M.PHIL. GAME THEORY: PROBLEMS

These problems are designed for discussion in the classes of Week 6 (half class) and Week 8 (full class) of Michaelmas Term:

Week 6: Problems 1–4.
Week 8: Problems 5–12.

1. A Simple Strategic Form Game. Consider the following symmetric 2×2 game:

	L	R
U	9 9	8 0
D	0 8	7 7

(1) By inspection, what are the pure-strategy Nash equilibria?

(2) Find the mixed-strategy equilibrium by using the fact that if players are willing to mix between two or more strategies, they must be indifferent between them.

(3) Draw the best-response correspondences. Where do they intersect?

2. Deletion of Dominated Strategies. Consider the following strategic-form game:

	L	C	R
T	0 2	1 1	2 4
M	4 3	2 1	3 2
B	3 1	2 0	0 3

(1) What strategies survive iterated elimination of strictly dominated strategies?

(2) What are the pure-strategy Nash equilibria?

3. Cournot Competition. Consider an n firm homogeneous product industry where firm i produces output q_i at cost cq_i. Price is

$$p = \alpha - \beta Q \quad \text{where} \quad Q = \sum_{i=1}^{n} q_i.$$

(1) What are the firms' outputs, prices and profits in the Cournot equilibrium? What happens as $n \to \infty$?

(2) Two firms merge. The merged firm has marginal costs of c, just as before. What happens to the merged firms' profits? What happens to the remaining firms' profits? Comment.

(3) Now suppose that any firm producing positive output incurs a fixed cost of F:

$$c_i(q_i) = F + cq_i \quad \text{if} \quad q_i > 0$$

and $c_i(0) = 0$. Let $n = 4$. Suppose F satisfies:

$$F = \frac{1}{\beta}\left[\frac{2(\alpha - c)}{9}\right]^2.$$

(a) What are the pure strategy equilibria?[1]

(b) Without calculations, do you think there may be any mixed equilibria?

4. Switching Costs. In many markets, consumers have *switching costs*.[2] Consider the following simple model of such a market: Two firms A and B simultaneously and non-cooperatively set prices in a single period for a commodity that they can each produce at zero cost. there are $n + s$ customers, where $n > 0$ and $s > 0$ all with reservation price R. Because of switching costs, $s/2$ customers can only buy from A and $s/2$ can only buy from B. The n "new" customers buy from the cheapest firm, if at all.

(1) Show that there are no pure-strategy equilibria.

(2) Find a mixed-strategy Nash equilibrium in which each firm chooses price p according to the distribution $F(p)$.[3]

[1]Hint: There may be equilibria with only $m \leq n$ active firms, so try each case $m = 1, \ldots, 4$. Start by looking at the case where one firm is producing the monopoly output and the other firms are producing nothing. Can this be a Nash equilibrium? Does a firm not producing in this situation have an incentive to deviate? Then look at the case when two firms are producing, and so on.

[2]Examples include the transactions costs of closing an account with one bank and opening another with a competitor, learning costs incurred by switching to a new make of computer after having learnt to use one make, the artificial switching costs caused by frequent flyer programmes, etc.

[3]Hint: First check that F is continuous, with no atoms. Now, for a given p_i, calculate expected profit. What can you say about all prices in the support of the mix? What value does p take when $F(p) = 1$? You should now be able to find $F(p)$. Over what interval of prices do the firms mix?