—or something similar. This indicates that the formal structure of this part of the theory—the relationship between statics and dynamics—may be generically different from that of the classical physical theories.

All these considerations illustrate once more what a complexity of theoretical forms must be expected in social theory. Our static analysis alone necessitated the creation of a conceptual and formal mechanism which is very different from anything used, for instance, in mathematical physics. Thus the conventional view of a solution as a uniquely defined number or aggregate of numbers was seen to be too narrow for our purposes, in spite of its success in other fields. The emphasis on mathematical methods seems to be shifted more towards combinatorics and set theory—and away from the algorithm of differential equations which dominate mathematical physics.

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Scaling of utilities and probability

JACOB MARSCHAK

The subject of the measurement of utility and preferences among uncertain events has given rise to a large, important, and highly technical literature in the past fifteen years. In this article Jacob Marschak presents, in as nontechnical a manner as is possible, the basic aspects of the measurement of utility. Although a reader with no mathematical training will have to read with care, this article is of considerable importance to those wishing to understand most of the recent work on decision making under uncertainty. The problems of utility measurement are not dealt with elsewhere in this volume; and in the literature, in general, it is difficult to find a basic exposition written with the degree of clarity of this one.

This is a revision in part from Marschak's paper "Mathematical Models and Experiments on Decision-making" (mimeographed), presented in February 1954 at a meeting sponsored by the Epidemiological Board, U. S. Army, and Dunlap and Associates, Inc., and devoted to mathematical models and human behavior. Other parts give a reformulation of Marschak's "Probability in the Social Sciences," in *Mathematical Thinking in the Social Sciences*, edited by Paul Lazarsfeld, Glencoe, Ill., The Free Press, 1954. Research undertaken by the Cowles Commission for Research in Economics under Contract Nonr-358 (OI), NR 047-006 with the Office of Naval Research. Produced here with the permission of the author.

One strategy is better than another if it brings the player more. More of what? Clearly, one uses some criterion of "utility," of something that is being maximized. Moreover, since the opponent's response or the future in general is uncertain, the player does not know a unique result of his strategy but rather expects various possible results, some with greater, some with smaller degrees of belief or "(subjective) probabilities."

If utilities and probabilities could be merely ranked (like degrees of patriotism) but not also scaled (like degrees of temperature), the theory of games could say rather little. Accordingly, von Neumann and Morgenstern revived the eighteenth century concept of a player who is concerned with his subjective status (so called "moral wealth"—the modern "utility"), a number which, under uncertainty, is a random one, and whose average (the "moral expectation"—the modern "expected Utility") the player tries to maximize. A lively discussion developed.[1] The most elegant restatement of the relevant postulates of rational behavior and a rigorous proof of the implied "Bernoulli theorem" was given by Herstein and Milnor.[2] In this discussion, the necessarily subjective nature of the probabilities was somewhat neglected. But this latter aspect (actually also traceable back to the eighteenth century) was revived by Ramsey and De Finetti and in L. J. Savage's *Foundations of Statistics*.[3]

The following exposition tries to appeal to the reader's intuition and cater to his laziness more than it would be permissible in a mathematical paper. A proof is merely sketched. It may be added, however, that the substance of the proof, too, is simpler than in my earlier papers on the subject.[4]

I shall often use the second person pronoun. In discussing rational behavior as in discussing logic, appeal is made to the reader's reasonable self: given time to think, how would you decide? I don't deal with another, quite a different question (as different as psychology is from logic): how often do you decide hurriedly and foolishly?

The Case of Three Outcomes

Suppose you, the reader, have to choose between two decisions (actions): to build or not to build a bomb-proof shelter for yourself and your family. To make the example drastic, let us suppose that you are rich but that the shelter would cost you practically the whole of your fortune; and that such a shelter is both indispensable and sufficient for life preservation in the case of war. Thus your two alternative decisions and the two alternative states of the world (war or peace) combine into the following "payoff matrix":

	States of World	
Decisions	Peace	War
Build shelter	Alive and poor (r_2)	Alive and poor (r_2)
Don't build shelter	Alive and rich (r_3)	Dead (r_1)

This is oversimplified indeed. Especially, we have neglected the difference between being alive and poor in peace, and being alive and poor in war (with shelterless neighbors dead). I need this particular simplification in order to have in my example just three possible outcomes. This will be a convenient introduction to more complicated cases. The three outcomes, or results, I have denoted by r_1 = "dead"; r_2 = "alive and poor"; r_3 = "alive and rich" I suppose you prefer r_3 to r_2 and r_2 to r_1. This ranking of results is sufficient to determine your choice of action if you are certain about the future. If you are certain that there will be war, you choose to build the shelter (because you prefer r_2 to r_1); if you are sure that there will be peace, you choose not to build the shelter (because r_3 is better than r_2).

[1] For example, at the colloquium on risk in econometrics, held in 1952; its Proceedings published by the Centre National de la Recherche Scientifique, Paris, 1953.

[2] I. N. Herstein and John Milnor, "An Axiomatic Approach to Measurable Utility," *Econometrica*, 1953.

[3] John Wiley and Sons, 1954.

[4] *Econometrica*, 1950; and Second Berkeley Symposium, ed. by J. Neyman, 1951.

But suppose you do not know whether there will be war or peace. Yet you have to take a decision! You will still choose not to build a shelter if your *degree of belief* in peace and your *degree of preference* for wealth relative to survival are sufficiently large. (Note that we now speak of degrees and not of mere ranks! Summer is warmer than spring, and spring is warmer than winter: this is ranking; but we can also measure degrees of temperature in each of the three seasons.)

The statement I have just made, about the degrees of your beliefs and preferences that will make you decide not to build a shelter though you are uncertain about the future, must be made more precise. Denote the degrees of belief (also called subjective probabilities) that you assign to peace and war, respectively, by p and $1 - p$. Calibrate your thermometer of preferences, or "utilities" as follows: assign utility zero to the worst of the three outcomes, and utility one to the best. Thus

$$u(r_1) = 0; \quad u(r_3) = 1$$

Then, since r_2 is better than r_1 and worse than r_3,

$$0 < u(r_2) < 1$$

Subject to these inequalities, you are still free to assign any number $u(r_2)$ to the outcome r_2. I now suggest that you can, without incurring any logical contradiction, determine the number $u(r_2)$ in such a way as to satisfy the following principle: "*of any two decisions, the one you* (*the reader*) *choose is the one that, in your opinion, results in the higher, or at least not lower, average utility.*" The average utility that results from the decision "don't build" is defined as the weighted average of $u(r_3)$ and $u(r_1)$, the weights being the corresponding degrees of belief, which we have denoted above by p and $1 - p$, respectively. Thus the average utility that results from "not building shelter" is

$$(1) \quad u(r_3) \cdot p + u(r_1) \cdot (1 - p) = 1 \cdot p + 0 \cdot (1 - p) = p$$

On the other hand, the average utility that results from the decision "to build" is

$$u(r_2) \cdot p + u(r_2) \cdot (1 - p) = u(r_2)$$

Now, suppose you have chosen to build. According to the above principle of choosing in favor of the decision that results in the higher average utility, your actual choice will imply that, in your opinion [and still using the scale where one had fixed $u(r_3) = 1$ and $u(r_1) = 0$],

$$u(r_2) \geq p$$

On the other hand, if you had chosen not to build, the above principle would imply [still using the same scale]

$$u(r_2) \leq p$$

You can now perform on yourself a mental experiment, by varying the number p and asking: "with this degree of belief into persistence of peace, do I still prefer to build (or not to build)?" You may come close enough to a value of p, at which you will be *indifferent* between building and not building. Call this hypothetical value: $p = p_0$. Then both statements are true: $u(r_2) \geq p_0$ and $u(r_2) \leq p_0$. That is

$$u(r_2) = p_0$$

You have now attached numerical utilities to all three outcomes considered. You have also attached numerical average utilities to the two alternative decisions, to build and not to build. Call these two decisions (actions) a_1 and a_2, respectively. Thus

$$u(r_1) = 0; \quad u(r_2) = p_0; \quad u(r_3) = 1;$$

$$\text{average utility if } a_1 \text{ is taken} = p_0$$

$$\text{average utility if } a_2 \text{ is taken} = p$$

Remember that p is your actual degree of belief in peace; while p_0 is that degree of belief in peace that would make you indifferent between the two decisions. If you have decided to build, this implies that, in your opinion, $p \geq p_0$. If you have decided not to build, this implies that, in your opinion, $p \leq p_0$.

We have had occasion to discuss your preferences between results (for example, better to be alive and poor than to be dead: choice between r_2 and r_1) as well as between decisions (better to build than not to build: choice between a_1 and a_2). It is thus natural to extend the concept of utility accordingly, and speak, not only of a utility of a result, but also of a utility of a decision. In fact a "result" can be regarded as a special case of a "decision," viz., a decision which can have only one result. Such was in fact the decision a_1 in our example; its unique result was r_2. Accordingly we might simply assign to it the utility $u(a_1) = u(r_2)$. On the other hand, decision a_2 is identified with a bundle of results (r_3 and r_1), each with a respective degree of belief (p and $1 - p$). We can call such a bundle a "prospect": it is a (subjective) probability distribution of results, a lottery. A prospect promising—as in the case of a_1—a single result, may be called a sure prospect. It is, in fact, *also* a (subjective) probability distribution, with one of the results having probability one. Thus, when you compared the two decisions, you did, in fact, compare the following two probability distributions (or prospects).

Decision:	Probability of		
	r_1	r_2	r_3
a_1	0	1	0
a_2	$1 - p$	0	p

One can say that, of two decisions (or of two corresponding prospects), you have chosen the one with a higher, or at least not lower, utility. If you choose a_1, then $u(a_1) \geq u(a_2)$. The principle of choosing the decision that results in a higher average utility can, then, be rephrased as follows: "the utility of a decision is the higher, the higher the average utility resulting from it." It will therefore not contradict this principle if you simply equate $u(a_1)$ and $u(a_2)$ with the corresponding average utilities. That is

$$u(a_1) = p_0; \quad u(a_2) = p$$

You have thus scaled the utilities of prospects, sure as well as non-sure ones:

$$u(r_1) = 0; \quad u(r_2) = u(a_1) = p_0; \quad u(a_2) = p; \quad u(r_3) = 1$$

It will prove more convenient to use a notation that unifies the sure and non-sure prospects, by listing the alternative results and their corresponding probabilities. In this notation, the equations just given become

$$u(r_1, r_2, r_3; 1, 0, 0) = 0;$$
$$u(r_1, r_2, r_3; 0, 1, 0) = u(r_1, r_2, r_3; 1 - p_0, 0, p_0) = p_0$$
$$u(r_1, r_2, r_3; 1 - p, 0, p) = p;$$
$$u(r_1, r_2, r_3; 0, 0, 1) = 1$$

It is easily checked that each of these equations satisfies the principle "utility of a prospect equals its average utility," i.e., the average of the utilities of the component outcomes (sure prospects) promised by the prospect in question.

Note that the zero and the unit of the scale of utilities were chosen arbitrarily, by assigning utility 0 to the worst, and utility 1 to the best of the results considered. To this extent, the scale is arbitrary, just as is a temperature scale. If, for example, we had to put $u(r_1) = -10$ and $u(r_3) = +100$, the principle of choosing the decision with the higher average utility would yield [compare equations (1)]:

$$u(r_3) \cdot p + u(r_1) \cdot (1 - p) = 100 \cdot p - 10 \cdot (1 - p) = 110p - 10$$

as the average utility for the decision a_2; the latter we have also denoted by $(r_1, r_2, r_3; 1 - p, 0, p)$. Thus if, as before, p_0 denotes the probability of peace that would make you indifferent between the two actions, our new scale will be (using both of the suggested notations):

$$u(r_1) = u(r_1, r_2, r_3; 1, 0, 0) = -10;$$
$$u(r_2) = u(a_1) = u(r_1, r_2, r_3; 0, 1, 0) = u(r_1, r_2, r_3; 1 - p_0, 0, p_0) = 110p_0 - 10;$$
$$u(a_2) = u(r_1, r_2, r_3; 1 - p, 0, p) = 110p - 10;$$
$$u(r_3) = u(r_1, r_2, r_3; 0, 0, 1) = 100$$

We see that, to convert from the old to the new utility scale, one has to perform a "linear transformation": that is, to multiply by a constant (110) and to add a constant (−10); just as we do in converting from Fahrenheit degrees to centigrades or from altitudes measured in feet over Lake Michigan level to altitudes in meters over sea level. In this sense, the utility scale that interprets your choices among decisions is "determinate up to an arbitrary linear transformation." (Even measurements of geometrical distances or of physical weights are determinate up to an arbitrary linear transformation, although in those cases only the multiplying constant is arbitrary: the "unit of measurement.")

So far, we have treated a case in which only three alternative outcomes (r_1, r_2, r_3) are possible. What have we shown? We have shown that your decisions are consistent with a scale of utilities that satisfies the following principle: *choose the decision that results, on the average, in the highest utility.* After fixing arbitrarily the utilities of two of the outcomes (assigning a higher utility to the better one), the application of this principle results in a definite utility number for the third outcome, and also in definite utility numbers for all prospects that promise the three outcomes with pre-assigned probabilities.

The General Case: "Bernoulli Norm"

Let us now relax the strait jacket of "three outcomes only." Let there be any (finite) number of outcomes, also called "sure prospects." Let us generalize even further, by considering prospects which promise, with preassigned probabilities, not only certain outcomes, but possibly also prospects. The case where a prospect is a lottery promising certain outcomes rather than a lottery promising certain lotteries is clearly a special one, with certain probabilities shrinking to zeros.

So far, the letters a_1, a_2 stood for decision or action, and each corresponded to a certain prospect. We shall continue to denote prospects by $a_1, a_2, \ldots$, but it will be convenient also to use the subsequent letters of the alphabet ($b, c, \ldots$) and also x. For probabilities, we shall use, as before, p; but q and π will also be needed.

I want to convince the reader that it will be possible for him to set up a scale of his utilities of all prospects, sure and otherwise, this scale satisfying the following property: if a prospect x is a lottery promising the prospects $a_1, a_2, \ldots, a_n$ with subjective probabilities $\pi_1, \ldots, \pi_n$, then

$$u(x) = \sum_{1}^{n} u(a_1)\pi_1 \qquad (2)$$

That is, again: the utility of a prospect should equal the average of promised utilities.

If it is possible to interpret your behavior as satisfying (2), we shall say that you obey the "Bernoulli Norm": see "Historical Note," below.

The technical term for "average of values of a variable, weighted by probabilities of their occurrences" is "mathematical expectation" of that variable, or, more briefly its "expected value." The expression on the right-hand side of (2) is, then, the "expected value of utility of the prospect x," or, still more briefly, "expected utility of x." What we try to show is: there exists a numerical scale of utility for all prospects, with the following property: the utility of a prospect equals its expected utility. Since you will decide in favor of a prospect with higher utility in preference to one with lower utility, you will choose a decision that maximizes the utility of a prospect; and, by the principle just mentioned (and that we are going to prove), this implies that you maximize the expected utility of a prospect.

Let a and b be two prospects (either sure or uncertain) facing the reader, and let him regard b as better than a. Consider the following classes of prospects:

1. a and b
2. all prospects that promise a or b
3. all prospects that are not better than b and not worse than a [Obviously (1) and possibly (2) are included in (3).]
4. all prospects that are better than b
5. all prospects that are worse than a.

The utilities in class (1) will be assigned arbitrarily, except for the condition that a is worse than b. We put $u(a) = 0$, $u(b) = 1$. [If classes (4) and (5) were empty, a might stand for "agony" and b for "bliss"!]

The utilities in class (2) will all lie between (and excluding) 0 and 1, for the following reason: Let a lottery c promise b if a possible event s happens, and a if it does not happen. Therefore, if you acquire c and s happens, you get something better than a; while if s does not happen you get a. Hence c is better than a. Similarly, c is worse than b. For, if you acquire c and s happens you get b; but if s does not happen you have something worse than b. Hence $u(a) < u(c) < u(b)$ and therefore $0 < u(c) < 1$, i.e., $u(c)$ is some proper fraction.

Moreover compare two lotteries in class (2): c_1 and c_2, where c_1 promises b if the event s_1 happens. Let p_1 be the probability of s_1 and suppose $p_1 < p_2$. Then $u(c_1) < u(c_2)$, for the following reason. The lottery c_2 promises b with larger probability than does lottery c_1; but with smaller probability than does a direct offer of b. Therefore c_2 can be conceived of as a lottery that promises c_1 and b with certain probabilities.[5] Hence, by a reasoning similar to the one made before, c_2 must be better than c_1 and worse than b.

It follows that you will rank the utilities of the various prospects in class (2) by assigning increasing proper fractions as the probability p of getting b increases.

It is permissible therefore to choose as the utility number for a lottery the fraction p if the lottery promises b with probability p. We have thus scaled all prospects of class (2), and this scale fits with the boundary values $u(a) = 0$ and $u(b) = 1$, since a and b can themselves be called lotteries, with $p = 0$ and $= 1$, respectively.

Before proceeding to the remaining classes of prospects, let us satisfy ourselves that our scale, so far, has the desired property. Let a prospect x be a lottery promising prospects [belonging to classes (1) and (2)] $c_1, \ldots, c_n$ with probabilities $\pi_1, \ldots, \pi_n$. Show that

$$(3) \qquad u(x) = \sum_1^n \pi_1 u(c_1)$$

To prove (3), we replace $u(x)$ by the probability with which the lottery x promises b. This probability is compounded from $p_1, \ldots, p_n$, where p_1 is the probability with which c_1 promises b. Hence $u(x) = \sum_1^n \pi_1 p_1$. But we have seen that $u(c_1) = p_1$. Hence (3) is true.

Consider now the class (3) of prospects. It includes class (1); and we have seen that it also includes class (2); but it includes more. We have covered those of the members of (3) that are a or b or lotteries promising a or b; and we were able to assign to each of these lotteries a utility p equal to the probability with which that lottery promises b. Of course, p ranges from 0 (the utility of a itself) to 1 (the utility of b itself). Now the utility of any member of class (3) must lie between 0 and 1 since it consists of prospects that are not better than b and not worse than a. Hence, any member of class (3)—say, d_1—that is not a lottery promising a or b has a utility that is equal to that of one of those lotteries—call it c_1—i.e., to some p, $0 \leq p \leq 1$. If we now form a lottery y that promises various members $d_1, \ldots, d_n$ of class (3) with probabilities $\pi_1, \ldots, \pi_n$, then y has the same utility as the lottery x considered in equation (3). For, the event s_1 (with probability π_1) will give the subject the prospect d_1 if he had chosen y; and the prospect c_1 if he had chosen x. And so for $s_2, \ldots, s_n$. But since we have seen each d_1 to have the same utility as the corresponding c_1 (this utility being equal to the probability with which c_1 promises b), it follows that the subject is indifferent between y and x. Thus (3) is extended to all

[5] Let these probabilities be q and $1 - q$, respectively, q is easy to find (though it is not really necessary for our purpose) by posing

$$q \cdot p_1 + (1 - q) \cdot 1 = p_2$$

since p_2, the chance of obtaining b, must be equal to the chance (in the new lottery) of getting it by virtue of having gotten c_1, *plus* the chance of getting it directly. We have $q = \dfrac{1 - p_2}{1 - p_1}$.

members of the class (3), since $u(y)$ can replace $u(x)$ and each $u(d_1)$ can replace $u(c_1)$.

This would complete the proof if we could assume that there exists for each subject a worst and a best prospect, "agony" and "bliss" (a and b). If this is not the case, i.e., utility is not bounded and classes (4) and (5) are not empty, equation (3) still holds good. To show this, it suffices to pick the worst and the best prospect—say, a' and b'—among those composing the particular lottery x [i.e., a' and b' will be among the $c_1, \ldots, c_n$ in (3)], and use a' and b' in the same way in which a and b were treated previously; i.e., create a new scale of utilities—say, u'—with $u'(a') = 0$, $u'(b') = 1$, $u'(a) = p < q = u'(b)$, where p and q are certain probabilities; a and b having, respectively, the same utilities as certain two lotteries, each promising a' or b'. We see by former reasoning, that in terms of this new scale, (3) will be valid, that is

$$u'(x) = \Sigma\pi_1 u'(c_1) \quad (4)$$

But this latter equation remains valid also if the function u' is replaced by any linear transform of it, $\alpha u' + \beta$ (this can be easily verified by substitution). Now, the functions u' and u are, in fact, linear transforms of each other, with (for any c) $u'(c) = (q - p)u(c) + p$ [i.e., the scale u' is obtained from the scale u by shifting the origin by p and multiplying the utility unit by $q - p$]. Hence the validity of (4) entails the validity of (3) for any prospects,[6] i.e., the validity of (2).

[6] With $u(a) = 0$ and $u(b) = 1$, the utility $u(e)$ of a member of class (4) and the utility $u(f)$ of a member of class (5) are easily shown to be

$$u(e) = \frac{1}{q} > 1; \quad u(f) = 1 - \frac{1}{r} < 0$$

where q and r are probabilities defined as follows: the subject is indifferent between b and a lottery promising e or with respective probabilities q and $1 - q$; and he is indifferent between a and a lottery promising f or b with respective probabilities r and $1 - r$. It is easily verified that with these definitions,

$$u(e) \cdot q + u(a) \cdot (1 - q) = u(b) \text{ and}$$
$$u(f) \cdot r - u(b) \cdot (1 - r) = u(a)$$

Historical Note

Suppose that you apply the utility scale just described to various amounts of monetary wealth, and discover that, for you, the utility can be regarded as proportional to money amount. Should your tastes happen to be of this kind then (since you have agreed that your choices are as described by the Bernoulli Norm and you therefore maximize the mathematical expectation of utility) you are a maximizer of the mathematical expectation of monetary wealth. And remember that this mathematical expectation was computed on the basis of your subjective probabilities.

Now, the idea that a consistent decision-maker chooses a bet that gives him the maximum expected monetary wealth, computed on the basis of his subjective probabilities, can be traced back to Thomas Bayes (eighteenth century). Daniel Bernoulli who lived in that same century was not so clear about the subjective nature of the probabilities that can be said to underline human choice. He assumed people to know the true odds in a game, and neglected the case when a man has no sufficient theory or no large sample to compute the odds with precision. But, on the other hand, Bernoulli was emphatic (as Bayes was not: remember that Bayes dealt with money instead of utilities) about the subjective nature of preferences. Unless your tastes are of a peculiar character, utilities are not proportional to money amounts (and moreover they can and should be attached to many other objects of choice besides money!), and one must be careful to state that the consistent decision-maker maximizes his expected utility, not his expected monetary wealth. This is what we called the Bernoulli Norm. In stating it, we did describe the probabilities used as subjective and called them sometimes degrees of belief. But we did not say how they too (like the equally subjective utility numbers) can be derived as characterizing a consistent decision-maker's behavior. This was done by the late Frank Ramsey in 1926, and more recently by De Finetti and by L. J. Savage. Their approach will be outlined in our concluding section.

I shall now try to convince you that your decisions, if consistent, can be interpreted in the following manner: there exist utility numbers, attached to outcomes of your actions, and there exist degrees of belief, attached to the future states of the world, with the following property: if for each of your possible actions the expected utility (i.e., the mathematical expectation of utility of outcomes) were computed on the basis of your degrees of belief, then the action chosen by you would be the one with the highest expected utility. We may call this the Ramsey Norm.

Imagine that the following eight actions $a_1, \ldots, a_8$ will have one of the two outcomes, Death (D) or Life (L), depending on whether the world will be in the state s_1 or s_2 or s_3 (mutually exclusive), as shown in the following table which, as you can convince yourself, exhausts all possible triplets of L- and D-symbols:

Actions	States of World s_1	s_2	s_3	Group
a_1	D	D	D	1
a_2	L	D	D	2
a_3	D	L	D	2
a_4	D	D	L	2
a_5	D	L	L	3
a_6	L	D	L	3
a_7	L	L	D	3
a_8	L	L	L	4

I assume that you prefer Life to Death. Then you will prefer a_8 to a_7 because a_8 has a better outcome that a_7 in state s_3, and the same outcome as a_7 otherwise. By this reasoning, you will prefer the action a_8 to any of the actions in group 3; you will prefer any action in 3 and any one in 2 to the action a_1.

Now suppose, in addition, that you happen to be indifferent between a_2, a_3, and a_4. If this is the case we shall say that your

degrees of belief in the occurrence of each of the states s_1, s_2, s_3 are $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$; and we shall assign to the occurrence of the state s_1 *or* s_2 (and similarly: of the state s_1 *or* s_3; and s_2 *or* s_3) the degree of belief $\frac{2}{3}$. Moreover: would it be reasonable for you to be indifferent within group 2 and not to be indifferent within group 3? Would this not mean that you grant the three possible future states of the world an equal degree of belief as long as rewards or punishments are attached to them in a certain way, but revise your beliefs when the rewards and punishments are interchanged? This would be unreasonable.

We thus have:

$$u(a_1) < u(a_2) = u(a_3) = u(a_4) < u(a_5) = u(a_6) = u(a_7) < u(a_8)$$

We now proceed to scale these utilities. We are free to fix the utility of Life (like the utility of "bliss" in an earlier part of the paper) at 1, and the utility of Death at 0. Then the utilities of prospects will also fall in their places, being equal to the probability of Life as promised in a given prospect. That is, we obtain $u(a_1) = 0$; $u(a_8) = 1$; $u(a_2) = u(a_3) = u(a_4) = $; $u(a_5) = u(a_6) = u(a_7) = \frac{2}{3}$. We shall find as before that this utility scale is consistent with your being a maximizer of expected utility.

Thus, not only your utilities but also your degrees of belief can be derived from your behavior—including the indifference which you have shown in choosing within group 2 of actions (and also—as consistency required—within the group 3 of actions).

We have thus shown that it was possible to interpret your choices as consistent with the existence of utilities and subjective probabilities and with the maximization of expected utility. Granted that the example was a special one. It can be easily extended to n (instead of 3) states of the world, and degrees of belief $0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1$ can then be defined—if one can catch you as being indifferent within a certain group of decisions. How to arrange for a demarkation of states of the world that would make this possible requires a more complete and rigorous logical analysis. (This has been done by L. J. Savage.) Here we have had to content ourselves with a mere sketch.