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Search prices are i.i.d. draws from distribution $F(p)$, supported on $[0, \bar{p}]$. $\underline{p}(n) \equiv \min\{p_1, \dots, p_n\}$.

$$E\underline{p}(n) = \int_0^{\bar{p}} [1-F(p)]^n dp$$ because $1-[1-F(p)]^n$ is the c.d.f. of $\underline{p}(n)$. ($Pr\{\underline{p}(n) \leq p\} \equiv 1 - Pr\{\underline{p}(n) > p\}$ $= 1 - Pr\{\text{all } n\ p_i > p\} = 1 - [1-F(p)]^n$.)

Stigler assumed that n had to be committed to in advance, so that a risk-neutral searcher solves

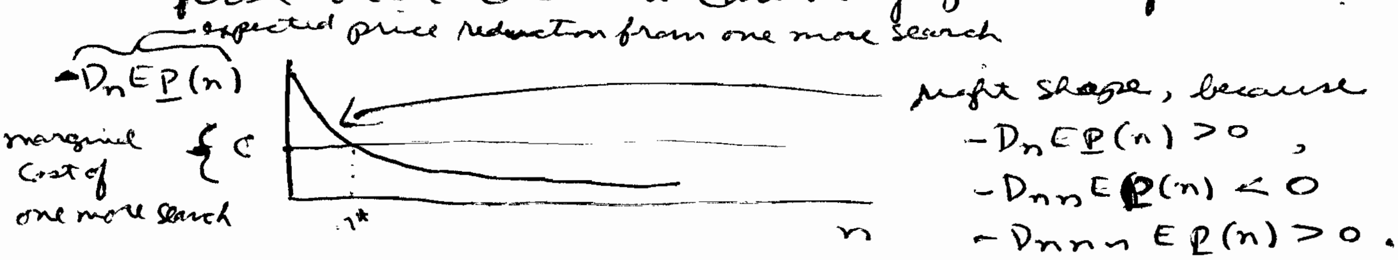
$$\min_n \; E\underline{p}(n) + cn$$, where c is the unit cost of search.

$$D_n E\underline{p}(n) = \int_0^{\bar{p}} [1-F(p)]^n \ln[1-F(p)]dp < 0 \quad (\text{since } D_n x^n \equiv x^n \ln x),$$

$D_{nn}E\underline{p}(n)$ is the same but w/ $\ln^2[1-F(p)]$, and $D_{nnn}E\underline{p}(n)$ is the same but w/ $\ln^3[1-F(p)]$. Thus $D_{nn}E\underline{p}(n) > 0$ and $D_{nnn}E\underline{p}(n) < 0$. Imagine that n is continuously variable. Then Stigler's problem has the first-order condition

$$D_n E\underline{p}(n) + c = 0$$

and the second-order condition $D_{nn}E\underline{p}(n) > 0$ (Yes, >0!)
The second-order condition is always satisfied, and the first-order condition can be graphed as follows:



It can be shown that for a one-dimensional optimization problem where the second-order condition is always satisfied, as here, the optimum respecting the discreteness of n is always one of the values adjacent to the optimum ignoring the discreteness; not characterized. The discrete optimum satisfies a discrete first-order condition (and locally, second-order condition) $E\underline{p}(n-1) - E\underline{p}(n) \geq c \geq E\underline{p}(n) - E\underline{p}(n+1)$.
It can be shown that this has a generically unique solution, and at most two solutions when the solution is not unique. Clearly this implies downward-sloping demand.

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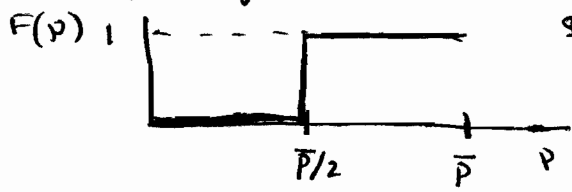
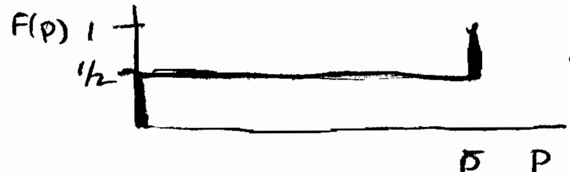
Search (continued) Clearly $\frac{dn^*}{dc} < 0$, as one would expect. Risk-neutral people prefer to search from riskier distributions, holding the mean constant. Intuitively, since you get to take the best price, and the maximum of linear functions is a convex function, you prefer more risk. "Formally", a mean-preserving spread in $F(\cdot)$ leaves $\int_0^{\bar{P}} [1-F(p)]\,dp$ unchanged, by definition, but it lowers $E\,\underline{P}(n) = \int_0^{\bar{P}} [1-F(p)]^n\,dp$ because it concentrates $F(p)$ as a function of p, and x^n is a convex function.

E.g., if p is concentrated at $\bar{P}/2$, you have



so that $F(p)$ is concentrated half at 0 and half at 1, maximally dispersed.

If, on the other hand, p is half at 0 and half at $\bar{P}$, you have



so that $F(p)$ is concentrated at 1/2, minimally dispersed.

In the first case $\int_0^{\bar{P}} [1-F(p)]^n\,dp = \int_0^{\bar{P}/2} 1^n\,dp + \int_{\bar{P}/2}^{\bar{P}} 0^n\,dp = \bar{P}/2$,

and in the second $\int_0^{\bar{P}} [1-F(p)]^n\,dp = \int_0^{\bar{P}} \left(\frac{1}{2}\right)^n dp = \bar{P}/2^n << \bar{P}/2, \forall n>1.$

In general, I think you can show that interchanging the roles of p and $F(p)$ (in effect, reflecting these graphs in the 45° line) reverses single-crossing changes and the integral condition.

Note that it's essential that p, not $F(p)$, is the variable of integration here. $F(p)$, viewed as a random variable defined as a function of the random variable p, is always (yes, always!) uniformly distributed. $(\Pr\{F(p) \le f\} \equiv \Pr\{p \le F^{-1}(f)\} \equiv F(F^{-1}(f))$. Note that $F^{-1}(f)$ is always well defined, even though in general $F(\cdot)$ is only weakly increasing.)

However, even though an increase in dispersion makes people better off, following Stigler's Rule they search less intensively (lower n's), not more. A mean-preserving spread lowers the expected gain from one more search,

$$-D_n E\underline{P}(n) = -\int_0^{\bar{P}} [1-F(p)]^n \ln[1-F(p)]\,dp$$

roughly, because it lowers $\int_0^{\bar{P}} [1-F(p)]^n\,dp$ and $\ln[1-F(p)] \leq 0$. (This is rough because the weights, $\ln[1-F(p)]$, are not uniform; can show it formally, I believe, by a tedious integration by parts argument using the differential version of the integral condition.)

<u>Search when not committed to n</u> (called "optimal" search in the literature, but which kind of rule is truly optimal depends on whether it's feasible (or at least desirable) to search sequentially; cf. Morgan and Manning reading. Sequential search would be silly in the academic labor market, and one can imagine consumer search environments where Stigler's assumption that n is committed to at the start is more realistic – e.g. searching additional stores is approximately free once you're at the mall, but going to the mall is costly, so your choice is essentially whether to go to a big (high-n) mall, a small (low-n) mall, or stay in your neighborhood (really low-n).)

Assume recall, and let s be the lowest price observed so far. Then the expected gain from one more search is

$$g(s) = \int_0^s \underbrace{(s-p)}_{\text{"u"}} \underbrace{f(p)dp}_{\text{"dv"}} = \Big[\underbrace{(s-p)}_{\text{"u"}}\underbrace{F(p)}_{\text{"v"}}\Big]_0^s + \int_0^s F(p)dp = \int_0^s F(p)dp.$$

The optimal policy is to set a reservation price R^* so that $c = \int_0^{R^*} F(p)dp$ and search till you found a price $p \leq R^*$. Proof: If $s > R^*$, there's a gain from searching at least once more. If $s < R^*$, there's a loss from one additional search, and you do not learn anything useful in the future and costs do not decrease. (Note $g'(s) = F(s) > 0$.)

thus if the immediate net gains are negative all future net gains will be too (under the maintained assumptions). Note how this solution makes search dependent on history.

Can show that not having recall doesn't change optimal policy here. ~~With~~ Can show that increasing costs are okay, with a "floating" reservation price $R^*(n)$!

It's harder if costs are decreasing or if $F(\cdot)$ is unknown. See problems, and Rothschild, "Searching for the lowest price ...".

Clearly, $\dfrac{dR^*}{dc} = \dfrac{1}{F(R^*)} > 0$, so people search less intensively, in the probabilistic sense appropriate here, when search is more costly. The number of searches, ~~is~~ n, is now a random variable, and when R^* goes up, n stochastically decreases in the sense of first-order stochastic dominance, because for any history of price observations, higher R^* makes you stop (weakly) sooner!

Clearly get downward-sloping demands as before. Now you search more intensively, in the sense of a lower R^*, in riskier distributions: as a mean-preserving spread in $F(\cdot)$ raises $\int_0^s F(p)dp$ for all s, so R^* must go down. However, whether the new distribution of n first-order stochastically dominates the old, or vice versa (must be one or the other, because Pr {accepting any given price} is constant over searches, so number of searches is geometrically distributed, w/ density $p(1-p)^n$, and these are FOSD-ordered by p) depends on whether new F(new R^*) $\lesseqgtr$ old F(old R^*), and this can go either way, with a presumption for <, since $F(p)$ stays the same on average, while R^* goes down.