in (1930). Joan Robinson's fine article is indeed the culmination of this whole line of reasoning, developing in much greater detail and in crystal-clear prose the mode of analysis that began with Harrod; but it is puzzling that Harrod (unlike Hicks, Marshall, Pigou, Robbins and Sraffa) is never mentioned, in spite of the striking similarities between the two analyses. An interesting sidelight is that, in a letter written soon after the appearance of her article and published in Robinson (1951, pp. 42–3), Keynes took a markedly general equilibrium approach to the problem.

Apart from relevant surveys of external economies by Ellis and Fellner (1943) and Chipman (1965, Section 2.8, pp. 736–49) there has been little further discussion of 'rising supply price', evidence perhaps that its nature is by now well understood. However, even as late as 1954, Scitovsky's well-received article with its Pigovian policy conclusions and remark that 'Pecuniary external economies clearly have no place in equilibrium theory' (1954, 149, 146), showed that confusion still existed. Maybe each generation of partial equilibrium theorists has to learn the lesson anew.

PETER NEWMAN

See also EXTERNAL ECONOMIES; GENERAL EQUILIBRIUM; PIGOU, ARTHUR CECIL; PECUNIARY AND NON-PECUNIARY ECONOMIES.

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risk. The phenomenon of *risk* (or alternatively, *uncertainty* or *incomplete information*) plays a pervasive role in economic life. Without it, financial and capital markets would consist of the exchange of a single instrument each period, the communications industry would cease to exist, and the profession of investment banking would reduce to that of accounting. One need only consult the contents of any recent economics journal to see how the recognition of risk has influenced current research in economics. In this entry we present an overview of the modern economic theory of the characterization of risk and the modelling of economic agents' responses to it.

RISK VERSUS UNCERTAINTY. The most fundamental distinction in this branch of economic theory, due to Knight (1921), is that of risk versus uncertainty. A situation is said to involve *risk* if the randomness facing an economic agent can be expressed in terms of specific numerical probabilities (these probabilities may either be objectively specified, as with lottery tickets, or else reflect the individual's own subjective beliefs). On the other hand, situations where the agent cannot (or does not) assign actual probabilities to the alternative possible occurrences are said to involve *uncertainty*.

The standard approach to the modelling of preferences under uncertainty (as opposed to risk) has been the *state preference approach* (e.g. Arrow, 1964; Debreu, 1959. ch. 7; Hirshleifer, 1965, 1966; Karni, 1985; Yaari, 1969). Rather than using numerical probabilities, this approach represents the randomness facing the individual by a set of mutually exclusive and exhaustive *states of nature* or *states of the world* $S = \{s_1, \ldots, s_n\}$. Depending upon the particular application, this partition of all possible futures may either be very coarse, as with the pair of states {it snows here tomorrow, it does not snow here tomorrow} or else very fine, so that the description of a single state might read 'it snows more than three inches here tomorrow *and* the temperature in Paris at noon is 73° *and* the price of platinum in London is over $700.00 per ounce'. The objects of choice in this framework consist of *state-payoff bundles* of the form $(c_1, \ldots, c_n)$, which specify the payoff that the individual will receive in each of the respective states. As with regular commodity bundles, individuals are assumed to have preferences over state-payoff bundles which can be represented by indifference curves in the *state-payoff space* $\{(c_1, \ldots, c_n)\}$.

Although this approach has led to important advances in the analysis of choice under uncertainty (see for example the above references), the advantages of being able to draw on the modern theory of probability has led economists to concentrate on the analysis of risk, where the consequences of agents' actions are alternative well-defined probability distributions over the random variables they face. An important justification for the modelling of randomness via formal probability distributions are those joint axiomatizations of preferences and beliefs which provide consistency conditions on preferences over state-payoff bundles sufficient to imply that they can be generated by a well-defined probability distribution over states of nature and a von Neumann–Morgenstern utility function over payoffs of the type described in the following section (e.g. Savage, 1954; Anscombe and Aumann, 1963; Pratt, Raiffa and Schlaifer, 1964; and Raiffa, 1968, ch. 5).

CHOICE UNDER RISK – THE EXPECTED UTILITY MODEL. For reasons of expositional ease, we consider a world with a single commodity (e.g. wealth). An agent making a decision under risk can therefore be thought of as facing a choice set of alternative univariate probability distributions. In order to consider both discrete (e.g. finite outcome) distributions as well as distributions with density functions, we shall represent each such probability distribution by means of its cumulative distribution functions $F(\cdot)$, where $F(x) \equiv \text{prob}(\tilde{x} \leqslant x)$ for the random variable $\tilde{x}$.

In such a case we can model the agent's preferences over alternative probability distributions in a manner completely analogous to the approach of standard (i.e. non-stochastic) consumer theory: he or she is assumed to possess a ranking $\succcurlyeq$ over distributions which is complete, transitive and continuous (in an appropriate sense), and hence representable by a real-valued *preference function* $V(\cdot)$ over the set of cumulative distribution functions, in the sense that $F^*(\cdot) \succcurlyeq F(\cdot)$ (i.e. the distribution $F^*(\cdot)$ is weakly preferred to $F(\cdot)$) if and only if $V(F^*) \geqslant V(F)$).

Of course, as in the non-stochastic case, the above set of assumptions implies nothing about the functional form of the preference functional $V(\cdot)$. For reasons of both normative appeal and analytic convenience, economists typically assume that $V(\cdot)$ is a *linear functional* of the distribution $F(\cdot)$, and hence takes the form

$$V(F) \equiv \int U(x)\,\mathrm{d}F(x) \qquad (1)$$

for some function $U(\cdot)$ over wealth levels x, where $U(\cdot)$ is referred to as the individual's *von Neumann–Morgenstern utility function*. (For readers unfamiliar with the *Riemann–Stieltjes integral* $\int U(x)\,\mathrm{d}F(x)$ it represents nothing more than the expected value of $U(\tilde{x})$, when $\tilde{x}$ possesses the cumulative distribution function $F(\cdot)$. Thus if $\tilde{x}$ took the values $x_1, \ldots, x_n$ with probabilities $p_1, \ldots, p_n$, $\int U(x)\,\mathrm{d}F(x)$ would equal $\Sigma\, U(x_i)p_i$, and if $\tilde{x}$ possessed the density function $f(\cdot) = F'(\cdot)$, $\int U(x)\,\mathrm{d}F(x)$ would equal $\int U(x) f(x)\,\mathrm{d}x$.

Since the right side of (1) may accordingly be thought of an the mathematical expectation of $U(\tilde{x})$, this specification is known as the *expected utility model* of preferences over random prospects (for a more complete statement of this model, see EXPECTED UTILITY HYPOTHESIS). Within this framework, an individual's attitudes toward risk are reflected in the shape of his or her utility function $U(\cdot)$. Thus, for example, an individual would always prefer shifting probability mass from lower to higher outcome levels if and only if $U(x)$ were an increasing function of x, a condition which we shall henceforth always assume. Such a shift of probability mass is known as a *first order stochastically dominating shift*.

RISK AVERSION. The representation of individuals' preferences over distributions by the shape of their von Neumann–Morgenstern utility functions provides the first step in the modern economic characterization of risk. After all, whatever the notion of riskier means, it is clear that bearing a random wealth $\tilde{x}$ is riskier than receiving a certain payment of $\bar{x} = E[\tilde{x}]$, i.e. the expected value of the random variable $\tilde{x}$. We therefore have from Jensen's inequality that an individual would be *risk averse*, i.e. always prefer a payment of $E[\tilde{x}]$ (and obtaining utility $U(E[\tilde{x}])$) to bearing the risk $\tilde{x}$ (and obtaining expected utility $E[U(\tilde{x})]$) if and only if his or her utility function were concave. This condition is illustrated in Figure 1, where the random variable $\tilde{x}$ is assumed to take on the values x' and x'' with respective probabilities 2/3 and 1/3.

Of course, not all individuals need be risk averse in the sense of the previous paragraph. Another type of individual is a *risk lover*. Such an individual would have a *convex* utility function, and would accordingly prefer receiving a random wealth $\tilde{x}$ to receiving its mean $E[\tilde{x}]$ with certainty. An example of such a utility function is given in Figure 2.

STANDARD DEVIATION AS A MEASURE OF RISK. While the above characterization of risk aversion (as well as its opposite) allows for the derivation of many results in the theory of behaviour under risk, it says nothing regarding which of a *pair* of non-degenerate random variables $\tilde{x}$ and $\tilde{y}$ is the most risky. Since real-world choices are almost never between risky and riskless situations but rather over alternative risky situations, such a means of comparison is necessary.

The earliest and best known univariate measure of the riskiness of a random variable $\tilde{x}$ is its *variance* $\sigma^2 = E[(\tilde{x} - \bar{x})^2]$ or alternatively its *standard deviation* $\sigma = \{E[(\tilde{x} - \bar{x}]^2\}^{1/2}$. The tractability of these measures as well as their well-known statistical properties led to the widespread use of *mean-standard deviation analysis* in the 1950s and 1960s, and in particular to the development of modern portfolio theory by Markowitz

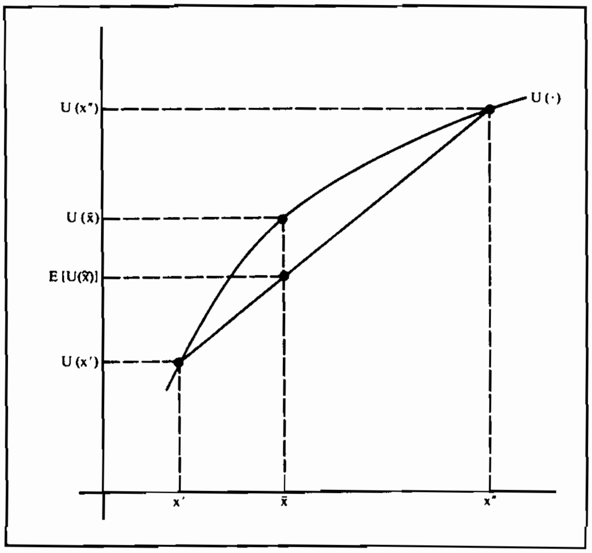

Figure 1 Von Neumann–Morgenstern Utility Function of a Risk Averse Individual

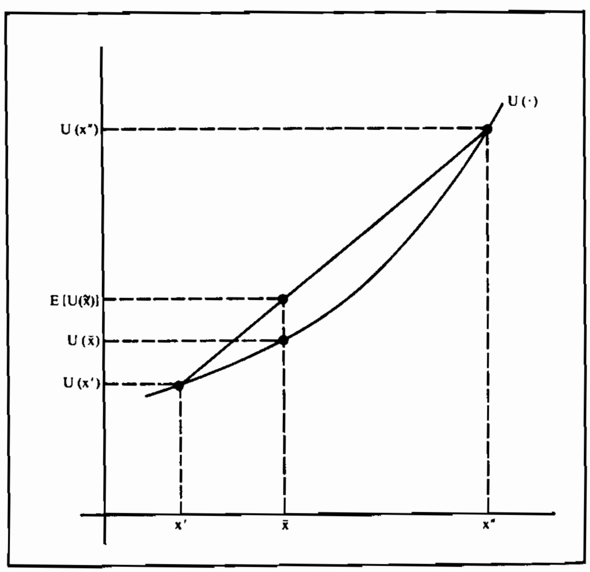

Figure 2 Von Neumann–Morgenstern Utility Function of a Risk Loving Individual

(1952, 1959), Tobin (1958) and others. As an example of this, consider Figure 3. Points A and B correspond to the distributions of a riskless asset with (per dollar) gross return r_0 and a risky asset with random return $\tilde{r}$ with mean $\mu\tilde{r}$ and standard deviation $\sigma\tilde{r}$. An investor dividing a dollar between the two assets in proportions $\alpha:(1-\alpha)$ will possess a portfolio whose return has a mean of $\alpha \cdot r_0 + (1-\alpha)\cdot \mu\tilde{r}$ and standard deviation $(1-\alpha)\cdot\sigma_{\tilde{r}}$, so that the set of attainable (μ, σ) combinations consists of the line segment connecting the points A and B in the figure. It is straightforward to show that if the individual were also allowed to *borrow* at rate r_0 in order to finance purchase of the risky asset (i.e. could sell the riskless asset short), then the set of attainable (μ, σ) combinations would be the ray emanating from A and passing through B.

If we then represent the individual's risk preferences by means of indifference curves in this diagram, we obtain their optimal portfolio (the example in the figure implies an equal division of funds between the two assets). In the more general case of choice between a pair of risky assets, the set of (μ, σ) combinations generated by alternative divisions of wealth between them will trace out a locus such as the one between points C and D in the diagram, with the curvature of this locus determined by the degree of statistical dependence (i.e. covariance) between the two random returns.

As mentioned, the representation and analysis of risk and risk-taking by means of the variance or standard deviation of a distribution proved tremendously useful in the theory of finance, culminating in the mean-standard deviation based *capital asset pricing model* of Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). However, by the late 1960s the mean-standard deviation approach was under attack for two reasons.

The first reason (known since the 1950s) was that the fact that an expected utility maximizer would evaluate all distributions solely on the basis of their means and standard deviations if and only if his or her von Neumann–Morgenstern utility function took the quadratic form $U(x) \equiv ax + bx^2$ for $b \lessgtr 0$. The sufficiency of this condition is established by noting that $E[U(\tilde{x})] = E[a\tilde{x} + b\tilde{x}^2] = a\bar{x} + b(\bar{x}^2 + \sigma^2)$. To prove necessity, note that the distributions which yield a 2/3:1/3 chance of the outcomes $x - \delta : x + 2\delta$ and a 1/3:2/3 chance of the outcomes $x - 2\delta : x + \delta$ both possess the same mean and variance for each x and δ, so that $(2/3)\cdot U(x-\delta) + (1/3)\cdot U(x+2\delta) \equiv (1/3)U(x-2\delta) + (2/3)U(x+\delta)$ for all x and δ. Differentiating with respect to δ and simplifying yields $U'(x+2\delta) + U'(x-2\delta) \equiv U'(x+\delta) + U'(x-\delta)$ for all x and δ. This implies that $U'(\cdot)$ must be linear and hence that $U(\cdot)$ must be quadratic.

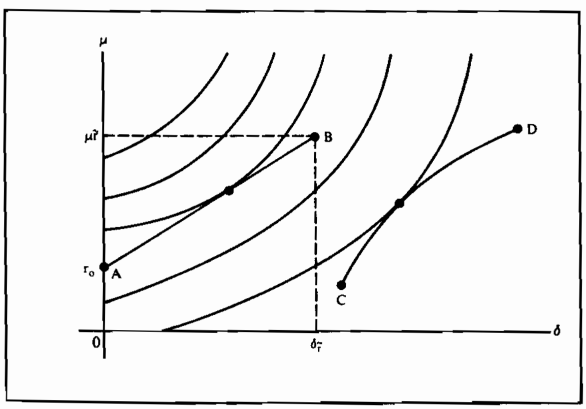

Figure 3 Portfolio Analysis in the Mean-Standard Deviation Diagram

The assumption of quadratic utility is objectionable. If an individual with such a utility function is risk averse (i.e. if $b < 0$), then (i) utility will decrease as wealth increases beyond $1/2b$, and (ii) the individual will be more averse to constant additive risks about high wealth levels than about low wealth levels – in contrast to the observation that those with greater wealth take greater risks (see for example Hicks (1962) or Pratt (1964)).

Borch (1969) struck the second and strongest blow to the mean-standard deviation approach. He showed that for any two points (μ_1, σ_1) and (μ_2, σ_2) in the (μ, σ) plane which a mean-standard deviation preference ordering would rank as indifferent, it is possible to find random variables $\tilde{x}_1$ and $\tilde{x}_2$ which possess these respective (μ, σ) values and where $\tilde{x}_2$ first order stochastically dominates $\tilde{x}_1$. However, *any* person with an increasing von Neumann–Morgenstern utility function would strictly prefer $\tilde{x}_2$ to $\tilde{x}_1$. In response to these arguments and the additional criticisms of Feldstein (1969), Samuelson (1967) and others, the use of mean-standard deviation analysis in economic theory waned. See, however, the recent work of Meyer (1987) for a partial rehabilitation of such two-moment models of preferences.

Besides the variance or standard deviation of a distribution, several other univariate measures of risk have been proposed. Examples include the *mean absolute deviation* $E[|\tilde{x} - \bar{x}|]$, the *interquartile range* $F^{-1}(0.75) - F^{-1}(0.25)$, and the classical statistical measures of *entropy* $\Sigma p_i \cdot \ln(p_i)$ or $\int f(x)\cdot \ln(f(x))\,dx$. Although they provide the analytical convenience of a single numerical index of riskiness, each of these measures are subject to problems of the sort encountered with the variance or standard deviation. In particular, the entropy measure can be particularly unresponsive to the values taken on by the random variable: the 50:50 gambles over the values \$50:\$51 and \$1:\$100 both possess the same entropy level.

INCREASING RISK. By the late 1960s, the failure to find a satisfactory univariate measure of risk led to another approach to this problem. Working independently, several researchers (Hadar and Russell, 1969; Hanoch and Levy, 1969; and Rothschild and Stiglitz, 1970, 1971) developed an alternative characterization of increasing risk. The appeal of this approach is twofold. First, it formalizes three different intuitive notions of increasing risk. Second, it allows for the straightforward derivation of comparative statics results in a wide variety of economic situations. Unlike the univariate measures described above, however, this approach provides only a partial ordering of random variables. In other words, not all pairs of random variables can be compared with respect to their riskiness.

We now state three alternative formalizations of the notion that a cumulative distribution function $F^*(\cdot)$ is riskier than another distribution $F(\cdot)$ with the same mean (in the following, all distributions are assumed to be over the interval $[0, M]$).

The first definition of increasing risk captures the notion that 'risk is what all risk averters hate'. Thus an increase in risk lowers the expected utility of all risk averters. Formally we may state this condition as:

(A) *$F^*(\cdot)$ and $F(\cdot)$ have the same mean and $\int U(x)\,dF^*(x) \leqslant \int U(x)\,dF(x)$ for all concave utility functions $U(\cdot)$.*

This criterion cannot be used to compare every pair of distributions with the same mean. However, if a pair of distributions

$F(\cdot)$ and $F^*(\cdot)$ do *not* satisfy condition (A) (in either direction), there must exist a risk averse (i.e. concave) utility function $U_a(\cdot)$ which prefers $F(\cdot)$ to $F^*(\cdot)$ and another risk averse function $U_b(\cdot)$ which prefers $F^*(\cdot)$ to $F(\cdot)$.

The second characterization of the notion that a random variable $\tilde{y}$ with distribution $F^*(\cdot)$ is riskier than a variable $\tilde{x}$ with distribution $F(\cdot)$ is that $\tilde{y}$ consists of the variable $\tilde{x}$ plus an additional noise term $\tilde{\varepsilon}$. One possible specification of this is that $\tilde{\varepsilon}$ be statistically independent of $\tilde{x}$. However, this condition is too strong in the sense that it does not allow the variance of $\tilde{\varepsilon}$ to depend upon the magnitude of $\tilde{x}$, as in the case of heteroskedastic noise. Instead, Rothschild and Stiglitz (1970) modelled the addition of noise by the condition:

(B) *$F(\cdot)$ and $F^*(\cdot)$ are the cumulative distribution functions of the random variables $\tilde{x}$ and $\tilde{x}+\tilde{\varepsilon}$, where $E[\tilde{\varepsilon}\,|\,x] \equiv 0$ for all x*

The third notion of increasing risk involves the concept (due to Rothschild and Stiglitz, 1970) of a *mean preserving spread.* Intuitively, such a spread consists of moving probability mass from the centre of a probability distribution to its tails in a manner which preserves the expected value of the distribution, as seen in the top panels of Figures 4 and 5. Formally we say

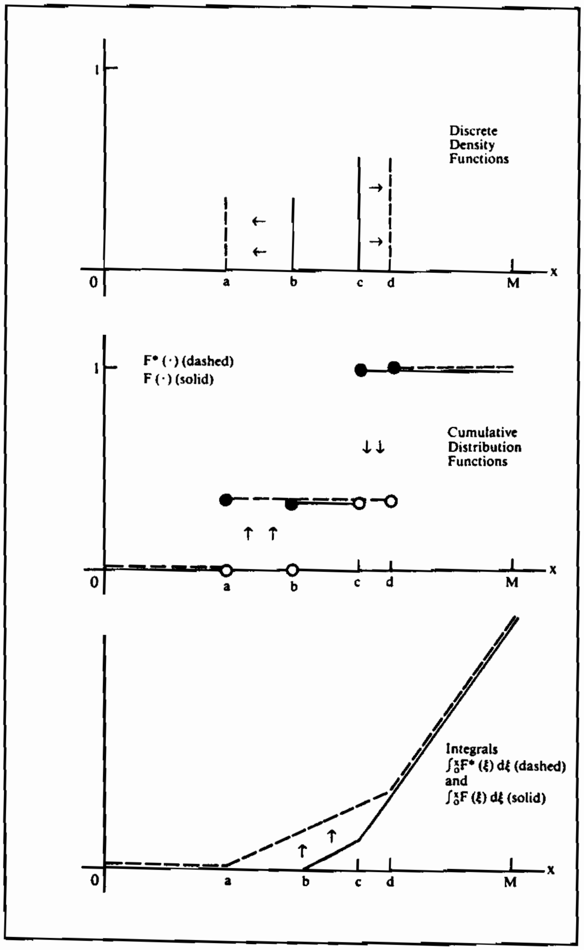

Figure 4 A Mean Preserving Spread of a Discrete Distribution

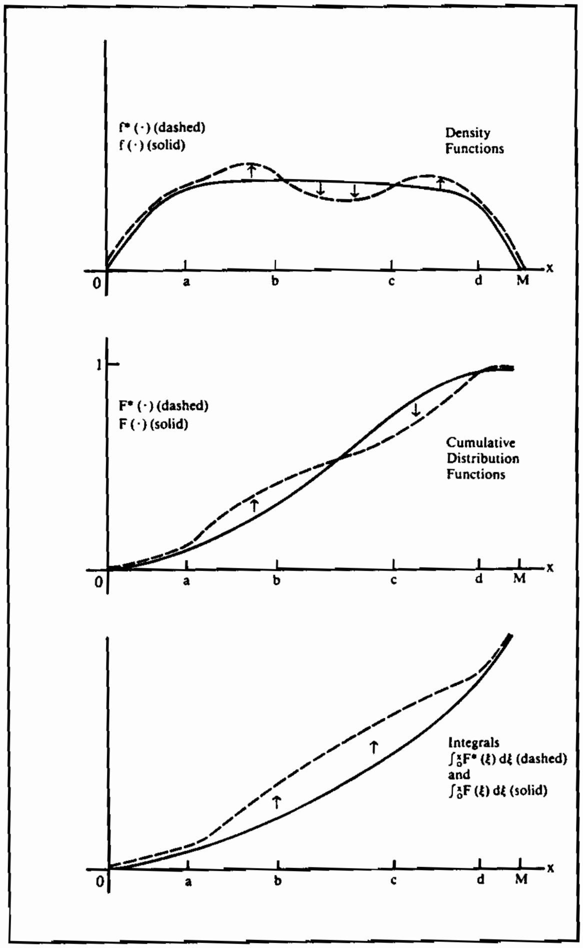

Figure 5 A Mean Preserving Spread of a Density Function

that $F^*(\cdot)$ differs from $F(\cdot)$ by a mean preserving spread if they have the same mean and there exists a single crossing point x_0 such that $F^*(x) \geqslant F(x)$ for all $x \leqslant x_0$ and $F^*(x) \leqslant F(x)$ for all $x \geqslant x_0$ (see the middle panels of these figures). Since it is clear that *sequences* of such spreads will also lead to riskier distributions, our third characterization of increasing risk is:

(C) *$F^*(\cdot)$ may be obtained from $F(\cdot)$ by a finite sequence, or as the limit of an infinite sequence, of mean preserving spreads.*

Although the single crossing property of the previous paragraph serves to characterize cumulative distribution functions which differ by a *single* mean preserving spread, distributions which differ by a sequence of such spreads will typically not satisfy the single crossing condition. If we consider the *integral* of the cumulative distribution function, however, we see from the bottom panels of Figures 4 and 5 that a mean preserving spread will always serve to raise or preserve the value of this integral for each x and (since $F^*(\cdot)$ and $F(\cdot)$ have the same mean) to preserve it for $x = M$. It is clear that this condition *will* continue to be satisfied by distributions which differ by a sequence of mean preserving spreads. Accordingly, we may rewrite condition (C) above by the analytically more convenient:

(C′) *The integral* $\int_0^x [F^*(\xi) - F(\xi)] \cdot d\xi$ *is non-negative for all* $x > 0$, *and is equal to* 0 *for* $x = M$.

Rothschild and Stiglitz (1970) showed that *these three concepts of increasing risk are the same* by proving that conditions (A), (B) and (C/C′) are equivalent. Thus, a single partial ordering of distribution functions corresponds simultaneously to the notion that risk is what risk averters hate, to the notion that adding noise to a random variable increases its risk, and to the notion that moving probability mass from the centre of a probability distribution to its tails increases the risk of that distribution.

This characterization of increasing risk permits the derivation of general and powerful comparative statics theorems concerning economic agents' response to increases in risk. The general framework for these results is that of an individual with a von Neumann–Morgenstern utility function $U(x, \alpha)$ which depends upon both the outcome of some random variable $\tilde{x}$ as well as a *control variable* α which the individual chooses so as to maximize expected utility $\int U(x, \alpha)\, dF(x; r)$, where the distribution function $F(\cdot\,; r)$ depends upon some exogenous parameter r (x for example might be the return on a risky asset, and α the amount invested in it). For convenience, we assume that $F(0; r) = \text{prob}(\tilde{x} \leqslant 0) = 0$ for all r. The first order condition for this problem is then:

$$\int U_\alpha(x, \alpha)\, dF(x; r) = 0 \qquad (2)$$

where $U_\alpha(x, \alpha) = \partial U(x, \alpha)/\partial \alpha$, and we assume that the second derivative $U_{\alpha\alpha}(x, \alpha) = \partial^2 U(x, \alpha)/\partial \alpha^2$ is always negative to ensure we have a maximum. Implicit differentiation of (2) then yields the comparative statics derivative:

$$\frac{d\alpha}{dr} = \frac{-\int U_\alpha(x, \alpha)\, dF_r(x; r)}{\int U_{\alpha\alpha}(x, \alpha)\, dF(x; r)} \qquad (3)$$

where $F_r(x; r) = \partial F(x; r)/\partial r$. Since the denominator of this expression is negative by assumption, the sign of $d\alpha/dr$ is given by the sign of the numerator $\int U_\alpha(x, \alpha)\, dF_r(x; r)$. Integrating by parts twice yields:

$$\int U_\alpha(x, \alpha)\, dF_r(x; r) = \int U_{xx\alpha}(x, \alpha) \cdot \left[\int_0^x F_r(\xi, r)\, d\xi\right] dx$$
$$= \int U_{xx\alpha}(x, \alpha) \cdot \left[\frac{d}{dr}\left(\int_0^x F(\xi, r)\, d\xi\right)\right] dx \qquad (4)$$

Thus, if increases in the parameter r imply increases in the riskiness of the distribution $F(\cdot\,; r)$, it follows from condition (D) that the signs of the square bracketed terms in (4) will be non-negative, so that the effect of r upon α depends upon the sign of $U_{xx\alpha}(x, \alpha) = \partial^3 U(x, \alpha)/\partial^2 x \partial \alpha$. Thus if $U_{xx\alpha}(x, \alpha)$ is uniformly negative, a mean preserving increase in risk in the distribution of x will lead to a fall in the optimal value of the control variable α and vice versa. Another way to see this is to note that if $U_\alpha(x, \alpha)$ is concave in x then a mean preserving increase in risk will lower the left side of the first order condition (2), which (since $U_{\alpha\alpha}(x, \alpha) \leqslant 0$) will require a drop in α to re-establish the equality. Economists routinely use this technique when analysing models involving risk; see for example Rothschild and Stiglitz (1971).

RELATED TOPICS. The characterization of risk outlined in the previous section has been extended along several lines. Diamond and Stiglitz (1974), for example, have replaced the notion of a mean preserving spread with that of a mean *utility* preserving spread to obtain a general characterization of a *compensated increase in risk*. They related this notion to the well-known Arrow–Pratt characterization of comparative risk aversion (see EXPECTED UTILITY HYPOTHESIS).

In addition, researchers such as Ekern (1980), Fishburn (1982), Fishburn and Vickson (1978), Hansen, Holt, and Peled (1978), Tesfatsion (1976), and Whitmore (1970) have extended the above work to the development of a general theory of *stochastic dominance*, which provides a whole sequence of similarly characterized partial orders on distributions, each presenting a corresponding set of equivalent conditions involving algebraic conditions on the distributions, types of spreads, and classes of utility functions which prefer (or are averse) to such spreads, etc. The comparative statics analysis presented above may be similarly extended to such characterizations (e.g. Machina, 1987). An extensive bibliography of the stochastic dominance literature is given in Bawa (1982). Finally, various extensions of the notions of increasing risk and stochastic dominance to the case of multivariate distributions may be found in Epstein and Tanny (1980), Fishburn and Vickson (1978), Huang, Kira and Vertinsky (1978), Lehmann (1955), Levhari, Parousch and Peleg (1975), Levy and Parousch (1974), Russell and Seo (1978), Sherman (1951), and Strassen (1965) (see also the mathematical results in Marshall and Olkin, 1979).

MARK J. MACHINA AND MICHAEL ROTHSCHILD

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See also EXPECTED UTILITY HYPOTHESIS; UNCERTAINTY.

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Rist, Charles (1874–1955). Born at Prilly, Switzerland, 1874; died at Versailles, 1955. Professor at Montpellier (1899–1912) and Paris (1913–33), Rist was the most notable and influential thinker and actor in the field of money in France in the first half of the 20th century. As a member of the *Comité des experts* (1926) and as a vice-governor of the Bank of France (1926–8), he took an active part in monetary reconstruction in the Twenties. He supported the novel idea of stabilization with devaluation (1926–8). He was also involved as an expert in monetary reforms in Rumania (1928), Austria, Turkey and Spain. He was France's delegate at the London Economic Conference (1933).

Although Rist is most widely known for his *History of Economic Doctrines*, written in cooperation with Charles Gide, his lasting claim to fame rests on his profound and consistent interpretation of monetary history and thought as demonstrated in his masterwork, *History of Monetary and Credit Theory*. Based on his first-hand experience in times of great instability, Rist's critical analysis of monetary thought from a long-run viewpoint provides an impressive perspective on the evolution of money. By emphasizing the 'store of value' function of money, and by postulating the inability of the State to safeguard it, Rist is critical of authors who supported some form of non-metallic currency, such as John Law, Smith, Ricardo, Wicksell, Knapp and Keynes. He is in sympathy with Cantillon, Galiani, Turgot, Thornton, Tooke and Walras. What he describes as the confusion between money and credit is to be dispelled by drawing a distinction between money proper (gold), credit instruments (convertible banknotes and deposits) and inconvertible paper money. In strong opposition to Keynesianism, Rist is a sceptic in regard of managed currencies and international agreements of the Bretton Wood type. Rist provides the key to the understanding of the French position in monetary matters as opposed to the typical Anglo-American stance in the past 60 years.

ROGER DEHEM

See also GIDE, CHARLES.

SELECTED WORKS

1915. (With Ch. Gide.) *A History of Economic Doctrines.* London: G. Harrap. 2nd edn, 1948.
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Robbins, Lionel Charles (1898–1984). Lionel Robbins, who in 1961 became Baron Robbins of Clare Market, was one of the major academic economists of the interwar period. He remained active after World War II but never really regained the centre of the stage that he had occupied. He was also a great public servant for his country, serving it well and loyally in many aspects of social, political and cultural life. He was truly a 'renaissance man'.

Robbins was born in 1898 in Middlesex, the son of Rowland Richard Robbins – for many years President of the National Farmers' Union – and Rosa Marion Robbins. He spent one year reading for an Arts degree at University College London and then volunteered for war service with the Royal Artillery. He saw active service on the Western Front, was wounded and invalided back to England in 1918. He was an undergraduate at the London School of Economics and Political Science from