss regime maps with constant sensitivity
and the associated global maps for alternative reference points



(a) (b)

Because $\boldsymbol{r}$ is unrestricted, each gain-loss regime’s preferences over $\boldsymbol{q}$ must be defined for the entire commodity space: Each value of $\text{sign}(\boldsymbol{q}-\boldsymbol{r})$ “switches on” a different regime’s preferences. With two goods, a reference point in the interior of commodity space divides it into four regimes. Figure 1’s panels show four regime indifference maps and the associated global indifference maps for reference points $\boldsymbol{r}$ and $\boldsymbol{r}'$. The shift from $\boldsymbol{r}$ to $\boldsymbol{r}'$ does not alter the regime maps, but as $\boldsymbol{r}$ varies, even locally, the shift alters how those maps connect across regimes as in Figure 1, and thereby alters the global map.

A useful concept that figures in some of our results is that of loss aversion. Loss aversion has strong experimental and empirical support. Generalizing TK's (pp. 1047-1048) definition for the two-good case, Definition 2 gives a nonparametric definition of loss aversion with constant sensitivity.[10]

DEFINITION 2: [Preferences with constant sensitivity and loss aversion.] Assume that reference-dependent preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *have constant sensitivity. A collection of gain-loss regime preferences over consumption bundles satisfies loss aversion if and only if, for any observation* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}$*, given* $\boldsymbol{r}_t$*, the preference ordering's global better-than-*$\boldsymbol{q}_t$ *set is weakly contained in each regime preference ordering's local better-than-*$\boldsymbol{q}_t$ *set.*

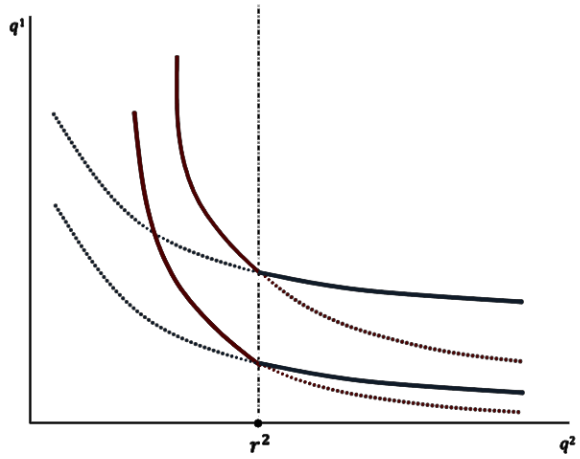
Because Definition 2's nesting of local and global better-than sets holds globally, loss aversion is equivalent to requiring that the gain-loss regimes' indifference maps satisfy a global single-crossing property: For any observation, across regimes that differ only in the gain-loss status of good *i*, the loss-side marginal rate of substitution between good *i* and any other good (generalized as needed for non-differentiable preferences) must be weakly more favorable to good *i* than the gain-side marginal rate of substitution. Neoclassical preferences are thus weakly loss averse. It is this single-crossing property, not the kinks in global indifference maps that it creates, that shapes loss aversion's nonparametric implications, which are testable with finite data. Loss aversion precludes nonconvex kinks, so if the regime maps all have convex better-than sets, then so do the associated global maps.

Figure 2 illustrates loss aversion with one active reference point and two gain-loss regimes. Loss aversion is a property of the relationship between

[10] The idea of loss aversion is still well defined with variable sensitivity, but formalizing it then is more complex, and Propositions 1 and 2 show that it would be nonparametrically irrefutable.

regimes' preferences over consumption bundles given a reference point; it is therefore independent of the reference points themselves.

Figure 2. Loss aversion with one active reference point
(solid curves for active parts of indifference maps,
dashed for inactive parts)



III. Nonparametric Implications of Neoclassical Preferences

This section reviews Afriat's (1967), Diewert's (1973), and Varian's (1982) classic nonparametric analyses of consumer demand in the neoclassical case where preferences respond only to levels of consumption, in preparation for our analysis of reference-dependent preferences. In the revealed-preference tradition of Samuelson (1948) and Houthakker (1950), they show that a price-taking consumer's demand behavior can be nonparametrically rationalized by the maximization of a nonsatiated utility function if and only if the data satisfy the Generalized Axiom of Revealed Preference ("GARP"). They also construct rationalizing utility functions in a way that is useful in our analysis.

DEFINITION 3: [Rationalization with neoclassical preferencess.] Preferences and an associated utility function $u(\boldsymbol{q})$ *rationalize the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ *if and only if* $u(\boldsymbol{q}_t) \geq u(\boldsymbol{q})$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

DEFINITION 4: [Generalized Axiom of Revealed Preference ("GARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *implies* $\boldsymbol{p}_t \cdot \boldsymbol{q}_t \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_s$, *where R indicates that there is some sequence of observations* $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \dots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i$, $\boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \dots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

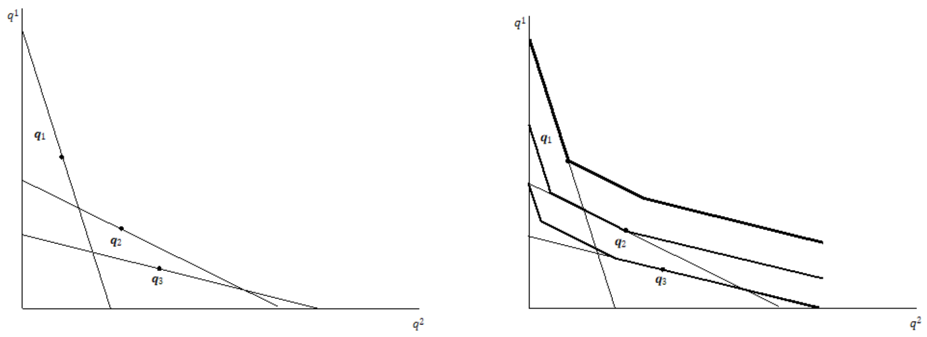
AFRIAT'S THEOREM: The following statements are equivalent:
[A] There exists a utility function $u(\boldsymbol{q})$ *that is continuous, non-satiated, and concave, and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.
[B] There exist numbers $\{\boldsymbol{U}_t, \lambda_t > 0\}_{t=1,\dots,T}$ *such that*
(1) $\quad U_s \leq U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q}_s - \boldsymbol{q}_t)$ *for all* $s, t \in \{1, \dots, T\}$
[C] The data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *satisfy GARP.*
[D] There exists a non-satiated utility function $u(\boldsymbol{q})$ *that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.

In the proof of Afriat's Theorem (Diewert 1973, Section 3; Varian 1982, Appendix I), which is constructive, for any t [B]'s inequalities (1) hold with equality for at least one $s \neq t$. This yields a canonical set of rationalizing preferences and an associated utility function, which we call the Afriat preferences and utility function. The Afriat utility function is piecewise linear, continuous, non-satiated, and concave. With finite data the Afriat preferences and utility function are only one of many possibilities for a rationalization (Varian 1982, Fact 4), but they play a central role in Section V's analysis.

DEFINITION 5: [Afriat preferences and Afriat utility function.] For data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *that satisfy GARP, or equivalently condition B) of Afriat's Theorem, the Afriat preferences are those represented by the Afriat utility function* $u(\boldsymbol{q}) = min_{t \in \{1,\dots,T\}}\{U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q} - \boldsymbol{q}_t)\}$, *where for any given* t *the* U_t *and* λ_t *are those that satisfy condition [B] with inequality (1) binding for at least one* $s \neq t$.

Figure 3 illustrates the Afriat preferences for a three-observation dataset that satisfies GARP. Figure 3a shows the observations' budget sets and consumption bundles. Figure 3b shows the associated Afriat indifference map, whose marginal rates of substitution are determined by the budget lines.

Figure 3. Neoclassical Afriat preferences for data that satisfy GARP



(a) (b)

The benefits of a nonparametric approach to neoclassical consumer demand are well understood. The theory's testable implications are inequality restrictions on observable, finite data, rather than shape restrictions on objects that are not directly observable. They can be checked directly without estimating econometric models of unobservable objects such as indifference, demand, or labor supply curves. The theory also largely avoids the need for the auxiliary statistical assumptions that structural econometric approaches require for consistent estimation. Measurement error is an exception, but it too can be handled nonparametrically (Varian 1985). Our analysis of reference-dependent consumer demand when it is empirically useful will preserve as many of these desiderata as possible.

IV. Nonparametric Implications of Reference-Dependent Preferences

This section begins our analysis of the nonparametric implications of reference-dependent preferences. Assuming that utility is additively separable across consumption and gain-loss utility, two factors determine whether the hypothesis of reference-dependent preferences can rationalize consumer demand behavior: whether sensitivity is constant or variable, and whether reference points are unmodelable or modelable. (None of our results depend on the interpretation of reference points.) Propositions 1 and 2 together show that unless reference points are modelable *and* sensitivity is constant, reference-dependent preferences are flexible enough to fit any data, with a minor qualification when constant sensitivity fails. Thus, without both modelable reference points and constant sensitivity, the hypothesis of reference-dependent preferences has no refutable implications.

A. Reference-dependent rationalization with unmodelable reference points

This section shows that if reference points are unmodelable, the hypothesis of reference-dependent preferences is nonparametrically irrefutable, even if Definition 4's GARP condition that is necessary (and sufficient) for a neoclassical rationalization is violated.

DEFINITION 6: [Rationalization with unmodelable reference points.] Reference-dependent preferences, an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$*, and hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\dots,T}$*, rationalize the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *if and only if* $u(\boldsymbol{q}_t, \boldsymbol{r}_t) \geq u(\boldsymbol{q}, \boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

PROPOSITION 1:[11] *[Rationalization with unmodelable reference points via preferences with variable or constant sensitivity.] For any data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *with unmodelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q}, \boldsymbol{r})$ *that are continuous, increasing in* $\boldsymbol{q}$*, and decreasing in* $\boldsymbol{r}$*, and a sequence of hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\dots,T}$*, that rationalize the data.*

Proof: Recall that we denote goods by superscripts, so that $\boldsymbol{q} \equiv (q^1, \dots, q^K)$, $\boldsymbol{q_t} \equiv (q_t^1, \dots, q_t^K)$, and so on. Let $a^k \equiv min_{t=1,\dots,T}\{p_t^k\} > 0$ for each *k* and *t*

[11] Gul and Pesendorfer (2006) and Freeman (2017) prove results with conclusions like Proposition 1's. However, Gul and Pesendorfer's rationalizing preferences do not satisfy KR's and our assumption of additive separability across consumption and gain-loss utility, and they allow the strength of loss aversion to vary wildly with the cardinality of their (finite) choice set. Freeman's Observation 1 does not restrict preferences, even to be monotonic. By contrast, Proposition 1's rationalizing preferences are credible candidates for an empirical explanation. We have found no informative results for cases with limited knowledge of reference points.

such that $q_t^k \geq r_t^k$; and $a^k \equiv max_{t=1,\dots,T}\{p_t^k\} > 0$ for each *k* and *t* such that $q_t^k < r_t^k$. Define the utility function $u(\boldsymbol{q}, \boldsymbol{r}) \equiv \sum_k a^k q^k + \sum_k a^k(q^k - r^k)$, which is strictly increasing in $\boldsymbol{q}$, strictly decreasing in $\boldsymbol{r}$, and satisfies constant sensitivity and Proposition 1's conditions for continuity. For observation *t*, set $\boldsymbol{r}_t = \boldsymbol{q}_t$ and consider any bundle $\boldsymbol{q} \neq \boldsymbol{q}_t = \boldsymbol{r}_t$ that (without loss of generality given strict monotonicity) exactly satisfies *t*'s budget constraint. For such bundles, $\sum_k p_t^k\,(q^k - q_t^k) = 0$ and, by the definition of the a^k,

(2) $\sum_k (a^k - p_t^k)(q^k - q_t^k) = \sum_k (a^k - p_t^k)(q^k - r_t^k) < 0$ and $\sum_k a^k(q^k - r_t^k) < 0$ and

(3) $u(\boldsymbol{q}, \boldsymbol{r}_t) - u(\boldsymbol{q}_t, \boldsymbol{r}_t) = 2\sum_k a^k(q^k - q_t^k) = 2\sum_k a^k(q^k - r_t^k) < 0,$

so $u(\boldsymbol{q}, \boldsymbol{r})$ rationalizes the choice of $\boldsymbol{q}_t$. Similarly for variable sensitivity. ■

With unmodelable reference points, Definition 6 allows a reference point to be chosen hypothetically for each observation—the nonparametric analogue of Farber's treatment of targets as latent variables. The proof of Proposition 1 hypothesizes a reference point for each observation with $\boldsymbol{r}_t = \boldsymbol{q}_t$ and preferences that, with those reference points, put the observation's consumption bundle at the kink of an approximately Leontief indifference curve (approximately to preserve strict monotonicity). Those preferences satisfy continuity, constant sensitivity, and Farber's, KR's, and CM's functional-form assumptions, so they are nonparametrically untestable as well. Because the rationalization works entirely by varying reference points across observations, it shows as directly as possible that the empirical usefulness of reference-dependent consumer theory depends on modeling reference points.

B. Reference-dependent rationalization with modelable reference points and variable sensitivity

This section shows that if reference points are modelable but sensitivity is variable, the hypothesis of reference-dependent preferences is refutable only via violations of GARP within subsets of observations that share *exactly* the same reference point. For such subsets, reference-dependent preferences reduce to neoclassical preferences. Either way, reference-dependence adds nothing empirically useful to the neoclassical model. Our results for this case are independent of how sensitivity varies, as long as it is not constant.

DEFINITION 7: [Rationalization with modelable reference points.] Reference-dependent preferences and an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$ *rationalize the data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points if and only if* $u(\boldsymbol{q}_t,\boldsymbol{r}_t) \geq u(\boldsymbol{q},\boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

PROPOSITION 2:[12] *[Rationalization with modelable reference points via preferences with variable sensitivity.] For any data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q},\boldsymbol{r})$ *that for each observation t and reference point* $\boldsymbol{r}_t$*, are continuous and strictly increasing in* $\boldsymbol{q}$ *and that rationalize the data, if and only if every subset of the data whose observations share exactly the same reference point satisfies GARP.*

Proof: Partition the observations into subsets $\tau^j, j = 1,\dots,J$, such that if and only if two observations $\{\boldsymbol{p}_s,\boldsymbol{q}_s,\boldsymbol{r}_s\}$ and $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}$ have the same reference point $\boldsymbol{r}_s = \boldsymbol{r}_t$, they are in the same subset. If there exists a reference-dependent utility function with the stated properties that rationalizes the data, then the data must satisfy GARP within any such subset, by Afriat's Theorem. Conversely, suppose the data within each such subset satisfies GARP. Let $b^k \equiv min_{t=1,\dots,T}\{p_t^k\}$, so that $0 < b^k \leq p_t^k$, and let $\boldsymbol{b} \equiv (b^1,\dots,b^K)$. For any

[12] As Proposition 2's proof shows, restricting sensitivity short of assuming that it is constant, for example by assuming diminishing sensitivity, still does not yield refutable implications. Unlike Proposition 1, Proposition 2 does *not* claim that $u(\boldsymbol{q},\boldsymbol{r})$ is continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$ or decreasing in $\boldsymbol{r}$. A rationalization might require discontinuous preferences if observations with nearby $\boldsymbol{r}$**'s** have very different budget sets. We have not tried to characterize rationalizability via a continuous $u(\boldsymbol{q},\boldsymbol{r})$. But for data generated by continuous preferences, Proposition 2's rationalizations should converge to a continuous limiting $u(\boldsymbol{q},\boldsymbol{r})$ as the data become richer.

subset τ^j and observation $t \in \tau^j$, let the indicator function $I_{\tau^j}(t) = 1$ if the observation $t \in \tau^j$ and $I_{\tau^j}(t) = 0$ otherwise, and let $u(\boldsymbol{q}, \boldsymbol{r}) \equiv \sum_j I_{\tau^j}(t) U^j(\boldsymbol{q}, \boldsymbol{r}_t)$, where $U^j(\boldsymbol{q}, \boldsymbol{r}_t) \equiv min_{\rho \in \tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} - \boldsymbol{b} \cdot \boldsymbol{r}_t$, which is Definition 5's Afriat utility function for observations in τ^j, with the U_ρ^j and λ_ρ^j taken from τ^j's binding condition B) inequalities (1) in Afriat's Theorem. If τ^j is a singleton subset, the terms in $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ follow observation *t*'s budget line. If not, those terms follow the minimum of τ^j's observations' budget lines, as in Figure 3b. Either way, $\boldsymbol{r}_t$ completely determines the $\boldsymbol{p}_\rho$ and $\boldsymbol{q}_\rho$ for all $\rho \in \tau^j$, as required to determine $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$. For each $\boldsymbol{r}_t$, $u(\boldsymbol{q}, \boldsymbol{r}_t)$ and $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ are continuous and increasing in $\boldsymbol{q}$. For any subset τ^j and observation $t \in \tau^j$ and any $\boldsymbol{q}$ with $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$, using τ^j's binding condition B) inequalities (1) for the preferences in that subset,

$$(4) \qquad U^j(\boldsymbol{q}, \boldsymbol{r}_t) \equiv min_{\rho \in \tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} - \boldsymbol{b} \cdot \boldsymbol{r}_t$$
$$\leq U_t^j + \lambda_t^j \boldsymbol{p}_t \cdot (\boldsymbol{q}_t - \boldsymbol{q}_t) - \boldsymbol{b} \cdot \boldsymbol{r}_t = U_t^j - \boldsymbol{b} \cdot \boldsymbol{r}_t \equiv U^j(\boldsymbol{q}_t, \boldsymbol{r}_t). \blacksquare$$

V. Characterizing Reference-dependent Preferences with Constant Sensitivity and Continuity

Sections IV's results show that nonparametrically refutable implications of reference-dependence depend on both modeling reference points and imposing constant sensitivity. To prepare for Section VI's analysis of rationalization in that case, this section characterizes reference-dependent preferences and utility functions with constant sensitivity and continuity.

Suppose that preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ satisfy: additive separability across consumption and gain-loss utility; constant sensitivity; and continuity in $\boldsymbol{q}$ and $\boldsymbol{r}$; with the number of goods $K \geq 2$ and reference-dependence active for all K goods;[13] and, for any $\boldsymbol{r}$, with the

[13] In riskless environments with convex budget sets, if $K = 1$ all monotone preferences are observationally equivalent, so reference-dependence cannot be empirically meaningful. And, as Proposition 3's wording suggests, its assumptions don't tie down the functional structure

induced preferences over $\boldsymbol{q}$ differentiable in the interior of each gain-loss regime, and marginal rates of substitution that differ across regimes throughout commodity space. Proposition 3 shows that the preferences must then be representable by a utility function $u(\boldsymbol{q}, \boldsymbol{r})$ with gain-loss utility functions that are additively separable across gain-loss regimes, across $\boldsymbol{q}$ and $\boldsymbol{r}$, and across goods within each regime; and whose good-by-good responses to reference points exactly mirror their responses to the components of consumption.

Let $G(\boldsymbol{q}, \boldsymbol{r})$ be a vector of binary numbers of length *K* with *k*th component 1 if $q^k \geq r^k$ and 0 otherwise. The gain-loss regime indicator $I_{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $\boldsymbol{g} = G(\boldsymbol{q}, \boldsymbol{r})$ and 0 otherwise; and the gain-loss indicators $G_+^k(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise.

PROPOSITION 3: [Preferences and utility functions with continuity and constant sensitivity.] Suppose there are K ≥ 2 goods, with reference-dependence active for all K goods, and that a reference-dependent preference ordering and an associated utility function have additively separable consumption utility and gain-loss utility components. Then the ordering satisfies constant sensitivity if and only if an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *can be written, for some consumption utility function* $U(\cdot)$ *and gain-loss regime utility functions* $V_{\boldsymbol{g}}(\cdot,\cdot)$ *and* $v_{\boldsymbol{g}}(\cdot)$*, as*

$$(5) \qquad u(\boldsymbol{q}, \boldsymbol{r}) \equiv U(\boldsymbol{q}) + \textstyle\sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{r}) V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}), \boldsymbol{r}).$$

Suppose further that the induced preferences over $\boldsymbol{q}$ *are differentiable in the interior of each regime, with marginal rates of substitution that differ across regimes throughout commodity space. Then the ordering satisfies constant sensitivity and continuity if and only if it is representable by a utility function*

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for goods for which reference-dependence is inactive. As we seek general characterizations, Propositions 4 and 5 take Proposition 3's conclusion, not its assumptions, as their premises.

$u(\boldsymbol{q},\boldsymbol{r})$ that can be written, for some consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$ (with the indicator functions $G_+^k(\cdot,\cdot)$ and $G_-^k(\cdot,\cdot)$ doing the work of the indicator $I_{\boldsymbol{g}}(\cdot,\cdot)$), as

(6) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

Conversely, any combination of induced regime preferences over $\boldsymbol{q}$ is consistent with continuity and constant sensitivity for some gain-loss utility functions.

Proof: The "if" part of each claim is immediate. The "only if" part regarding (5) follows from Definition 1 via the standard characterization of additively separable preferences. To prove the "only if" part regarding (6), note that $u(\boldsymbol{q},\boldsymbol{r})$ in (5) is continuous if and only if

(7) $$V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big) = V_{\boldsymbol{g}'}\big(v_{\boldsymbol{g}'}(\boldsymbol{q}),\boldsymbol{r}\big)$$

for any $\boldsymbol{q}$, $\boldsymbol{r}$, and *i* with $q^i = r^i$ and any gain-loss regimes $\boldsymbol{g}$ and $\boldsymbol{g}'$ that differ in the gain-loss status of good i. But (7) can hold under those conditions only if each regime's $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ is additively separable in the components of $\boldsymbol{q}$ and, for component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$, $k = 1, \dots, K$,

(8) $$\textstyle\sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big) \equiv \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

First suppose that (7) is satisfied for some $\boldsymbol{q}$, $\boldsymbol{r}$, and *i* with $q^i = r^i$. If $\partial V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \neq 0$, (7) implies that $\partial V_{\boldsymbol{g}'}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \neq 0$ as well. Adding $U(\boldsymbol{q})$ to each side of (7), partially differentiating each side with respect to $q^j$and then q^i, with $r^i = q^i$, and taking ratios would then show that the marginal rates of substitution between goods *i* and *j* are equal across regimes $\boldsymbol{g}$ and $\boldsymbol{g}'$ for all $q^i = r^i$, a contradiction. Thus with $q^i = r^i$, $\partial V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \equiv \partial V_{\boldsymbol{g}'}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \equiv 0$ for any $j \neq i$, and standard characterization results show that for a regime $\boldsymbol{g}$, $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ is additively separable across the components of $\boldsymbol{q}$. Given that, changing the gain-loss status of a good j with $q^i = r^i$ would violate (7) and therefore continuity, unless for some functions $w_+^k(\cdot)$ and $w_-^k(\cdot)$, $k = 1,\ldots, K$,

(9) $\sum_g I_g(\boldsymbol{q},\boldsymbol{r})V_g(v_g(\boldsymbol{q}),\boldsymbol{r}) \equiv \sum_k[G_+^k(\boldsymbol{q},\boldsymbol{r})\, w_+^k(q^k,\boldsymbol{r}) + G_-^k(\boldsymbol{q},\boldsymbol{r})w_-^k(q^k,\boldsymbol{r})]$.

Finally, unless the $w_+^k(\cdot,\cdot)$ and $w_-^k(\cdot,\cdot)$ are also additively separable in $\boldsymbol{r}$, with good-by-good responses to reference points that exactly mirror their good-by-good responses to bundles as in (8) (with $w_+^k(q^k,\boldsymbol{r}) \equiv \{v_+^k(q^k) - v_+^k(r^k)\}$ and $w_-^k(q^k,\boldsymbol{r}) \equiv \{v_-^k(q^k) - v_-^k(r^k)\}$), for some $\boldsymbol{q}$, $\boldsymbol{r}$, and k, changing q^k and r^k with $r^k = q^k$would induce different changes in $V_g\big(v_g(\boldsymbol{q}),\boldsymbol{r}\big)$ and $V_{g'}\big(v_{g'}(\boldsymbol{q}),\boldsymbol{r}\big)$, violating (7) and continuity. The contradiction establishes our claim regarding (8) and completes the proof of (6). A similar argument shows that any combination of induced regime preferences over $\boldsymbol{q}$ is consistent with continuity and constant sensitivity for some gain-loss utility functions. ■

Proposition 3's preferences nest the functional structure assumptions with constant sensitivity maintained in TK's and KR's analyses and all previous empirical studies. Their gain-loss utility component resembles KR's assumption regarding the structure of the gain-loss utility function, which Proposition 3 shows is necessary for continuity. However, that resemblance is limited to gain-loss utility. Under KR's and our assumption that preferences are additively separable across consumption and gain-loss utility, continuity does not restrict the structure of the consumption utility function, which does not vary across gain-loss regimes. Thus, with nonparametric flexibility, Proposition 3's characterization enables us to relax KR's implied restrictions that the sum of consumption and gain-loss utility that determines a consumer's demand is additively separable across goods and their restrictions on how that sum's marginal rates of substitution vary across gain-loss regimes (CM, Table 1). Both relaxations are likely to be important empirically.

Proposition 3's characterization (6) plays an important role in Proposition 4's and 5's conditions for a rationalization with modelable reference points and constant sensitivity. With constant sensitivity a consumer's induced preferences over $\boldsymbol{q}$ and her/his optimal choice of $\boldsymbol{q}$ are independent of $\boldsymbol{r}$ within a gain-loss regime, but the maximized value of $u(\boldsymbol{q}, \boldsymbol{r})$ still varies with $\boldsymbol{r}$ within a regime. (6)'s terms in $v_{+}^{k}(r^k)$ and $v_{-}^{k}(r^k)$ ensure continuity of $u(\boldsymbol{q}, \boldsymbol{r})$ despite such variation, by subtracting a regime-by-regime "loss cost". Because the loss costs depend on $\boldsymbol{r}$, not $\boldsymbol{q}$, a consumer faces a menu of fixed, exogenous regime charges, which influence her/his incentive to defect from an observation's consumption bundle to bundles in other regimes. This incentive constraint figures in Proposition 4's and 5's conditions for a rationalization.

VI. Nonparametric Implications of Reference-Dependent Preferences with Constant Sensitivity and Continuity

This section uses Proposition 3's characterization of reference-dependent preferences that satisfy constant sensitivity and continuity to derive nonparametric conditions for a reference-dependent rationalization in that case. (It seems clear from the literature and our own efforts that there are no simple combinatoric conditions that are necessary and sufficient for a reference-dependent rationalization, as GARP is in the neoclassical case.)

With modelable reference points, the observations' consumption bundles can be objectively sorted into gain-loss regimes. By Afriat's Theorem, GARP for each regime's observations is necessary for a rationalization, because it is required for the existence of preferences that preclude defections from an observation's bundle to affordable bundles within the same regime.

However, GARP for each regime's observations is not sufficient for a rationalization, for two reasons. First, the gain-loss regime utility functions that rationalize the consumer's choices within each regime must satisfy Proposition 3's restrictions that their component utility functions must be the same across all regimes, and GARP regime-by-regime does not ensure that that is possible. Second, the rationalizing regime utility functions must also prevent defections from an observation's bundle to affordable bundles in other regimes, in which preferences may differ. This involves Section V's loss costs, which are determined by the rationalizing regime utility functions.

Another difficulty in deriving conditions for a rationalization is that there is normally a range of rationalizing gain-loss regime utility functions, as in the neoclassical case (Varian (1982, Fact 4, Figure 3). Choosing among them involves complex trade-offs, because a choice that lowers the gain from defecting *from* bundles in a regime raises the gain from defecting *to* them.

Propositions 4 and 5 approach these difficulties in two steps. Proposition 4 derives benchmark necessary and sufficient conditions for a rationalization, conditional on the choice of rationalizing gain-loss regime utility functions. Because those conditions are conditional, they are not directly applicable.

Proposition 5 derives directly applicable sufficient (but not generally necessary) conditions based on rationalizing regime utility functions like Definition 3's Afriat utility functions. Because other rationalizing regime utility functions usually exist, those conditions are not necessary; but with rich enough data they should be asymptotically necessary, as explained below.

Let $\Gamma(g;\boldsymbol{r})\}$ be the set of $\boldsymbol{q}$ in regime g for $\boldsymbol{r}$. Let $\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g) \equiv \{t \in \{1,\dots,T\} \mid \boldsymbol{q}_t \in \Gamma(g;\boldsymbol{r}_t)\}$ be the set of t with $\boldsymbol{q}_t$ in regime g for $\boldsymbol{r}_t$.

PROPOSITION 4: [Rationalization with modelable reference points via preferences and utility functions with constant sensitivity.] Suppose that reference-dependent preferences and an associated utility function are defined over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function satisfies Proposition 3's (6). Consider data $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points. Then the statements [A] and [B] are equivalent:*

[A] There exists a continuous reference-dependent utility function $u(\boldsymbol{q},\boldsymbol{r})$ *that satisfies constant sensitivity; is strictly increasing in* $\boldsymbol{q}$ *and strictly decreasing in* $\boldsymbol{r}$*; and that rationalizes the data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$.

[B] Each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over consumption bundles, with continuous, strictly increasing consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$, such that, for any regime g and any pair of observations $\sigma, \tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)$ (with the indicator functions $G_+^k(\cdot,\cdot)$ and $G_-^k(\cdot,\cdot)$ again doing the work of $I_{\boldsymbol{g}}(\cdot,\cdot)$),

$$(10) \qquad U(\boldsymbol{q}_\sigma) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_+^k(q_\sigma^k) + G_-^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_-^k(q_\sigma^k)] \\ \leq \mathrm{U}(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)] + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau)$$

and for each observation $\{\boldsymbol{p}_\tau, \boldsymbol{q}_\tau, \boldsymbol{r}_\tau\}_{t=1,\ldots,T}$ with $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)$ and each $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ with $g' \neq g$ for which $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$,

$$(11) \quad U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] \\ \leq U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].$$

Proof: That [B] implies [A] is immediate. To prove that [A] implies [B], take the rationalizing regime preferences represented by $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$, which satisfy (10). Use Proposition 3 to write the condition preventing defections from the bundle of observation $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)$ in regime g to a bundle $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ in regime $g' \neq g$ for $\boldsymbol{r}_\tau$ with $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$:

$$u(\boldsymbol{q}, \boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau) \equiv U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_{\tau_\tau})\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\boldsymbol{r}_\tau)$$
$$(12) \equiv \{U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_+^k(q^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_-^k(q^k)]\} - \{U(\boldsymbol{r}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_-^k(r_\tau^k)]\}$$
$$\leq \{U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)]\} - \{U(\boldsymbol{r}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(r_\tau^k)]\}$$
$$\equiv U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}] \equiv u(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau).$$

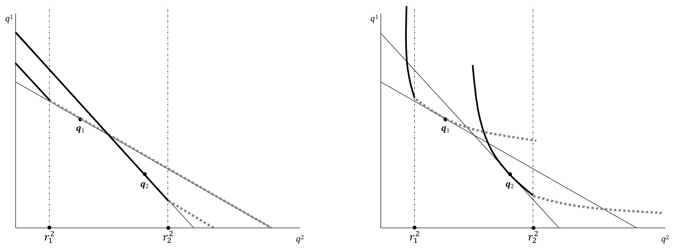
(12)'s central inequality can then be rearranged to yield (11). ■

Proving Proposition 4 requires linking Section V's loss costs to things that can be estimated from the data, not only at given points but as functions of $\boldsymbol{r}$. The proof shows that this can be done, as in (12).

Figures 4 and 5 illustrate Proposition 4. In each case the entire dataset violates GARP, with observation 1's consumption bundle chosen in 1's budget set over observation 2's bundle, and vice versa. In each case the observations' reference points put their bundles in different gain-loss regimes, so constant sensitivity allows different preferences for each observation. And in each case each regime's single observation trivially satisfies GARP within its regime.

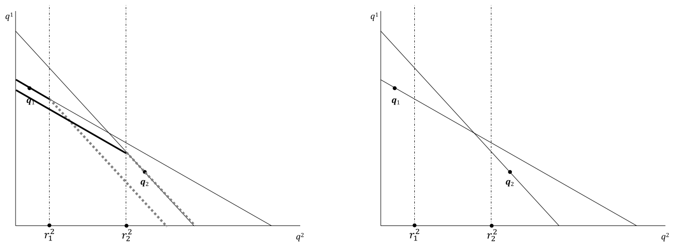
Figures 4a-b depict Afriat and non-Afriat rationalizing regime preferences. In each case condition (11) is satisfied, so that a rationalization is possible.

Figure 4. Rationalizing data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for loss maps, dashed lines for gains maps)



(a) (b)

Figure 5. Failing to rationalize data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for (a)'s loss maps, dashed lines for (a)'s gains maps)



(a) (b)

By contrast, in Figure 5a Afriat rationalizing regime preferences do not satisfy (11) and Figure 5b shows that there can be no choice of rationalizing regime preferences (Afriat or not) for which (11) is satisfied, so that a rationalization is impossible. A rationalization in Figure 5b would require regime preferences that connect a loss-regime indifference curve through observation 1's bundle to a gain-regime curve that cuts into observation 2's budget set and stays outside observation 1's budget set, thus passing northeast of 2's bundle; and also loss- and gain-regime indifference curves satisfying the analogous conditions interchanging observations 1 and 2. Such curves are inconsistent with optimality of each observation's consumption bundle.

The difference between Figure 4's and Figure 5's examples can be understood in terms of loss aversion (Definition 2). The change in rationalizing Afriat preferences across the gain-loss regimes in Figure 4a is consistent with loss aversion, but the analogous change in Figure 5a is not.

A Corollary shows that if the rationalizing regime preferences satisfy loss aversion, Proposition 4's no-cross-regime-defections constraints (11) must be satisfied, so that its conditions (10) are then sufficient for a rationalization.

Recall that the gain-loss indicator functions $G_+^k(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise; and that $\Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g) \equiv \{t \in \{1, \dots, T\} \,|\, \boldsymbol{q}_t \in \Gamma(g; \boldsymbol{r}_t)\}$ is the set of observation indicators t for which $\boldsymbol{q}_t$ is in regime *g* for $\boldsymbol{r}_t$.

COROLLARY*: [Rationalization with modelable reference points via preferences and utility functions with constant sensitivity that satisfy a condition weaker than loss aversion.] Suppose that reference-dependent preferences and an associated utility function are defined over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function therefore satisfies Proposition 3's (6). Consider data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points. If each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over consumption bundles, with continuous, strictly increasing consumption utility function* $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$*, such that, for any regime* g *and any pair of observations* $\sigma, \tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ *for which* $\boldsymbol{p}_\tau \cdot \boldsymbol{q}_\sigma \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *doing the work of a regime indicator function* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

(13) $$U(\boldsymbol{q}_\sigma) + \sum_k [G_+^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_+^k(q_\sigma^k) + G_-^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_-^k(q_\sigma^k)]$$
$$\leq \mathrm{U}(\boldsymbol{q}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)] + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau),$$

and there are no observations for which $\boldsymbol{q}_t$ *is* not *on the boundary of the convex hull of* $\boldsymbol{q}_t$*'s upper contour set for the associated candidate global preference ordering for* $\boldsymbol{r}_t$*, then the consumption utility function* $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$ *rationalize the data.*

Proof: As in Proposition 4, by Afriat's Theorem, the hypothesized combination of preferences over bundles with consumption utility function $U(\cdot)$ and gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$ prevent defections from any observation's consumption bundle to any affordable bundle in the same own gain-loss regime. If the hypothesized preferences are such that there are no observations *t* for which $\boldsymbol{q}_t$ is *not* on the boundary of the convex hull of the better-than-$\boldsymbol{q}_t$ set for the candidate global preference ordering given $\boldsymbol{r}_t$, then we can assume that they satisfy loss aversion without loss of generality. For, the candidate global ordering can then be replaced by a convexified ordering whose better-than-$\boldsymbol{q}_t$ sets are the convex hulls of the candidate global ordering, without changing any observation's optimal bundle. Definition 2 then implies that $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$ also prevent defections from any observation's bundle to any affordable bundle in a different regime. Alternatively, consider a defection from $\boldsymbol{q}_\tau \in \Gamma(g; \boldsymbol{r}_\tau)$ to some $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ with $g' \neq g$ and $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$. If $\boldsymbol{q}$ were in regime g, we would have, by Afriat's Theorem,

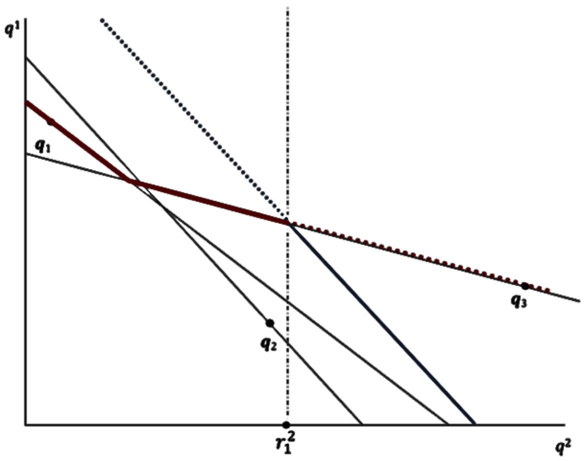
(14) $$U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_-^k(q^k) - v_-^k(r_\tau^k)\}]$$
$$\leq U(\boldsymbol{q}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].$$

Given that $\boldsymbol{q}$ is actually in regime g', the interpretation of loss aversion in terms of marginal rates of substitution implies that the left-hand side of (14) is lower or at least no higher than if $\boldsymbol{q}$ were in regime g. (14) thus prevents defections from $\boldsymbol{q}_\tau$ to affordable bundles in different regimes. ■

Loss aversion is an empirically well-supported assumption known to have important implications, but to our knowledge it has not previously been linked to the *existence* of a reference-dependent rationalization. As the proof suggests, loss aversion's testability is limited for the same reason that the convexity of neoclassical preferences is not nonparametrically testable.

The Corollary's final "no observations for which $\boldsymbol{q}_t$ is *not* on the boundary" condition rules out bunching of consumption bundles in regions of commodity space where the rationalizing regime preferences violate loss aversion and is vacuously satisfied for preferences that satisfy loss aversion. Such restrictions on bunching are unusual in a nonparametric analysis.

Figure 6. Rationalizing data that violate GARP when preferences violate loss aversion but satisfy the Corollary's sufficient conditions for a rationalization (solid curves for active parts of indifference maps, dashed for inactive parts)



In Figure 6 the entire dataset violates GARP, the Afriat gain-loss regime preferences violate loss aversion, but the data satisfy the Corollary's final conditions, thus allowing a rationalization. Only reference point $\boldsymbol{r}_1$ is shown and observation 1 is in the good-2 loss regime. Assume that $\boldsymbol{r}_2 = [0, 0]$, so that observation 2's budget set is entirely in the good-2 gain regime; and that

$\boldsymbol{r}_3 = [0, m]$, where m is large enough that observation 3's budget set is entirely in the good-2 loss regime. The Afriat regime preferences yield a candidate for global preferences that make all three observations' consumption bundles optimal: Observations 2's and 3's budget sets are entirely in their regimes (good-2 gain and good-2 loss, respectively), so their bundles' optimality in their regimes suffices for global optimality. Observation 1's bundle is optimal for its good-2 loss regime preferences and Corollary 1 ensures that its bundle's optimality extends to its entire budget set.

As already noted, Proposition 4's necessary and sufficient conditions for a rationalization are not directly applicable because they are conditional on the choice of rationalizing gain-loss regime utility functions. Proposition 5 derives directly applicable sufficient conditions by specifying rationalizing regime utility functions in the style of the regime's Afriat utility functions (Definition 5). Those conditions include inequalities like (1) in Afriat's Theorem or Proposition 4's (10), which prevent defections from an observation's consumption bundle to affordable bundles in the same gain-loss regime, while enforcing Proposition 3's restrictions (6) on the component gain-loss utility functions. The conditions also include inequalities like Proposition 4's (11), which prevent defections to affordable bundles in other regimes.

PROPOSITION 5: [Sufficient conditions for rationalization with modelable reference points, via reference-dependent preferences and utility function with constant sensitivity and continuity.] The following conditions are sufficient for the existence of continuous reference-dependent preferences and utility function with constant sensitivity $u(\boldsymbol{q}, \boldsymbol{r})$ *that rationalize data with modelable reference points* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$*: There exist numbers* U_t, v_{t+}^k, v_{t-}^k, *and* $\lambda_t > 0$ *for each* $k = 1, \ldots, K$ *and* $t = 1, \ldots, T$ *such that:*

[A] For any gain-loss regime g *and any pair of observations* $\sigma, \tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *again doing the work of* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

$$
\begin{aligned}
(15) \qquad & U_\sigma + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_{\sigma+}^k + G_-^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_{\sigma-}^k] \\
& \leq U_\tau + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_{\tau+}^k + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_{\tau-}^k] \; + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau).
\end{aligned}
$$

[B] For observations σ, τ, $q_\sigma^k \geq q_\tau^k$ for k = 1,..., K, $U_\sigma \geq U_\tau$; and for observations σ, τ and any k = 1,..., K, $q_\sigma^k \geq q_\tau^k$, $v_{\sigma+}^k \geq v_{\tau+}^k$, and $v_{\sigma-}^k \geq v_{\tau-}^k$.

[C] For any pair of regimes g and $g' \neq g$, observation $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)$, and bundle $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ for which $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$,

$$
\begin{aligned}
& min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g')} \{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} \\
(16) \qquad & -min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g')} \{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\} \\
& \leq min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)} \{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_\tau - \boldsymbol{q}_\rho)\} \\
& -min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)} \{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\}.
\end{aligned}
$$

Proof: Given choices of U_t, v_{t+}^k, v_{t-}^k, and λ_t, t = 1,…, T, that satisfy [A] and [B], let $u^g(\boldsymbol{q}, \boldsymbol{r})$ denote the rationalizing Afriat regime utility function for regime g, including (6)'s loss costs, which exists regime by regime by Afriat's Theorem. For $\boldsymbol{q} \in \Gamma(g; \boldsymbol{r})$, using (10) as in the proof of Afriat's Theorem:

$$
\begin{aligned}
& u^g(\boldsymbol{q}, \boldsymbol{r}) - U(\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}] - U(\boldsymbol{r}) \\
& \equiv \left\{U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}) v_+^k(q^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}) v_-^k(q^k)]\right\} - \left\{U(\boldsymbol{r}) + \sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r})\{v_+^k(r^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}) v_-^k(r^k)]\right\} \\
(17) \qquad & \equiv min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)} \{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} \\
& - min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)} \{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r} - \boldsymbol{q}_\rho)\}.
\end{aligned}
$$

The rationalizing reference-dependent utility function, including loss costs, is then $u(\boldsymbol{q}, \boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_g I_{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{r})\, u^g(\boldsymbol{q}, \boldsymbol{r})$. By construction, $u(\boldsymbol{q}, \boldsymbol{r})$ is continuous, strictly increasing in $\boldsymbol{q}$, and strictly decreasing in $\boldsymbol{r}$.

For observations $\sigma, \tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g)$ in the same gain-loss regime g, with $\boldsymbol{p}_\tau \cdot \boldsymbol{q}_\sigma \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$, loss costs cancel out and (16) reduces to the usual Afriat inequalities (with the Afriat utilities expressed not as single numbers but as sums of consumption plus gain-loss utilities). Thus by Afriat's Theorem, [A] prevents defections to affordable bundles in the same regime.

For gain-loss regimes g and $g' \neq g$, observation $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$, and bundle $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ with $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_t$,

$$u(\boldsymbol{q},\boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau) \equiv U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r}_{\tau_\tau})\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\boldsymbol{r}_\tau)$$

$$\equiv \left\{U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r}_\tau)v_+^k(q^k) + G_-^k(\boldsymbol{q},\boldsymbol{r}_\tau)v_-^k(q^k)]\right\} - \left\{\mathrm{U}(\mathbf{r}_\tau) + \sum_k [\mathrm{G}_+^k(\mathbf{q},\mathbf{r}_\tau)\{\mathrm{v}_+^k(\mathrm{r}_\tau^k) + \mathrm{G}_-^k(\mathbf{q},\mathbf{r}_\tau)\mathrm{v}_-^k(\mathrm{r}_\tau^k)]\right\}$$

$$\equiv min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\}$$

$$-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\}$$

(18)

$$\leq min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_\tau - \boldsymbol{q}_\rho)\}$$

$$-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\}$$

$$\equiv \left\{U(\boldsymbol{q}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_-^k(q_\tau^k)]\right\} - \left\{U(\boldsymbol{r}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_-^k(r_\tau^k)]\right\}$$

$$\equiv u(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)) - U(\boldsymbol{r}_\tau),$$

which prevents defections across regimes. ■

Proposition 5 depends on the choice of Afriat rationalizing regime utility functions.[14] As other choices might also suffice, its sufficient conditions are not necessary. For example, the Afriat regime preferences in Figure 7a do not yield a rationalization but the non-Afriat regime preferences in Figure 7b do.

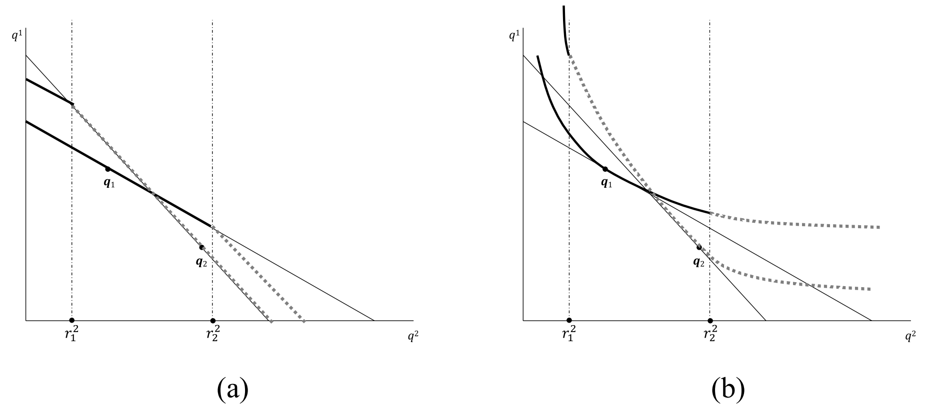
Although Proposition 5's sufficient conditions are not necessary, Mas-Colell's (1978) and Forges and Minelli's (2009) results for the neoclassical case suggest a sense in which they should be asymptotically necessary. In the neoclassical case, they study the limit as the data become rich in the sense that as $T \to \infty$ the data come to include {reference point×budget set} combinations as close as desired to any possible combination, showing that the range of convexified rationalizing preferences then collapses on Definition 5's Afriat preferences.[15] With constant sensitivity this result cannot be immediately applied gain-loss regime by regime, because of Proposition 3's constraint that

[14] Varian's (1982, Fact 4) bounds for the neoclassical case don't imply that all rationalizing preferences are convex, but examples show that requiring such convexity involves a loss of generality for some rationalizing regime preferences in Proposition 4. Proposition 5 avoids that difficulty by using the Afriat regime preferences, which are convex by construction.

[15] Also requiring richness of consumption bundles would rule out non-convex preferences.

the component gain-loss utility functions must be the same in all regimes. But it is a plausible conjecture that in the limit, if the Afriat regime preferences do not yield a rationalization, neither can any other regime preferences, so that Proposition 5's sufficient conditions are asymptotically necessary.

Figure 7. A rationalization may require non-Afriat rationalizing regime preferences (solid lines for the loss map, dashed for the gain map)



Proposition 5 immediately suggest a procedure for nonparametrically estimating a continuous reference-dependent model with constant sensitivity:

(i) Use the observations' modeled reference points to sort their consumption bundles into gain-loss regimes.

(ii) Pooling the data from all regimes, use linear programming to find Afriat numbers U_t, v_{t+}^k, v_{t-}^k, and $\lambda_t > 0$ for each $k = 1, \dots, K$ and $t = 1, \dots, T$ that satisfy [A]'s Afriat inequalities (15).

(iii) Use the fact that for each observation in a regime, (15) can hold with equality for another observation in the regime, to choose numbers so that for observation *t* in regime *g*, the rationalizing Afriat utilities are given as in (17) in the proof of Proposition 5:

$$U_t = u^g(\boldsymbol{q}_t, \boldsymbol{r}_t) \equiv min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)} \left\{ U_\rho + \sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_t - \boldsymbol{q}_\rho) \right\}.$$

(iv) Use (ii)'s Afriat numbers U_t, v_{t+}^k, and v_{t-}^k to check that [B]'s monotonicity restrictions are satisfied.

(v) Use (iii)'s rationalizing Afriat utilities to check, regime by regime and observation by observation, that [C]'s conditions (16) are satisfied by scanning along the budget surface.

Proposition 5's conditions (15) involve linear inequalities in a finite number of variables; and its conditions (16) involve nonlinear inequalities in a continuum of $\boldsymbol{q}$ values. Both sets of inequalities are finitely parameterized by the U_t, v_{t+}^k, v_{t-}^k, and λ_t that satisfy [A]'s (15). Thus our procedure satisfies most of the desiderata of and should inherit much of the tractability of Diewert's (1973) and Varian's (1982) methods for the neoclassical case.

VII. Conclusion

This paper presents a nonparametric analysis of the theory of consumer demand or labor supply with reference-dependent preferences. Our nonparametric model of preferences closely follows KR's structural analysis, maintaining their and all others' assumption that preferences are additively separable across components of consumption and gain-loss utility and relaxing KR's and all others' unnecessarily restrictive assumptions on functional structure and other studies' assumptions on functional form.

Propositions 1 and 2 show that unless reference points are precisely modelable or observable and sensitivity is constant, reference-dependent models of consumer demand are flexible enough to fit any data (with a minor exception when sensitivity is variable). Proposition 1 also suggests that analyses that, like Farber's, treat reference points as latent variables may be as heavily influenced by the constraints they impose in estimating reference points as they are by reference-dependence per se.

Assuming modelable reference points and constant sensitivity, Proposition 3 characterizes preferences that are continuous, deriving KR's assumption that gain-loss utility is determined by the good-by-good differences between realized and reference utilities from continuity, while relaxing KR's parallel functional structure assumption on consumption utility, which is not required for continuity because the consumption utility function is constant across gain-loss regimes. This allows us to relax KR's assumption's unnecessarily restrictive implication that the sum of consumption and gain-loss utility that determines consumer demand is additively separable across goods, and the associated restrictions on how its marginal rates of substitution vary across regimes. Blow and V. P. Crawford's (2024) preliminary analysis of Farber's (2005, 2008) data suggests that these relaxations are empirically important.

Propositions 4 and 5 use Proposition 3's characterization to derive directly applicable sufficient and, with rich enough data, asymptotically necessary conditions for a reference-dependent rationalization with continuity and constant sensitivity. Our results suggest methods for either structural or nonparametric estimation of reference-dependent consumer demand, using data like Farber's (2005, 2008, 2015), CM's, Thakral and Tô's (2021), Andersen et al.'s (2023), or Brandon et al.'s (2023), and using natural sample proxies like CM's for KR's rational-expectations model of the targets. Such analyses should reveal the extent to which previous econometric analyses' unnecessarily restrictive functional-structure assumptions bias their results.

We hope that our analysis shows that reference-dependent models of consumer demand are a useful addition to the consumer demand toolkit.

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