Economics 172A: Introduction to Operations Research **Winter 2008**
Supplementary handout on problem formulation

A UCSD degree … and your own MS-burger (pronounced "Messburger") franchise! You have three "profit centers":

● the MS-Burger ("a quarter of a quarter of a pound of USDA choice beef on a fresh-baked bun")

● the Beefburger ("for the total carnivore, a USDA choice beef pattie on a 'bun' also made entirely of beef"), and

● the Breadburger ("a fresh-baked bun in … what else? ... another fresh-baked bun")

Producing x_1 MS-burgers, x_2 Beefburgers, and x_3 Breadburgers yields total profit (taking costs into account) of $60x_1 + 50x_2 + 10x_3$.

MS-burger CEO Joel Watson sends you $b_1 > 0$ quarter-of-a-quarter-of-a-pound units of beef and $b_2 > 0$ buns every month; it takes one unit of beef and one bun to make an MS-burger, two units of beef (and no buns) to make a Beefburger, and two buns (and no beef) to make a Breadburger.

Formulate the linear programming problem that determines the profit-maximizing use of your monthly supply of beef and buns, assuming that the x_i must be nonnegative, but ignoring integer restrictions. Do not assume that all the beef or all the buns must be used.

Choose x_1, x_2, x_3 to solve Max $60x_1 + 50x_2 + 10x_3$ subject to $x_1 + 2x_2 + 0x_3 \le b_1$ (p_1)
$x_1 + 0x_2 + 2x_3 \le b_2$ (p_2)
The dual prices of the primal constraints (the dual control variables) are in parentheses.

Formulate the dual, under the same assumptions.

Let p_1 be the dual price of beef and p_2 be the dual price of buns. Then the dual is

Choose p_1, p_2 to solve Min $b_1p_1 + b_2p_2$ subject to

$$\begin{aligned} p_1 + p_2 &\ge 60 \ (x_1) \\ 2p_1 + 0p_2 &\ge 50 \ (x_2) \\ 0p_1 + 2p_2 &\ge 10 \ (x_3) \end{aligned}$$

The dual prices of the dual constraints (the primal control variables) are in parentheses.

The dual has the interpretation of choosing prices to minimize the value of the resources (beef and buns) subject to the constraint that the firm cannot make a profit at those prices. (Similar but not identical to the story we told for the dual of the diet problem; the difference is because the diet problem's primal was a minimization problem.)

If we chose numerical values for b_1 and b_2, we could solve the dual graphically, because it (unlike the primal) has only two control variables.