A Dual Dutch Auction in Taipei: The Choice of Numeraire and Auction Form in Multi-Object Auctions with Bundling

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Taipei Auction: auctioneer announces a fixed money price for a basket, then puts a series of identical fish into it until some buyer signals that he is willing to pay the price

Unusual in two respects: bundling, with units of money rather than fish bundled, and fish as the numeraire rather than money; because the price of fish is the quantity of money exchanged for a unit of fish, the Taipei Auction is dual to a conventional Dutch auction, with the roles of quantity and price reversed

Zoë Crawford's photogaphs of the Hu-Lin Street evening market, including the dual Dutch auctioneer and the numeraire, can be viewed at http://weber.ucsd.edu/~vcrawfor/Photos.html#EveMkt

We study the choice of numeraire and auction form in multi-object auctions with bundling, to learn how they interact and why duality is so rarely observed

ASSUMPTIONS

One seller, with F fish, and $n \geq 2$ buyers; index seller $i = 0$ and buyers $i = 1,\ldots,n$; fish and money are homogeneous and perfectly divisible; resale is impossible

Agents have quasilinear von Neumann-Morgenstern indirect (reflecting future auctions) utility functions, additively separable, linear in money, but strictly concave in fish

Agent *i*'s utility is $u_i(f_i, m_i) = v_i g(f_i) + m_i$, $i = 0,\ldots n$, where $g(\cdot)$ is increasing, strictly concave, and differentiable, and $v_0 = 1$; $g(\cdot)$ satisfies Inada condition $\lim_{f\to\infty} g'(f) = \infty$, so seller's optimal bundle is always interior (can allow seller to have different function $g(\cdot)$ than the buyers, and sometimes can dispense with strict concavity)

v^i determines buyer *i*'s reservation price in money for fish bundles and in fish for money bundles; "higher value" means "higher value of fish"; $v_i > 1$, $i = 1,\ldots,n$, so efficiency requires trade; and $v_1 > v_2 > \ldots > v_n > 1$, so buyer 1 prefers fish more than buyer 2, etc.

Ex ante, v_i are i.i.d., with distribution $H(\cdot)$, with finite support $[v^{min}, v^{max}]$, where $v^{min} > 1$; buyers symmetric, with independent private values (for fish in money and money in fish)

Structure otherwise common knowledge. Allow buyers' values first to be common knowledge to the seller as well as the buyers; then to be common knowledge among buyers but unknown to the seller; and finally, to be privately known by each buyer

Focus on symmetric Nash or Bayesian equilibria throughout

ROLE FOR BUNDLING

First discuss possible role for bundling, taking the choice of numeraire as given

Two buyers, one seller, two identical fish

$v_0 = 0$, so seller values only money, easily relaxed

$g(1) = 2$, $g(2) = 3$, $v_1 = 5$, and $v_2 = 2$, so buyer 1's reservation prices are \$10 for one fish and \$15 for two, and buyer 2's reservation prices are \$4 for one fish and \$6 for two

Compare a sequence of two conventional auctions of one fish each with a single, bundled auction of both fish

When buyers' values are common knowledge, a conventional auction, English or Dutch, is always won by the highest valuer, at a price at least approximately equal to the second-highest value

In sequential auctions, the buyers' subgame-perfect equilibrium bidding strategies must reflect their rational anticipations of how the outcome of the current auction will influence the outcome of subsequent auctions

In example, in a sequence of two auctions of one fish at a time, buyer 1 must win the second auction because his value is higher, but at a price that depends on whether buyer 2 won the first auction, due to the diminishing marginal value of fish

There is a unique subgame-perfect equilibrium, in which buyer 2 wins the first auction despite his lower value for fish, paying \$3, and buyer 1 then wins the second auction, paying \$2. In this equilibrium buyer 1's utility is 8; buyer 2's utility is 1; and the seller's utility is 5

In a single, bundled auction of both fish, buyer 1 wins and pays \$6, and his utility is 9; buyer 2 loses, pays nothing, and his utility is 0; and the seller receives \$6, and his utility is 6

Bundled auction both allocates the fish more efficiently and yields the seller higher revenue: Everyone but buyer 2 is better off, and compensation could yield a Pareto-improvement

From now on, take bundling as given and study the effect of auction form and choice of numeraire on allocation generated by a single, bundled auction under different information conditions

Assume that seller and buyers ignore strategic interactions with any subsequent auctions, and that their preferences over outcomes of current auction can be described by von Neumann-Morgenstern indirect utility functions

Assume that seller chooses the optimal bundle, given the auction form

First buyers' values are common knowledge to the seller as well as the buyers; then common knowledge among buyers but unknown to the seller; and finally, privately known by each buyer

The possible relationships among the seller's equilibrium expected utilities under different information conditions can be summarized as follows:

Conventional Dutch	=	**Dual Dutch**	**Conventional Dutch**	>	**Dual Dutch**	**Conventional Dutch**	$\gtrless$	**Dual Dutch**
$\|$		$\|$	$\|$		$\|$	$\|$		**V**
Conventional English	=	**Dual English**	**Conventional English**	>	**Dual English**	**Conventional English**	>	**Dual English**
Values commonly known			**Values unknown to seller**			**Values privately known**		

Our observations in the Hu-Lin Street evening market make it intriguing that the potential for improving upon conventional auctions depends on the auction being both dual and Dutch; our results provide a possible rationale for the Taipei auction, and suggest that its conjunction of duality and Dutchness was not coincidental

BUYERS' VALUES COMMONLY KNOWN TO SELLER AS WELL AS BUYERS

Given seller's choice of bundle, an English or Dutch auction, conventional or dual, is a standard auction of a single, indivisible object

In equilibrium each auction is won by the highest valuer (of fish), who pays money for the bundle of fish or receives fish in exchange for the bundle of money in an amount that makes the second-highest valuer indifferent between winning and losing

Given this, in a conventional English or Dutch auction the seller's optimal fish bundle, f^c, solves:

(1) $$\max g(F-f)+m \quad \text{s. t.} \quad m = v_2 g(f).$$

f^c then determines the money price, m^c, via the auction

In a dual auction the seller's optimal money bundle, m^d, also solves problem (1). m^d then determines the amount of fish received, f^d, via the auction

PROPOSITION 1: *Suppose that the buyers' values are common knowledge. Then, in an English or Dutch auction, conventional or dual, the highest valuer wins the auction, paying money or receiving fish according to the second-highest value. The seller's optimal fish bundle in a conventional auction equals the amount of fish received by the winning buyer in its dual counterpart; the seller's optimal money bundle in a dual auction equals the money paid by the winning buyer in its conventional counterpart; and all four auctions yields the same outcome.*

PROOF: The constraint in (1) makes m a known, increasing function of f, or vice versa, so it doesn't matter whether the seller chooses f^c, determining m^c, or m^c, determining f^c

Thus, when buyers' values are common knowledge, the rarity of dual auctions cannot be explained by the asymmetry in how fish and money enter agents' preferences

Even with complete information, the seller's choice of bundle causes inefficiency:

PROPOSITION 2: *Suppose that the buyers' values are common knowledge. Then, in an English or Dutch auction, conventional or dual, the seller's optimal bundle is too small and the volume of trade is too low for efficiency.*

PROOF: By Proposition 1, all four auctions yield the same exchange of money for fish, and the seller's optimal bundle maximizes $g(F-f) + v_2\, g(f)$, in a conventional auction directly by choice of f^c, and in a dual auction indirectly by choice of m^d. Maximizing $g(F-f) + v_2\, g(f)$, the surplus from a hypothetical exchange between the seller and the second-highest valuer, yields a bundle too small to maximize $g(F-f) + v_1\, g(f)$, the surplus from the actual exchange between the seller and the highest valuer.

BUYERS' VALUES COMMONLY KNOWN TO BUYERS, BUT NOT SELLER

In this case, for a given choice of numeraire, English and Dutch auctions are both won by the highest valuer, who pays money or receives fish according to the second-highest value. Thus, we need only distinguish between conventional and dual auctions

In a conventional auction, seller's optimal fish bundle, f^c, is the value of f that solves:

(2) $$\max E[g(F-f)+m] \qquad \text{s. t.} \qquad m = v_2 g(f).$$

Seller's choice of f^c and the realization of v_2 together determine m^c, via the auction. E refers to *un*conditional distribution of v_2, the second-highest value in n independent draws from the common distribution $H(\cdot)$

In a dual auction, seller's optimal money bundle, m^d, is the value of m that solves (2)

But now seller's choice of m^d and the realization of v_2 together determine f^d, via the auction; and the expectation is taken over the distribution of f^d induced by that of v_2

In a conventional auction f^c is deterministic and m^c is random, while in a dual auction m^d is deterministic and f^d is random. As a result, even though f^c and m^d solve the "same" problem with different forms of uncertainty, and in equilibrium each auction yields an exchange between the buyer and the highest valuer, conventional and dual auctions yield different volumes of trade and expected utilities

Given that $g(\cdot)$ is concave and satisfies an Inada condition, the second-order conditions of problem (2) are always satisfied, and its solutions are always interior

f^c is determined by the first-order condition

(3) $$\psi(f)\frac{1}{Ev_2} = 1, \text{ where } \qquad \psi(f) \equiv \frac{g'(F-f)}{g'(f)},$$

and m^d is determined by the first-order condition

(4) $$E\left[\psi(g^{-1}[m/v_2])\frac{1}{v_2}\right] = 1.$$

$f^c \leq F$ and, for all realizations of v_2, $f^d \leq F$, so $m^d \leq v^{min} g(F)$. Given the Inada condition, (3) rules out violations of the first constraint; and (4) rules out violations of the second

$g^{-1}(\cdot)$ is positive, increasing, and convex, and $\psi(\cdot)$ is positive and increasing; our assumptions on $g(\cdot)$ do not determine whether $\psi(\cdot)$ is concave or convex, but $\psi(\cdot)$ is convex for many parameterizations, and this appears to be the normal case

PROPOSITION 3: *Suppose that the buyers' values are common knowledge among the buyers but unknown to the seller. Then, in an English or Dutch auction, conventional or dual, the highest valuer wins the auction, paying money or receiving fish according to the second-highest value. A conventional English and a conventional Dutch auction yield the same outcome, and a dual English and a dual Dutch auction yield the same outcome. However, a conventional auction always yields the seller higher expected utility than its dual counterpart.*

PROOF: First parts are immediate. To prove the last part, note that in a conventional auction seller can set $f = Ef^d$. f^c must therefore yield him an expected utility at least as high as Ef^d and the distribution of $m = v_2\, g(Ef^d)$ it induces. Thus,

(5) $$E[g(F-f^c)+m^c] \geq g(F-Ef^d)+E[v_2 g(Ef^d)] = g(F-Ef^d)+[Ev_2][g(Ef^d)]$$
$$> Eg(F-f^d)+[Ev_2][Eg(f^d)] > Eg(F-f^d)+E[v_2 g(f^d)] = Eg(F-f^d)+m^d,$$

where the inequalities follow from revealed preference, the strict concavity of $g(\cdot)$ and Jensen's inequality, and the fact that v_2 and f^d are negatively correlated.

REMARK: Seller prefers conventional auctions because from his point of view, a conventional auction induces uncertainty only about the allocation of money, which is costless, while a dual auction induces costly uncertainty about the allocation of fish. However, the proof is not a direct translation of this insurance intuition, and shows that the seller's preference requires only that either the seller or the winning buyer is strictly risk averse in the relevant range, even though the buyers bear no uncertainty. In fact the seller's preference extends to the case where both he and the buyers are risk-neutral, where his welfare increases with volume of trade. In a conventional auction he can set $f^c = F$, realizing expected utility Ev_2F. In a dual auction, because $g(\cdot)$ no longer satisfies the Inada condition we must impose $m^d \leq v^{min}F$ to ensure that $f^d = m^d/v_2 \leq F$. He therefore sets $m^d = v^{min}F$, realizing expected utility less than Ev_2F. Thus, with risk-neutrality Proposition 3's conclusion remains valid because first inequality in (5) is strict.

PROPOSITION 4: *Suppose that the buyers' values are common knowledge among the buyers but unknown to the seller. Then the same buyer wins the auction, whether it is conventional or dual, and losing buyers are indifferent between conventional and dual auctions. f^c, the seller's optimal fish bundle in a conventional auction, is larger than $g^{-1}(m^d/v^{max})$, and therefore larger than the amount of fish received by a buyer in its dual counterpart who wins when $v_2 \approx v^{max}$; m^d, the seller's optimal money bundle in a dual auction, is smaller than $m^c = g(f^c)v^{max}$, and therefore smaller than the amount of money paid by a buyer in its conventional counterpart who wins when $v_2 \approx v^{max}$; and such a buyer prefers the outcome of a conventional auction to that of its dual counterpart. If, in addition, the function $\psi(\cdot)$ is convex, then for any $v_2 \geq Ev_2$, in a conventional auction $f^c > g^{-1}(m^d/v_2)$, the amount of fish received by a buyer in its dual counterpart who wins when the second-highest value is v_2; in a dual auction $m^d < m^c = v_2\, g(f^c)$, the money paid by a buyer in its conventional counterpart who wins when the second-highest value is v_2; and any buyer who wins when $v_2 \geq Ev_2$ prefers the outcome of a conventional auction to that of its dual counterpart.*

PROOF: Highest valuer still wins the auction, paying money or receiving fish according to the second-highest value, and losing buyers are indifferent between auctions. In this version of our model the buyers know all buyers' values, and therefore bear no uncertainty in equilibrium. In a conventional auction, the winning buyer pays money price $m^c = v_2 g(f^c)$ for fish bundle f^c, realizing utility $(v_1 - v_2)g(f^c)$. In a dual auction, the winning buyer receives $f^d = g^{-1}(m^d/v_2)$ units of fish in exchange for money bundle m^d, realizing utility $(v_1-v_2)g(f^d)$. Thus, to show that a buyer who wins when $v_2 \approx v^{max}$ realizes higher utility in a conventional auction, and to justify the comparisons of the amounts exchanged in this case, we need only show that $g(f^c) > m^d/v^{max}$ (= $g(f^d)$ when $v_2 = v^{max}$). Suppose, per contra, that $g(f^c) \leq m^d/v^{max}$, so that $f^c \leq g^{-1}(m^d/v^{max})$. Then

(6)

$$1 = \psi(f^c)\frac{1}{Ev_2} \leq \psi(g^{-1}[m^d/v^{\max}])\frac{1}{Ev_2} < \psi(g^{-1}[m^d/v^{\max}])E\left[\frac{1}{v_2}\right] < E\left[\psi(g^{-1}[m^d/v_2])\frac{1}{v_2}\right] = 1,$$

where the equalities are from the first-order conditions (3) and (4) and the inequalities follow from the facts that $\psi(\cdot)$ and $g^{-1}(\cdot)$ are increasing, and from Jensen's inequality. The contradiction in (6) establishes the results for $v_2 \approx v^{max}$.

To show that if $\psi(\cdot)$ is convex, a buyer who wins when $v_2 \geq Ev_2$ realizes higher utility in a conventional auction, and to justify the comparisons of the amounts exchanged in this case, it suffices to show that $g(f^c) > m^d/Ev_2$, because if $v_2 \geq Ev_2$, $g(f^d) \leq m^d/Ev_2$. Suppose, per contra, that $g(f^c) \leq m^d/Ev_2$. Then

(7)
$$1 = E\left[\psi(g^{-1}[m^d/v_2])\frac{1}{v_2}\right] \geq E\left[\psi(g^{-1}[g(f^c)Ev_2/v_2])\frac{1}{v_2}\right]$$
$$> E[\psi(g^{-1}[g(f^c)Ev_2/v_2])]E\left|\frac{1}{v_2}\right| > \psi(f^c)\frac{1}{Ev_2} = 1,$$

where the inequalities follow from the facts that $\psi(\cdot)$ and $g^{-1}(\cdot)$ are increasing, that the random variables in brackets at the end of the first line are positively correlated, from Jensen's inequality, and from the convexity of $\psi(\cdot)$.

REMARK: Without further restrictions, Proposition 4's comparisons appear to be ambiguous for a buyer who wins when $v_2 < Ev_2$, who might prefer the outcome of a conventional auction or its dual counterpart. The welfare comparison for buyers is also ambiguous ex ante, where comparing $E[(v_1 - v_2)g(f^c)]$ and $E[(v_1 - v_2)g(f^d)]$ is further complicated by the correlation between $(v_1 - v_2)$ and f^d. The ambiguity also extends to the case where the buyers' values are privately known.

BUYERS' VALUES PRIVATELY KNOWN

In an dual English auction, buyers' uncertainty about others' values has no effect:

PROPOSITION 5: *Suppose that the buyers' values are privately known. Then, in an English auction, conventional or dual, the seller's optimal bundle and the auction outcome are the same as when the buyers' values are common knowledge among the buyers but unknown to the seller. Thus, the highest valuer wins the auction, paying money or receiving fish according to the second-highest value; a conventional English auction always yields the seller higher expected utility than a dual English auction; and Proposition 4's comparisons of the buyers' welfares and the amounts exchanged remain valid for English auctions.*

PROOF: In an English auction it is a dominant strategy for a buyer to bid his true value. The seller's optimal bundle is therefore still determined by problem (2), the outcome is the same as when buyers' values are common knowledge among the buyers but unknown to the seller, and the conclusions of Propositions 3 and 4 remain valid.

By contrast, in a Dutch auction with privately known values, buyers bear uncertainty about each other's bids, and the outcome can differ from the outcome when the buyers' values are common knowledge among the buyers but unknown to the seller. In a conventional Dutch auction, the effect of this difference is limited:

PROPOSITION 6: *Suppose that the buyers' values are privately known. Then, in a conventional Dutch auction, the highest valuer wins the auction, at a money price equal to the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest. For any given bundle, the seller's expected revenue and utility are the same as in a conventional English auction. A conventional Dutch auction therefore has the same optimal bundle as a conventional English auction and yields the seller the same expected utility, which is higher than his expected utility in a dual English auction.*

PROOF: Given seller's fish bundle, conventional Dutch auction is equivalent to a single-object auction with risk-neutral seller and buyers. The bundle is always won by the highest valuer, who pays the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest. From the point of view of the seller, the expectation of this price is $E(E[v_2 g(f^c)|v_1]) = Ev_2 g(f^c)$. Thus, for any given bundle the seller's expected revenue (and utility) are the same, an instance of the Revenue Equivalence Theorem. Conventional English and Dutch auctions therefore have the same optimal bundles and yield the seller the same expected utility. By Propositions 3 and 5, this expected utility is the same as in a conventional English or Dutch auction when the buyers' values are common knowledge among the buyers but unknown to the seller, and it is higher than the seller's expected utility in a dual English auction when the buyers' values are either common knowledge among the buyers or privately known.

In a dual Dutch auction, buyers' uncertainty about each other's bids has a significant effect on the outcome, which gives a dual Dutch auction a potential advantage over a conventional English or Dutch auction or, a fortiori, a dual English auction

A dual auction effectively converts the buyers from risk-neutral to risk-averse, and in a dual Dutch auction with privately known values, risk-averse buyers' uncertainty about other buyers' bids induces them to bid more aggressively than if they were risk neutral, raising the seller's expected utility is higher, other things equal. The insurance advantage of conventional auctions persists with privately known values, and the seller can prefer a conventional English or Dutch auction or a dual Dutch auction, depending on whether the benefits of more aggressive bidding outweigh the benefits of insurance:

PROPOSITION 7: *Suppose that the buyers' values are privately known. Then, in a dual Dutch auction, the highest valuer wins the auction; but the bidding is more aggressive than if each buyer's bid were directly determined by the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest, with the result that the winning buyer receives less fish for a given money bundle. This more aggressive bidding benefits the seller, other things equal, and can outweigh the insurance advantage of conventional auctions. The seller's expected utility is always higher in a dual Dutch auction than in a dual English auction, but it can be either higher or lower than in a conventional English or Dutch auction.*

PROOF: Given the seller's money bundle, a dual Dutch auction with privately known values is a standard single-object auction with risk-averse seller and buyers. All buyers are equally risk averse, so in symmetric equilibrium the highest valuer still wins the auction. With money bundle m, the winning (lowest) fish bid is $f(v_1) = g^{-1}(m/b(v_1))$, where $b(\cdot)$ is an increasing, continuous, and differentiable function. The bidding is strictly more aggressive than with risk-neutral buyers, so there exists a $b^{\min} > 0$ such that

$$(8) \qquad b(v_1) \geq b^{\min} + E(v_2 \mid v_1) \text{ for all } v_1.$$

The seller's optimal money bundle, m^d, is then the value of m that solves the problem:

$$(9) \qquad \max E[g(F - g^{-1}[m/b(v_1)]) + m],$$

where the expectation is taken with respect to the unconditional distribution of v_1. Because $g(\cdot)$ satisfies Inada condition, the solution of (9) is interior; and given the fact that $\psi(\cdot)$ and $g^{-1}(\cdot)$ are increasing, m^d is uniquely determined by the first-order condition

$$(10) \qquad E\left[\psi(g^{-1}[m/b(v_1)])\frac{1}{b(v_1)}\right] = 1.$$

To see that the seller's expected utility in a dual Dutch auction can be lower than in a conventional English or Dutch auction, suppose that $g(\cdot)$ is linear, so that the seller and buyers are risk-neutral. This violates our assumptions, but can be smoothed so that $g(\cdot)$ is slightly strictly concave everywhere but near 0, where it is concave enough to satisfy our Inada condition; a continuity argument will then yield the desired conclusion. When $g(\cdot)$ is linear, the seller's welfare increases with the volume of trade. We impose the constraints $f^c \le F$ and $m^d \le v^{min}F$ to ensure that $f^d = m^d/v_2 \le F$. In a conventional auction, the seller has a boundary maximum at $f^c = F$, receiving expected revenue Ev_2F for the fish bundle F, and realizing expected utility Ev_2F. In a dual Dutch auction, the seller again has a boundary maximum at $m^d = v^{min}F$. The winning buyer receives $f^d = m^d/E(v_2|v_1)$ units of fish and the seller's expected utility is

$$(11) \qquad \begin{aligned} E(F - v^{\min}F/E(v_2 \mid v_1)) + v^{\min}F &< E(F - Ev_2F/E(v_2 \mid v_1)) + Ev_2F \\ &< E(F - Ev_2F/E[E(v_2 \mid v_1)]) + Ev_2F = Ev_2F, \end{aligned}$$

where the inequalities follow from the fact that his expected utility is increasing in m^d, and from Jensen's inequality. Thus, with risk-neutrality, the seller strictly prefers a conventional English or Dutch auction to a dual Dutch auction.

To see that the seller's expected utility in a dual Dutch auction can be higher than in a conventional auction, suppose that $g(\cdot)$ is strictly concave, and that the support of the distribution $H(\cdot)$, $[v^{min}, v^{max}]$, satisfies $v^{min} = v^{max} - 1$. Imagine that the distribution $H(\cdot)$ shifts rightwards, preserving its shape, with $v^{min}, v^{max} \to \infty$. It is clear from Proposition 6 and (3) that in a conventional Dutch auction, $f^c \to F$, so that the seller's equilibrium expected utility approaches $g(0) + Ev_2\, g(F)$ in the limit. In a dual Dutch auction, as v^{min}, $v^{max} \to \infty$, it is clear that $b(v_1) \to \infty$ for all v_1. It then follows from (10) that $\psi(g^{-1}[m^d/b(v_1)]) \to \infty$ for all v_1, because, by the monotonicity of $\psi(\cdot)$, $g(\cdot)$, and $b(\cdot)$,

$$(12) \qquad \psi(g^{-1}[m/b(v^{\max})])\frac{1}{b(v^{\max})} < \psi(g^{-1}[m/b(v_1)])\frac{1}{b(v_1)} < \psi(g^{-1}[m/b(v^{\min})])\frac{1}{b(v^{\min})},$$

and $\psi(g^{-1}[m/b(v^{\max})]) \to \infty$ if and only if $\psi(g^{-1}[m/b(v^{\min})]) \to \infty$. This implies that $f^d = g^{-1}(m^d/b(v_1)) \to F$ for all v_1, so $m^d \to Eb(v_1)g(F)$. Because the winning buyer's bid maximizes the probability of winning times his utility when he wins, $v_1g(f) - m$, as $m^d \to Eb(v_1)g(F)$, $b(\cdot) - v^{max}$, $b^{min} - v^{max}$, and $E(v_2|v_1) - v^{max}$ all converge to limits for which (8) holds. Taking expectations in (8), the seller's equilibrium expected utility in a dual Dutch auction approaches $g(0) + Eb(v_1)g(F) \ge g(0) + [b^{min} + Ev_2]\, g(F)$ in the limit, which is strictly greater than the seller's equilibrium expected utility in a conventional auction.

REMARKS: It may seem puzzling that in the last part of the proof, the support of the distribution of $f^{d} = g^{-1}(m^{d}/b(v_1))$ collapses on F, while the winning fish bid has a nonnegligible effect on the seller's welfare. The reason is that in the limit, the buyers compete by tiny variations in their bid amounts of extremely valuable fish, which despite their size have nonnegligible effects on the seller's money revenue and expected utility.

Finally, our limiting argument in the second part of the proof is just a device to show that it is possible for the seller to prefer a dual Dutch auction; there is no reason to suppose that for low values, the seller must prefer a conventional auction. For example, when $H(\cdot)$ is uniform, with $v^{min} = v^{max} - 1$; $F = 100$, and $g(f) \equiv f^{1/k}$, numerical solutions using Mathcad yield expected utilities for the seller in a conventional English or Dutch auction, a dual English auction, and a dual Dutch auction, respectively, as follows:

v^{min} \ k	2	3	4
2	25.39, 24.65, 24.92	12.77, 12.34, 12.51	9.10, 8.75, 8.86
3	33.33, 33.96, 34.19	17.12, 16.68, 16.84	12.10, 11.74, 11.85
4	43.33, 43.50, 43.71	21.57, 21.12, 21.26	15.13, 14.78, 14.89
5	53.33, 53.18, 53.37	26.08, 25.60, 25.74	18.21, 17.85, 17.95
6	63.33, 62.93, 63.10	29.40, 30.12, 30.25	20.03, 20.93, 21.03
7	73.33, 72.73, 72.90	34.04, 34.66, 34.79	23.19, 24.03, 24.12

Thus, the seller can prefer a dual Dutch auction to a conventional English or Dutch auction (or, a fortiori, a dual English auction) for quite moderate values of v^{min}, although it is apparent from the table that the relationship is complex and nonmonotonic.