Answers to Game Theory and IO Practice Problems for Classes in Weeks 6 and 8
First-Year M. Phil Microeconomics, Michaelmas Term 2011
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To be worked and handed in for the Week 6 class:

1. Consider the following two-person game:

		Player 2 L	Player 2 R
Player 1	U	1, 2	0, 1
	D	3, 0	x, 1

Assume that both players know the value of x, and both know that they know, and so on.

(a) For what values of x (if any) is there a Nash equilibrium in which Player 2 chooses R with probability one? Explain, and describe the equilibrium or equilibria in different cases.
(a) x greater than or equal to 0. Choosing R yields Player 2 a payoff of 1 no matter what Player 1 chooses, so Player 2 can choose R only if he thinks the probability that Player 1 will choose D is at least ½ (so that 1 > ½ 0 + ½ 2). But if Player 2 chooses R with probability one, Player 1 will choose D in equilibrium if and only if x is greater than or equal to 0. Thus, whenever x is greater than or equal to 0, there is an equilibrium in which Player 2 chooses R with probability one. If instead $x < 0$, Player 1's best response to R is U, to which Player 2's best response is L, so there is no such equilibrium.

(b) For what values of x (if any) does decision R for Player 2 survive iterated deletion of strictly dominated strategies? Explain.
(b) Any x. If $x > 0$, D strictly dominates U for Player 1, and once U is deleted, R dominates L for Player 2. If x is less than or equal to 0, there is no strict dominance, so both strategies for both players survive iterated deletion of strictly dominated strategies.

2. Two players, Row and Column, are driving toward each other on a one-lane road. Each player chooses simultaneously between going straight (S), swerving left (L), and swerving right (R). If one player goes straight while the other swerves, either right or left, the one who goes straight gets payoff 3 while the other gets –1. If each player swerves to his left, or each swerves to his right, then each gets 0 (remember, they are going in opposite directions). If both go straight, or if one swerves to his left while the other swerves to his right, then the cars crash and each gets payoff –4.

(a) Write the payoff matrix for this game.
(a)

			Column	
		L	**S**	**R**
	L	**0, 0**	**-1, 3**	**-4, -4**
	S	**3, -1**	**-4, -4**	**3, -1**
Row	**R**	**-4, -4**	**-1, 3**	**0, 0**

(b) Find all of the game's rationalizable strategies for each player.
(b) L, S, R are rationalizable for each player.

(c) Find all of the game's Nash equilibria in pure strategies.
(c) (S,L), (L,S), (S,R), (R,S)

(d) Find a Nash equilibrium in which Row uses a pure strategy and Column mixes between two of his strategies. Clearly identify which strategy or strategies have positive probabilities for each player, and what Column's mixing probabilities are. (Hint: Which of Row's pure strategies could make Column willing to put positive probability on two of Column's pure strategies?)
(d) Only S for Row can make Column willing to mix on two strategies, L and R, because Column has a unique best response to L or R for Row. Given that –4 < –1 for Column, all that is needed for an equilibrium of this kind is that Row has S as a best response. But this is true because 3 > 0 > -4, for *any* mixing probabilities for Column.

(e) Find a Nash equilibrium in which both Row and Column mix between two of their strategies. Clearly identify which strategies have positive probabilities for each player, and their mixing probabilities are. (Hint: Pick two pure strategies for each player—because the game is symmetric, it's natural to try the same two strategies for each—and figure out what the mixing probabilities would have to be on just those two strategies. Then compare each player's expected payoff with what he could get by switching to his third strategy.)
(e) In one equilibrium of this kind, each player mixes 0.5 – 0.5 on L and S. This yields expected payoff –1/2 for each, which is higher than the payoff –2.5 of switching to R. By symmetry, there is another equilibrium in which each player mixes 0.5 – 0.5 on R and S. But mixing on L and R doesn't work, because the mix would have to be 0.5 – 0.5, and then switching to S would be better for either player.

(f) Find the (unique) Nash equilibrium where each player uses all three of his strategies in a mixture. (Hint: First prove that the probabilities of L and R must be equal in the equilibrium mixture. Then show that for each player the probability of S must be 5/8.)

(f) The first is true by symmetry, or by looking at the algebra. The probability of L or R must also be the same for both players by symmetry. Call it p. Then, for Row to be willing to mix on all three pure strategies, we must have 0p –1(1-2p) –4p = 3p – 4(1-2p)+3p = -4p –1(1-2p) + 0p, or –1 – 2p = 14p – 4 = –1 – 2p, or p = 3/16.

3. Suppose three identical, risk-neutral firms must decide simultaneously and irreversibly whether to enter a new market which can accommodate only two of them. If all three firms enter, all get payoff 0; otherwise, entrants get 9 and firms that stay out get 8.

(a) Identify the unique mixed-strategy equilibrium and describe the resulting probability distribution of the total ex post number of entrants. (You are not asked to show this, but the game also has three pure-strategy equilibria, in each of which exactly two firms enter; but these equilibria are arguably unattainable in a one-shot game in the absence of prior agreement or precedent. The mixed-strategy equilibrium is symmetric, hence attainable.)
(a) In a mixed-strategy equilibrium, each of the firms enters with probability p, where p equalizes the expected payoffs of entering and staying out: $9(1-p^2) = 8$, which reduces to p = 1/3. The binomial equilibrium probabilities that exactly 0, 1, 2, or 3 firms enter are then 8/27, 12/27 = 4/9, 6/27 = 2/9, or 1/27 respectively.

4. In each of the following games, graph players' best-response curves, letting p be the probability that Row plays Up and q be the probability that Column plays Left. Then use your best-response curves to find all the equilibria in each game, whether in pure or mixed strategies. (Games (a) and (b) are zero-sum and only Row's payoffs are shown; games (c) and (d) are non-zero-sum and both players' payoffs are shown.)

(a) Column

		Left	Right
Row	Up	- 3	1
	Down	0	- 5

(b) Column

		Left	Right
Row	Up	5	- 4
	Down	1	2

(c) Bill

		B1	B2
Ann	A1	4 , 2	- 5, 6
	A2	- 1, 5	0 , - 2

(d) Bill

		B1	B2
Ann	A1	3 , 2	0 , 1
	A2	1, 0	4 , 6