MEANINGFUL THEOREMS: NONPARAMETRIC ANALYSIS OF REFERENCE-DEPENDENT PREFERENCES

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Summary

This paper derives nonparametric sufficient (and with rich data, necessary) conditions for the existence of reference-dependent preferences that can rationalize some patterns of consumer demand behavior that do not allow a neoclassical rationalization.

We build on Kőszegi and Rabin's (2006; "KR") parametric implementation of Kahneman and Tversky's (1979; "KT") and Tversky and Kahneman's (1991) theory of reference-dependent preferences.

KR's preferences respond to levels of consumption, as in neoclassical consumer theory, and changes in consumption relative to a reference point, as in KT's theory.

We show that unless KT's and KR's notion of "sensitivity" is constant (Tversky and Kahneman's "sign-dependence"; KR's assumption A3′) and reference points are precisely modelable or observable (henceforth "modelable"), the hypothesis of reference-dependent preferences has virtually no useful nonparametrically refutable implications.

That is the grain of truth in the common belief that allowing preferences to be reference-dependent destroys the parsimony of neoclassical consumer theory.

However, we also show that with constant sensitivity and modelable reference points, the model does have useful nonparametrically refutable implications, which we characterize.

Our characterization relaxes two strong functional-structure assumptions KR maintained:

- That the preferences that determine consumer demand are additively separable across goods
- That those preferences' marginal rates of substitution satisfy particular knife-edge constraints on how they vary with the reference point

Both of the relaxed restrictions are strongly violated in our empirical illustration.

We illustrate our characterization by revisiting Farber's (2005, 2008) and Crawford and Meng's (2011; "CM") empirical analyses of cabdrivers' labor supply, using Farber's data.

We control for neoclassical and reference-dependent models' different flexibilities using Beatty and Crawford's (2011, pp. 2786-87) proximity-based variant of Selten and Krischker's (1983) and Selten's (1991) nonparametric measure of predictive success, which judges a model's flexibility by how likely random data are to be consistent with it.

Relaxing KR's functional structure assumptions greatly increases predictive success.

For a substantial number of Farber's drivers, a relaxed reference-dependent model has a higher measure of predictive success than a comparably relaxed neoclassical model.

Empirical Background

Reference-dependent consumer theory has played an important role in empirical analyses of workers', consumers', and investors' choice behavior since Camerer et al.'s (1997) analysis of the daily labor supply of New York City cabdrivers.

A neoclassical model of labor supply is analogous to a model of consumer demand for earnings and leisure.

It therefore predicts a positive elasticity of hours worked with respect to the wage unless there are very large income effects.

But Camerer et al., taking drivers' earnings per hour as analogous to a wage, estimate a strongly negative elasticity.

To explain the negative elasticity, Camerer et al. propose a model in which drivers have daily earnings targets like KT's reference points, treated econometrically as latent variables.

Experiments and analyses of field data suggest that most people are loss-averse—more sensitive to changes below their reference points ("losses") than above them ("gains").

This creates kinks in preferences that make a driver's optimal earnings tend to bunch around his earnings target, thus working less on days when earnings per hour are high.

Depending on the details, such bunching can yield a negative overall wage elasticity of hours, despite the positive incentive effect of anticipated higher wages.

Farber (2005, 2008) analyzes a newer dataset on New York City cabdrivers, following Camerer et al. in allowing earnings targeting with targets treated as latent variables.

In his data, as in Camerer et al.'s, a reference-dependent model fits better than a neoclassical model, and his estimates of the wage elasticity are again negative.

But his estimates of the earnings targets are unstable, which he argues precludes a useful reference-dependent model of drivers' labor supply.

Farber (2015) analyzes a much larger dataset on New York City cabdrivers, again concluding that reference-dependence is not useful in explaining labor supply.

In each case Farber concludes that most of his drivers are irrational. E.g. Farber (2008):

"This [earnings-targeting] is clearly nonoptimal from a neoclassical perspective, since it implies quitting early on days when it is easy to make money and working longer on days when it is harder to make money. Utility would be higher by allocating time in precisely the opposite manner."

Inspired by Camerer et al.'s and Farber's analyses, KR bring KT's theory of reference-dependent preferences closer to economic applications.

KR assume rationality but expand the domain of preferences, allowing preferences to respond to changes in consumption, relative to a reference point, as well as levels.

Expanding the domain of preferences is a slippery slope, but the slippage here is disciplined by the idea of reference-dependence and supported by a large body of evidence.

● KR's utility function has separate, additively separable components of neoclassical utility of consumption levels and reference-dependent "gain-loss" utility of consumption changes

● Unlike Camerer et al. and Farber, KR allow reference-dependence for all goods

● KR close their model by setting reference points equal to their rational expectations

Crawford and Meng (2011; "CM") use KR's model to reconsider Farber's econometric analyses, using Farber's (2005, 2008) data.

As in KR's theoretical analysis and all previous empirical work on this topic, CM assume:

- The utility function is additively separable across consumption and gain-loss utility

- Preferences have constant sensitivity (Tversky and Kahneman's 1991 "sign-dependence"; KR's assumption A3'):

  A reference point divides consumption space into gain-loss regimes, such as "earnings loss, hours gain" in labor supply. With constant sensitivity preferences over consumption bundles must be the same throughout a regime but may vary freely across regimes.

- Utility is continuous, with gain-loss utility determined by the good-by-good differences between realized and reference consumption utilities

Econometrically, CM model KR’s rational-expectations reference points via natural sample proxies, instead of treating them as latent variables as Camerer et al. and Farber did.

This avoids the instability of Farber’s estimated earnings targets, and appears to yield a useful reference-dependent model of drivers’ labor supply.

For anticipated changes in earnings and hours, gain-loss utility drops out of the model, which then coincides with a neoclassical model, in which higher wages increase labor supply. But for unanticipated changes, loss aversion creates kinks in preferences that allow a rationality-based explanation of Camerer et al.’s and Farber’s negative wage elasticities.

CM’s Figure 1:

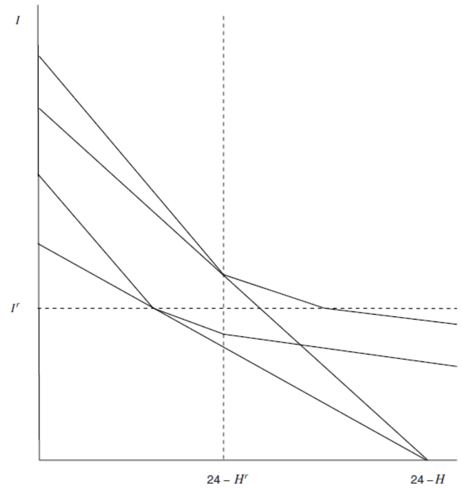


Figure 1. A Reference-Dependent Driver Stops at the Second Target He Reaches: Income on a Bad Day (*lower budget line*), Hours on a Good Day (*upper budget line*)

Despite reference-dependent models' successes in empirical analyses of workers', consumers', and investors' choice behavior, several factors have limited their appeal.

Because they expand the domain of preferences, some researchers doubt that they yield any testable implications—Samuelson's (1947) "meaningful theorems".

Such doubts are exacerbated when reference points are not modeled or observed.

Further, empirical implementations have relied on parametric structural assumptions that are not directly supported by theory or evidence, and are not entirely natural:

- KR's assumption that the sum of consumption and gain-loss utility that determines consumer demand is additively separable across goods

- KR's constant-sensitivity restrictions on how that sum's marginal rates of substitution vary across gain-loss regimes, as in CM's Table 1 for the two-good case:

TABLE 1—MARGINAL RATES OF SUBSTITUTION WITH REFERENCE-DEPENDENT PREFERENCES BY DOMAIN

	Hours gain ($H < H^r$)	Hours loss ($H > H^r$)
Income gain ($I > I^r$)	$-U_2'(H)/U_1'(I)$	$-[U_2'(H)/U_1'(I)][1-\eta+\eta\lambda]$
Income loss ($I < I^r$)	$-[U_2'(H)/U_1'(I)]/[1-\eta+\eta\lambda]$	$-U_2'(H)/U_1'(I)$

- Strong (though standard) assumptions regarding the forms of utility functions

A Nonparametric Model of Reference-dependent Preferences

Our model of reference-dependent preferences follows KR's and CM's models and encompasses Camerer et al.'s and Farber's, but without imposing KR's functional structure assumptions or Farber's and CM's functional form assumptions.

Like KR we maintain rationality, while expanding the domain of preferences to include an additively separable component of gain-loss utility as well as consumption utility.

Our theory applies to a single consumer or (as in most empirical studies) a group assumed to have homogeneous preferences, but we'll speak of a single consumer.

The consumer is a price-taker, who chooses among consumption bundles $\boldsymbol{q} \in \mathbb{R}_+^K$, where goods are indexed *k* = 1,…, *K*.

Preferences are represented by a family of utility functions $u(\boldsymbol{q}, \boldsymbol{r})$, where $\boldsymbol{r} \in \mathbb{R}_+^K$ is an exogenous reference point, conformable to a *K*–good consumption bundle.

$u(\boldsymbol{q}, \boldsymbol{r})$ is continuous, increasing in $\boldsymbol{q}$, and decreasing in $\boldsymbol{r}$.

This is as flexible as a general increasing function of levels $\boldsymbol{q}$ and changes $\boldsymbol{q} - \boldsymbol{r}$.

It nests the neoclassical case where preferences respond only to levels of consumption; KT's case where preferences respond only to changes; and Farber's, KR's, and CM's cases where preferences respond to both.

Index observations t = 1,…,T.

When reference points are unmodelable (and unobservable), the data are prices and quantities $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ and hypothetical reference points are denoted $\{\boldsymbol{r}_t\}_{t=1,\dots,T}$.

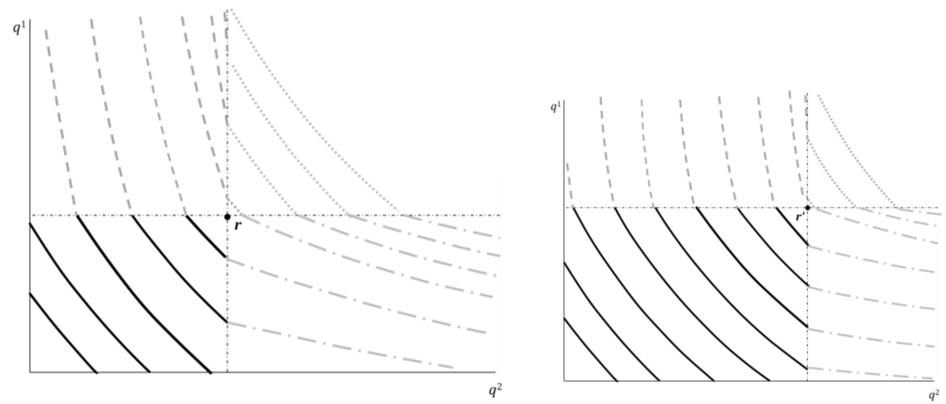
When reference points are modelable (or observable), the data are prices, quantities, and reference points $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$.

Goods are sometimes denoted as scalars indexed by subscripts: for k = 1,…, K, $\boldsymbol{q} \equiv (q^1, \dots, q^K)$; and for t = 1,…, T, $\boldsymbol{q_t} \equiv (q_t^1, \dots, q_t^K)$, with analogous notation for $\boldsymbol{p}$, $\boldsymbol{p_t}$, $\boldsymbol{r}$, $\boldsymbol{r_t}$.

A reference point divides commodity space into gain-loss regimes, such as “earnings-loss and hours-gain” in labor supply.

Constant sensitivity (Tversky and Kahneman’s 1991 “sign-dependence”; KR’s A3') requires preferences over consumption bundles to be the same for all bundles in a gain-loss regime but leaves them free to vary across regimes. (The general case is “variable sensitivity”.)

Figure 1. A set of regime maps with constant sensitivity
and the associated global map for alternative reference points



(a) (b)

Each regime map is defined for the entire space: different $\boldsymbol{r}$’s “switch on” different maps.

With $u(\boldsymbol{q},\boldsymbol{r})$ decreasing in $\boldsymbol{r}$, its *level* varies with $\boldsymbol{r}$ even though a regime’s map is constant.

Neoclassical Rationalization

Our analysis builds on Afriat's (1967), Diewert's (1973), and Varian's (1982) nonparametric analyses of the neoclassical case where preferences respond only to consumption levels.

They show that the demand behavior of a price-taking consumer can be rationalized by the maximization of a nonsatiated neoclassical utility function over levels if and only if the data satisfy the Generalized Axiom of Revealed Preference ("GARP").

DEFINITION 2: [Generalized Axiom of Revealed Preference ("GARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *implies* $\boldsymbol{p}_t \cdot \boldsymbol{q}_t \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_s$, *where R indicates that there is some sequence of observations* $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \ldots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i, \boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \ldots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

This reduces the theory's testable implications to a set of inequality restrictions on the observable, finite data, rather than shape restrictions on objects that are not directly observable like indifference curves, demand curves, or labor supply curves.

AFRIAT'S THEOREM: [Afriat 1967, Diewert 1973, Varian 1982.] The following statements are equivalent:

[A] There exists a utility function $u(\boldsymbol{q})$ *that is continuous, non-satiated, and concave, and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.

[B] There exist numbers $\{\boldsymbol{U}_t, \lambda_t > 0\}_{t=1,\dots,T}$ *such that*

(1) $U_s \leq U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q}_s - \boldsymbol{q}_t)$ *for all* $s, t \in \{1, \dots, T\}$

[C] The data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *satisfy GARP.*

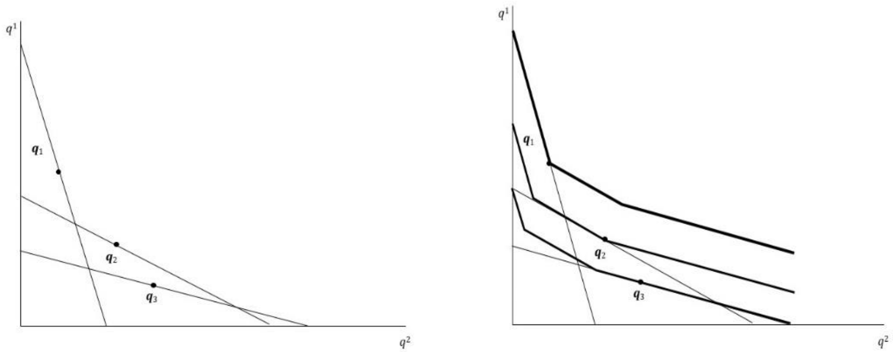
[D] There exists a non-satiated utility function $u(\boldsymbol{q})$ *that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.

In the proof of Afriat's Theorem, [B]'s inequalities (1) hold with equality for at least one $s \neq t$. This yields canonical "Afriat" rationalizing preferences and utility function.

DEFINITION 3: [Afriat preferences and utility function.] For data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *that satisfy GARP, or equivalently condition [B] of Afriat's Theorem, the Afriat preferences are those represented by the Afriat utility function* $u(\boldsymbol{q}) = min_{t\in\{1,\dots,T\}}\{U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q} - \boldsymbol{q}_t)\}$*, where the* U_t *and* λ_t *are those that satisfy the binding condition [B] inequalities (1) in Afriat's Theorem.*

Figure 2 illustrates Afriat preferences for a three-observation dataset that satisfies GARP. Figure 2a shows observations' budget sets and consumption bundles. Figure 2b shows the Afriat indifference map, whose marginal rates of substitution are determined by the budget lines. The Afriat utility function is piecewise linear, continuous, non-satiated, and concave.

Figure 2. Afriat preferences for data that satisfy GARP



(a) (b)

With finite data the Afriat preferences are only one of many possibilities for a rationalization (Varian 1982, Figure 3, Fact 4).

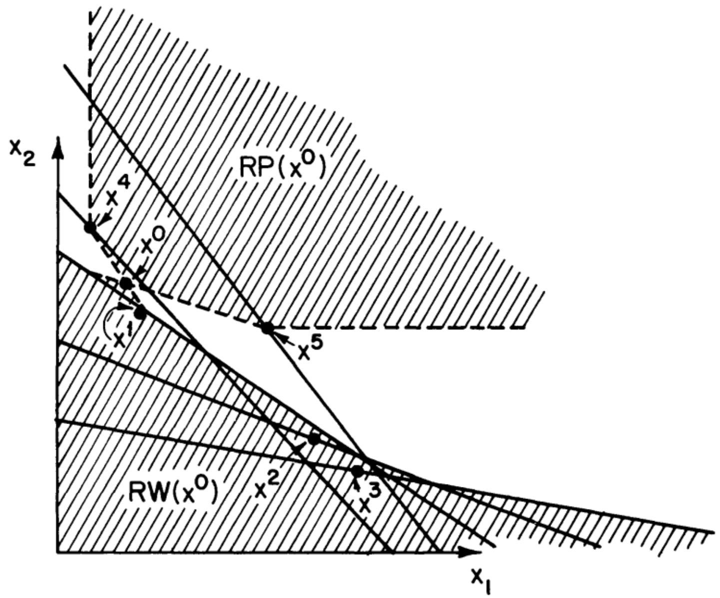


FIGURE 3.

However, their reference-dependent generalization plays a central role in our analysis.

Reference-dependent Rationalization

A neoclassical nonparametric analysis makes essential use of rationality.

We can adapt its methods because KR's theory of reference-dependent preferences maintains rationality in a larger domain.

Even so, our analysis raises new issues because the consumer chooses levels and changes bundled and priced together, and his choices can influence reference-dependent preferences by changing how consumption relates to the reference point.

The existence of a reference-dependent rationalization depends on two factors:

- Whether reference points are unmodelable (as assumed in Camerer et al.and Farber) or modelable (as assumed in KR and CM)

- Whether sensitivity is constant (as assumed in Farber, sometimes KR, and CM) or variable

We now consider these cases in turn.

DEFINITION 4: [Rationalization with unmodelable reference points.] Reference-dependent preferences, an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$, and hypothetical reference points $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$, rationalize the data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ if and only if $u(\boldsymbol{q}_t, \boldsymbol{r}_t) \geq u(\boldsymbol{q}, \boldsymbol{r}_t)$ for all $\boldsymbol{q}$ and $\boldsymbol{t}$ such that $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

PROPOSITION 1: [Rationalization with unmodelable reference points via preferences with variable or constant sensitivity.] For any data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ with unmodelable reference points, there exist reference-dependent preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ that are continuous, increasing in $\boldsymbol{q}$, and decreasing in $\boldsymbol{r}$, and a sequence of hypothetical reference points $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$, that rationalize the data.

The proof hypothesizes a reference point for each observation that coincides with its consumption bundle, and preferences that with those reference points put each observation’s bundle at the kink of an approximately Leontief indifference curve.

That the rationalization works entirely by varying reference points across observations shows that the parsimony of reference-dependent consumer theory depends on modeling (or observing) reference points.

Analyses that treat reference points as latent variables may be as heavily influenced by the constraints they impose in estimating reference points as by reference-dependence per se.

DEFINITION 5: [Rationalization with modelable reference points.] Reference-dependent preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *rationalize the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$ *with modelable reference points if and only if* $u(\boldsymbol{q}_t, \boldsymbol{r}_t) \geq u(\boldsymbol{q}, \boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

PROPOSITION 2: [Rationalization with modelable reference points via preferences with variable sensitivity.] For any data $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$ *with modelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q}, \boldsymbol{r})$ *that for each observation t and reference point* $\boldsymbol{r}_t$*, are continuous and strictly increasing in* $\boldsymbol{q}$ *and that rationalize the data, if and only if every subset of the data whose observations share the same reference point satisfies GARP.*

Proposition 2 shows that with modelable reference points and variable sensitivity, the hypothesis of reference-dependent preferences is nonparametrically refutable only via violations of GARP within subsets of observations that share the same reference point.

The proof adapts the standard proof of Afriat's Theorem, showing that variable sensitivity allows preferences that rationalize choices in any subsets of the data whose observations share the same reference point can be extended to rationalize the entire dataset.

Thus, reference-dependence with variable sensitivity adds nothing to the neoclassical model in the way of refutable implications.

Characterizing Reference-dependent Preferences That Satisfy Constant Sensitivity and Continuity

Assuming modelable reference points and constant sensitivity, we now characterize preferences and utility functions that are continuous even across gain-loss regimes.

Let $G(\boldsymbol{q},\boldsymbol{r})$ be a vector of binary numbers of length *K* with *k*th component 1 if $q^k \geq r^k$ and 0 otherwise.

The gain-loss regime indicator $I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) = 1$ if $\boldsymbol{g} = G(\boldsymbol{q},\boldsymbol{r})$ and 0 otherwise; and the gain-loss indicators $G_+^k(\boldsymbol{q},\boldsymbol{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\boldsymbol{q},\boldsymbol{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise.

PROPOSITION 3: [Preferences and utility functions with continuity and constant sensitivity.] Suppose there are K ≥ 2 goods, with reference-dependence active for all K goods, and that a reference-dependent preference ordering and an associated utility function have additively separable consumption utility and gain-loss utility components. Then the ordering satisfies constant sensitivity if and only if an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$ *can be written, for some consumption utility function* $U(\cdot)$ *and gain-loss regime utility functions* $V_{\boldsymbol{g}}(\cdot,\cdot)$ *and* $v_{\boldsymbol{g}}(\cdot)$*, as*

(5) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}).$$

Suppose further that the induced preferences over $\boldsymbol{q}$ *are differentiable in the interior of each regime, with marginal rates of substitution that differ across regimes throughout some open neighborhood of commodity space. Then the ordering satisfies constant sensitivity and continuity if and only if it is representable by a utility function* $u(\boldsymbol{q},\boldsymbol{r})$ *that can be written, for some consumption utility function* $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *doing the work of* $I_{\boldsymbol{g}}(\cdot,\cdot)$*), as*

(6) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

Conversely, any combination of induced regime preferences over $\boldsymbol{q}$ *is consistent with continuity and constant sensitivity for some gain-loss utility functions.*

Proposition 3 derives, from continuity, KR's and others' functional-structure assumption that gain-loss utility is determined, additively separably across goods, by the good-by-good differences between realized and reference consumption utilities. Informally,

(5) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}).$$

is continuous if and only if for any $\boldsymbol{q}$, $\boldsymbol{r}$, and *i* with $q^i = r^i$ and any gain-loss regimes $\boldsymbol{g}$ and $\boldsymbol{g}'$ that differ in the gain-loss status of good i

(7) $$V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big) = V_{\boldsymbol{g}'}\big(v_{\boldsymbol{g}'}(\boldsymbol{q}),\boldsymbol{r}\big).$$

A change in one good's consumption can change the gain-loss regime, which unless each regime's $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ is additively separable in the components of $\boldsymbol{q}$, can violate (7).

Unless gain-loss utility is determined by the good-by-good differences between realized and reference consumption utilities as in

(6) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}]$$

changing q^k and r^k with $r^k = q^k$ can violate (7).

Proposition 3 allows consumption utility, and thus the sum of consumption and gain-loss utility that determines consumer demand, *not* to be additively separable across goods.

Proposition 3 also allows the preferences over consumption bundles induced by consumption plus gain-loss utility to vary as freely as possible across gain-loss regimes while preserving continuity, thereby relaxing the knife-edge cross-regime links between marginal rates of substitution implied by KR's assumption that consumption and gain-loss utility have the same additively-separable-across-goods functional form. CM's Table 1:

TABLE 1—MARGINAL RATES OF SUBSTITUTION WITH REFERENCE-DEPENDENT PREFERENCES BY DOMAIN

	Hours gain ($H < H^r$)	Hours loss ($H > H^r$)
Income gain ($I > I^r$)	$-U_2'(H)/U_1'(I)$	$-[U_2'(H)/U_1'(I)][1-\eta+\eta\lambda]$
Income loss ($I < I^r$)	$-[U_2'(H)/U_1'(I)]/[1-\eta+\eta\lambda]$	$-U_2'(H)/U_1'(I)$

Thus Proposition 3 unbundles KR's functional-structure assumptions from their strong assumptions of additive separability and on marginal rates of substitution.

Proposition 3's unbundling could be used in a structural or nonparametric analysis.

Recall that with $u(\boldsymbol{q}, \boldsymbol{r})$ decreasing in $\boldsymbol{r}$**,** its *level* varies with $\boldsymbol{r}$ even within a gain-loss regime.

Proposition 3's equation (6)

(6) $$u(\boldsymbol{q}, \boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

assigns each gain-loss regime *g* a "loss cost" (the parts depending on $\boldsymbol{r}$**)**, incurred whenever any bundle $\boldsymbol{q}$ in regime *g* is chosen but otherwise independent of $\boldsymbol{q}$ within the regime.

A full rationalization with observable reference points and constant sensitivity depends on specifying loss costs as a function of $\boldsymbol{q}$ and $\boldsymbol{r}$**,** because they determine a consumer's incentive to "defect" from an observation's consumption bundle to some bundle in its budget set in another gain-loss regime, with possibly different preferences.

Although a consumer's choices do not reveal loss costs directly, Propositions 4 and 5 use Proposition 3's characterization to show that they can be inferred from the sum of consumption and gain-loss utility that rationalize his choices within each gain-loss regime.

Rationalization with Modelable Reference Points and Constant Sensitivity

Proposition 4 translates the requirements for a rationalization with modelable reference points and constant sensitivity into the language of Proposition 3, showing that necessary and sufficient conditions for a rationalization are the existence of continuous, strictly increasing consumption utility function and gain-loss good-by-good component utility functions that preclude, for any observation and consumption bundle:

● defections from its bundle to any bundle in the same gain-loss regime in its budget set

These conditions parallel the inequalities in Afriat's Theorem, while imposing the restriction that the good-by-good component utility functions are constant across gain-loss regimes.

(Thus, GARP for each regime's observations is necessary for a rationalization, but not sufficient even if the next group of conditions are satisfied.)

and

● defections from its bundle to any bundle in another regime in its budget set

Let $\Gamma(g; \boldsymbol{r})\}$ be the set of $\boldsymbol{q}$ in regime *g* for $\boldsymbol{r}$. Let $\Theta\left(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g\right) \equiv \{t \in \{1, \dots, T\} \,\middle|\, \boldsymbol{q}_t \in \Gamma(g; \boldsymbol{r}_t)\}$ be the set of t with $\boldsymbol{q}_t$ in regime *g* for $\boldsymbol{r}_t$.

PROPOSITION 4: [Rationalization with modelable reference points via preferences and utility functions with constant sensitivity.] Suppose that reference-dependent preferences and an associated utility function are defined over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function satisfies Proposition 3's (6). Consider data $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points. Then the statements [A] and [B] are equivalent:*

[A] There exists a continuous reference-dependent utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *that satisfies constant sensitivity; is strictly increasing in* $\boldsymbol{q}$ *and strictly decreasing in* $\boldsymbol{r}$*; and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$.

[B] Each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over consumption bundles, with continuous, strictly increasing consumption utility function $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$*, such that, for any regime* g *and any pair of observations* $\sigma, \tau \in \Theta\left(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g\right)$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *again doing the work of* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

(10)
$$U(\boldsymbol{q}_\sigma) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_+^k(q_\sigma^k) + G_-^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_-^k(q_\sigma^k)]$$
$$\leq \mathrm{U}(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)] + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau)$$

and for each observation $\{\boldsymbol{p}_\tau, \boldsymbol{q}_\tau, \boldsymbol{r}_\tau\}_{t=1,\dots,T}$ *with* $\tau \in \Theta\left(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g\right)$ *and each* $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ *with* $g' \neq g$ *for which* $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$*,*

(11)
$$U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_-^k(q^k) - v_-^k(r_\tau^k)\}]$$
$$\leq U(\boldsymbol{q}_{\boldsymbol{\tau}}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_{\boldsymbol{\tau}}, \boldsymbol{r}_{\boldsymbol{\tau}})\left\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\right\} + G_-^k(\boldsymbol{q}_{\boldsymbol{\tau}}, \boldsymbol{r}_{\boldsymbol{\tau}})\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].$$

The proof operationalizes conditions (11) by taking the rationalizing regime preferences represented by $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$, which satisfy (10), and using them to write the condition preventing defections from the bundle of observation $\tau \in \Theta\big(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g\big)$ in regime g to a bundle $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ in regime $g' \neq g$ for $\boldsymbol{r}_\tau$ with $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$:

$$u(\boldsymbol{q}, \boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau) \equiv U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_{\tau_\tau})\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\boldsymbol{r}_\tau)$$

$$(12) \equiv \{U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_{\boldsymbol{\tau}}) v_+^k(q^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_{\boldsymbol{\tau}}) v_-^k(q^k)]\} - \{U(\boldsymbol{r}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_{\boldsymbol{\tau}})\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_{\boldsymbol{\tau}}) v_-^k(r_\tau^k)]\}$$

$$\leq \{U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_{\boldsymbol{\tau}}, \boldsymbol{r}_{\boldsymbol{\tau}}) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_{\boldsymbol{\tau}}, \boldsymbol{r}_{\boldsymbol{\tau}}) v_-^k(q_\tau^k)]\} - \{U(\boldsymbol{r}_{\boldsymbol{\tau}}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_{\boldsymbol{\tau}}, \boldsymbol{r}_{\boldsymbol{\tau}})\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}_{\boldsymbol{\tau}}, \boldsymbol{r}_{\boldsymbol{\tau}}) v_-^k(r_\tau^k)]\}$$

(12)’s central inequality can then be rearranged to yield (11).

Figures 3 and 4 illustrate Proposition 4.

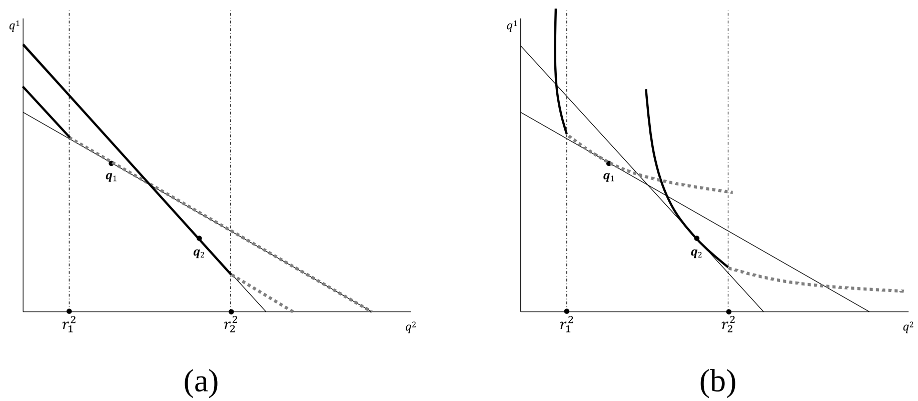
In each case the entire dataset violates GARP, with observation 1’s consumption bundle chosen in 1’s budget set over observation 2’s bundle, and vice versa.

In each case each regime’s single observation trivially satisfies GARP within its regime.

And in each case the observations’ reference points put their bundles in different gain-loss regimes, so constant sensitivity allows different preferences for each observation.

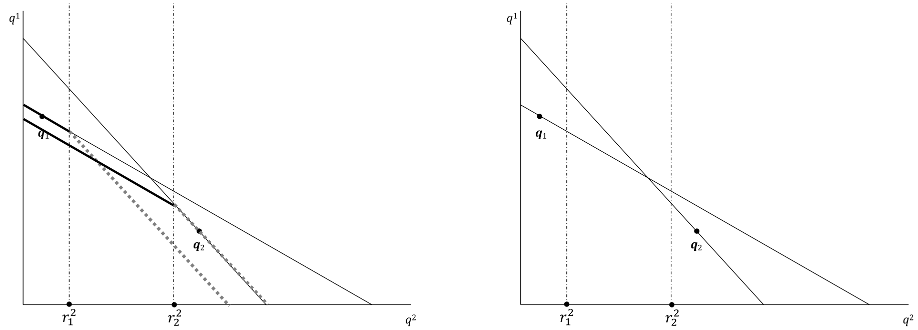
Figure 3a depicts Afriat rationalizing regime preferences and Figure 3b depicts non-Afriat rationalizing regime preferences. With condition (11) satisfied, a rationalization is possible.

Figure 3. Rationalizing data that violate GARP via reference-dependent preferences with constant sensitivity (solid lines for loss maps, dashed lines for gains maps)



In Figure 4a Afriat rationalizing regime preferences do not satisfy condition (11). Figure 4b shows more generally that there can be no choice of rationalizing regime preferences for which (11) is satisfied, so a rationalization, Afriat or not, is not possible.

Figure 4. Failing to rationalize data that violate GARP via reference-dependent preferences with constant sensitivity (solid lines for loss maps, dashed lines for gains maps)



(a) (b)

The difference between Figure 3's and Figure 4's examples can be understood in terms of loss aversion. The change in Afriat preferences across regimes in Figure 3a is consistent with loss aversion, but not the change in Figure 4a.

Online Appendix A shows that if the rationalizing regime preferences (Afriat or not) satisfy loss aversion, Proposition 4's conditions (11) are automatically satisfied; but that loss aversion is not quite necessary for a rationalization.

As already noted, Proposition 4's necessary and sufficient conditions for a rationalization are not directly applicable because with finite data there is a normally range of preferences that rationalize a gain-loss regime's data (Varian 1982, Fact 4) and Proposition 4's condition [B] rests on an unspecified choice among those rationalizing regime preferences.

Finding a choice that precludes beneficial cross-regime defections involves complex trade-offs, because preferences that reduce the gain from defecting *from* bundles in a regime increase the gain from defecting *to* bundles in the regime.

Proposition 5 uses Proposition 4's conditions to derive directly applicable sufficient conditions by specializing the choice of rationalizing gain-loss regime utilities to a reference-dependent generalization of Definition 3's Afriat regime utilities.

Recall that $\Gamma(g;\boldsymbol{r})\}$ is the set of $\boldsymbol{q}$ in regime *g* for $\boldsymbol{r}$. And $\Theta\left(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g\right) \equiv \{t \in \{1,\ldots,T\} \,\middle|\, \boldsymbol{q}_t \in \Gamma(g;\boldsymbol{r}_t)\}$ is the set of t with $\boldsymbol{q}_t$ in regime *g* for $\boldsymbol{r}_t$.

PROPOSITION 5: [Sufficient conditions for rationalization with modelable reference points, via reference-dependent preferences and utility function with constant sensitivity and continuity.] The following conditions are sufficient for the existence of continuous reference-dependent preferences and utility function with constant sensitivity $u(\boldsymbol{q}, \boldsymbol{r})$ that rationalize data with modelable reference points $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$: There exist numbers U_t, v_{t+}^k, v_{t-}^k, and $\lambda_t > 0$ for each $k = 1,\ldots,K$ and $t = 1,\ldots,T$ such that:

[A] For any gain-loss regime g and any pair of observations $\sigma, \tau \in \Theta\left(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g\right)$ (with the indicator functions $G_+^k(\cdot,\cdot)$ and $G_-^k(\cdot,\cdot)$ again doing the work of $I_{\boldsymbol{g}}(\cdot,\cdot)$),

(13)
$$U_\sigma + \sum_k [G_+^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_{\sigma+}^k + G_-^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_{\sigma-}^k]$$
$$\leq U_\tau + \sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_{\tau+}^k + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_{\tau-}^k] + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau).$$

[B] For observations σ, τ, $q_\sigma^k \geq q_\tau^k$ for k = 1,…, K, $U_\sigma \geq U_\tau$; and for observations σ, τ and any k = 1,…, K, $q_\sigma^k \geq q_\tau^k$, $v_{\sigma+}^k \geq v_{\tau+}^k$, and $v_{\sigma-}^k \geq v_{\tau-}^k$.

[C] For any pair of regimes g and $g' \neq g$, observation $\tau \in \Theta\left(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}; g\right)$, and bundle $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ for which $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$,

$$\begin{aligned}
&min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g')}\left\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\right\} \\
(14) \qquad &-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g')}\left\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\right\} \\
&\leq min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g)}\left\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_\tau - \boldsymbol{q}_\rho)\right\} \\
&-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g)}\left\{U_\rho + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\right\}.
\end{aligned}$$

Proposition 5's conditions (14) precluding beneficial defections across gain-loss regimes again requires linking Proposition 3's loss costs to things that can be estimated from the data, not only at particular points but as functions of $\boldsymbol{r}$. This is done just as in Proposition 4, but now using the Afriat rationalizing regime utilities.

Although continuity requires gain-loss utility to be additively separable across goods by Proposition 3, Proposition 5 does not require consumption utility, or therefore the sum that determines consumer demand, to be additively separable across goods as KR assumed.

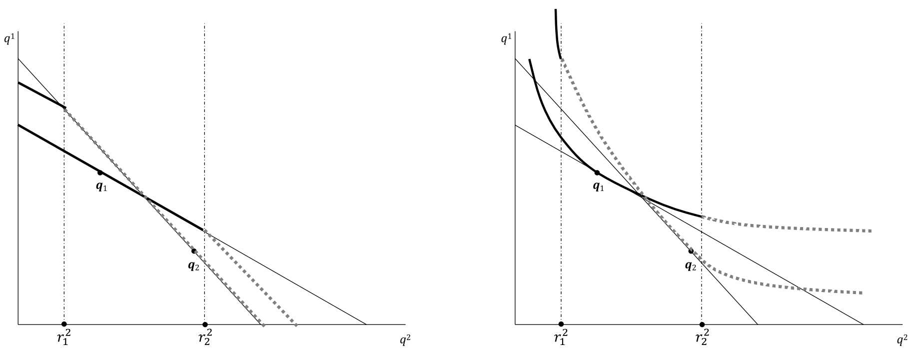
Neither does Proposition 5 require KR's constant-sensitivity cross-regime links between marginal rates of substitution (CM's Table 1).

Both generalizations appear to be empirically important.

Proposition 5 does rely on the choice of Afriat rationalizing regime utility functions. As other choices might also suffice, its sufficient conditions are not necessary.

For example, the Afriat regime preferences in Figure 5a do not yield a rationalization but the non-Afriat regime preferences in Figure 5b do.

Figure 5. A rationalization may require non-Afriat rationalizing regime preferences (solid lines for the loss map, dashed for the gain map)



(a) (b)

Although Proposition 5's sufficient conditions are not necessary, Mas-Colell's (1978) and Forges and Minelli's (2009) analyses of the neoclassical case suggest that in the limit as the data become rich, so each regime's range of convexified rationalizing regime preferences collapses on its Afriat preferences, those conditions are asymptotically necessary.

EMPIRICAL ILLUSTRATION: FARBER'S AND CM'S CABDRIVERS REVISITED

We now illustrate the empirical potential of our characterization of continuous reference-dependent preferences with modelable reference points and constant sensitivity.

We use Proposition 5's conditions to revisit Farber's (2005, 2008) and CM's (2011) analyses of cabdrivers' labor supply nonparametrically.

DATA

Like CM (2011) we use Farber's (2005, 2008) original dataset, with two changes.

- We use the NY/NJ urban CPI to control for price level changes in the sample period.

- We replace Farber's hourly wage variable, income per hour spent working, where working time is defined as the sum of time spent driving with a fare-paying passenger plus time spent waiting for the next passenger, with earnings per hour spent driving.

  With Farber's wage variable, shift-to-shift wage variation makes each observation's budget line pivot around its zero-earnings end; thus a driver's budget lines never cross, he satisfies GARP trivially, and a nonparametric analysis gives only a meaningless recapitulation of his data.

  With our wage variable, waiting time is a fixed cost, varying exogenously from shift to shift, with weather, the frequency of customers, etc.; thus a driver's budget lines cross frequently (Appendix B, Figure B.1), which is essential for a meaningful analysis.

We use static sample proxies like CM’s for KR’s rational-expectations reference points.

We follow CM’s econometric analysis closely, with these exceptions:

- We estimate preferences for each driver separately, allowing full preference heterogeneity (our theoretical analysis also covers the case of homogeneity)
- We compare alternatives to CM’s rational-expectations reference points: three expectations-based and three recent experience-based, crossed with reference-dependence in hours alone, earnings alone, or both earnings and hours, both unconditional and conditioned on weather (rain, snow, or dry) or time of day (day or night) (18 alternative reference-point models per driver)
- We compare the neoclassical model with models in which reference-dependence is with respect to hours only, earnings only, or both hours and earnings; and models that do or do not impose additive separability across goods
- We include 6 drivers Farber and CM excluded due to small (≤ 10) sample sizes

Proposition 5's sufficient conditions for a reference-dependent rationalization in this case suggest a simple nonparametric estimation procedure, which we apply driver by driver and model by model (with modifications when GARP is not always satisfied):

(i) Use the observations' modelled reference points to sort their consumption bundles into gain-loss regimes.

(ii) Pooling the data from all regimes, use linear programming to find Afriat numbers U_t, v_{t+}^k, v_{t-}^k, and $\lambda_t > 0$ for each $k = 1, \dots, K$ and $t = 1, \dots, T$ that satisfy [A]'s Afriat inequalities (13).

(iii) Use the fact that for each observation in a regime, (13) can hold with equality for at least one other observation in the regime, to choose numbers so that for observation *t* in regime *g*, the rationalizing Afriat utilities are given by

$$U_t = u^g(\boldsymbol{q}_t, \boldsymbol{r}_t) \equiv min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)} \left\{ U_\rho + \sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_t - \boldsymbol{q}_\rho) \right\}$$

as in (15) in the proof of Proposition 5.

(iv) Use (ii)'s Afriat numbers U_t, v_{t+}^k, and v_{t-}^k to check that [B]'s monotonicity restrictions are satisfied.

(v) Use (iii)'s rationalizing Afriat utilities to check, regime by regime and observation by observation, that [C]'s conditions (14) are satisfied by scanning along the budget surface.

This procedure inherits most of the simplicity and computational tractability of Diewert's and Varian's linear-programming methods for the neoclassical case.

Although Proposition 5's no-defection conditions (14) involve the entire Afriat regime utility functions, those functions are finitely parameterized by the U_t^g and λ_t^g from step (ii) and the $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}$. Thus the procedure involves a finite number of inequalities in a finite number of variables, and should be computationally feasible even in large datasets.

In our two-good illustration, GARP (Definition 3) reduces to the Weak Axiom of Revealed Preference, which is then necessary and sufficient for a neoclassical rationalization:

DEFINITION 6: [Weak Axiom of Revealed Preference ("WARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *and* $\boldsymbol{q}_s \neq \boldsymbol{q}_t$ *implies not* $\boldsymbol{q_t} R \boldsymbol{q_s}$*, where R indicates that there is some sequence of observations* $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \ldots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i, \boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \ldots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

MODEL COMPARISONS: PASS RATES AND SELTEN MEASURES

We compare models using Beatty and Crawford's (2011, pp. 2786-87) proximity-based variant of Selten and Krischker's (1983) and Selten's (1991) measure of predictive success, which levels the playing field across models of varying flexibility and sample size.

For a given model, Selten and Krischker's original measure is $m(\pi^i, a^i) \equiv \pi^i - a^i$, the difference between driver *i*'s pass rate π^i, the proportion of observations that fit the model exactly, and the "area" a^i, the probability that random data would fit the model exactly.

As $m \to 1$, a model's restrictions become tighter and yet behavior satisfies them: a highly successful model.

As $m \to -1$, the restrictions become vacuous and yet behavior fails to satisfy them: a pathologically bad model.

As $m \to 0$, the restrictions approach random compliance: a harmless but useless model.

Beatty and Crawford’s proximity-based variant of the pass rate replaces Selten and Krischker’s pass rate π^i with a proximity: one minus the Euclidean distance, rescaled as a proportion of the maximum possible distance, between the data and a model’s target area.

This measure, like π^i, lies in [0, 1], with higher values for more successful models.

Beatty and Crawford’s variant of the Selten measure is the difference between their rescaled proximity and Selten and Krischker’s original area a^i.

Like Selten’s measure, this lies in [-1, 1], with higher values for more successful models.

From now on we use “Selten measure” for Beatty and Crawford’s proximity-based variant.

Given Propositions 5's gap between the sufficient and necessary conditions for a reference-dependent rationalization, we bound reference-dependent models' (proximity-based) pass rates and Selten measures as follows.

No bounds are needed for a neoclassical model because Afriat's Theorem shows that GARP is necessary and sufficient for a rationalization without regard to Varian's Fact 4.

Imposing Proposition 5's within-regime conditions [A] ((13)) and monotonicity conditions [B], but not its cross-regime no-defection conditions [C] ((14)), yields an approximate upper bound on the pass rate.

"Approximate" because Proposition 5's within-regime conditions [A] assume the Afriat regime utilities and are sufficient but not necessary, so the pass rate could be higher.

Imposing all of Proposition 5's conditions yields an approximate lower bound on the pass rate, which could again be higher for the same reason as above.

The approximate lower and upper bounds on the Selten measures follow similarly.

In each case the one-sided approximation suffices for our purposes.

ADDITIVE SEPARABILITY ACROSS GOODS

In the literature on reference-dependent models, additive separability across goods has almost always been assumed.

It has also been assumed in neoclassical models of cabdrivers' labor supply, but not for all other applications of neoclassical demand theory.

Figures 6-9 give the empirical cumulative distribution functions, aggregated across reference point models and drivers, of proximity-based pass rates and Selten measures for neoclassical and reference-dependent models, with or without additive separability.

For neoclassical (Figure 6) or reference-dependent (Figure 8) models, the pass rate increases a great deal when additive separability across goods is relaxed.

In each case (Figures 7 and 9) the Selten measures show that the increase justifies the extra flexibility.

Accordingly, from now on we relax additive separability across goods.

Figure 6: Empirical CDFs of Pass Rates for Neoclassical Models

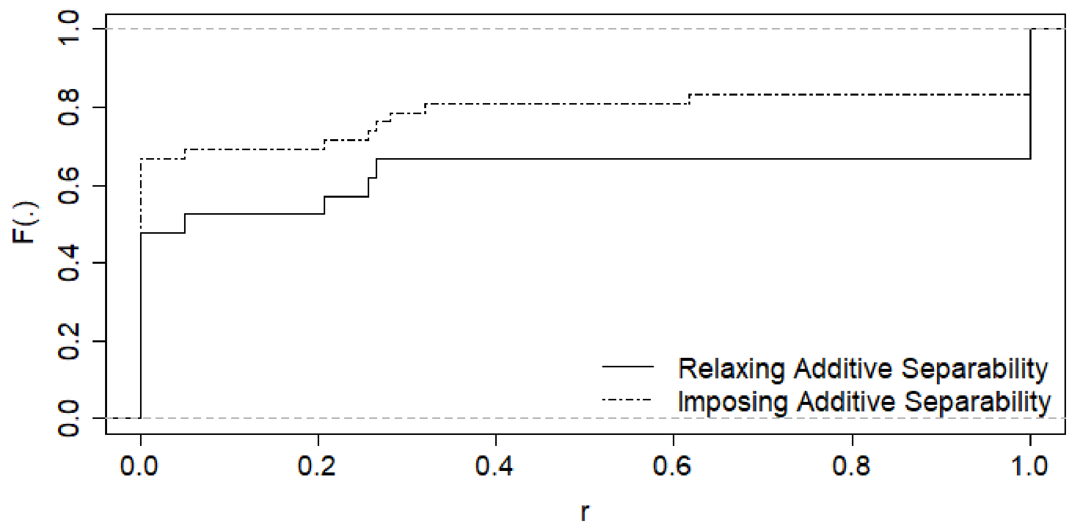


Figure 7: Empirical CDFs of Selten Measures for Neoclassical Models

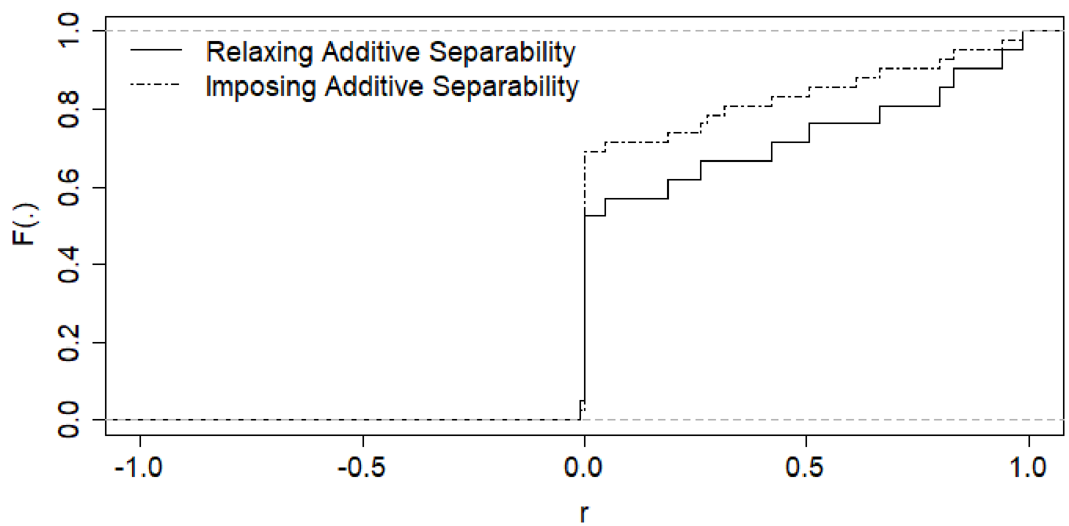


Figure 8: Empirical CDFs of Pass Rates for Reference-dependent Models

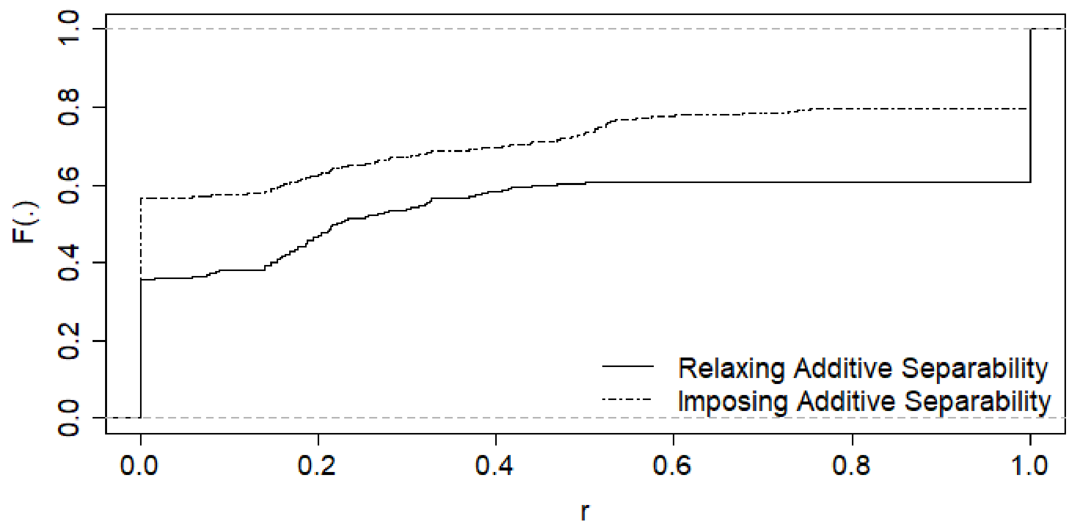
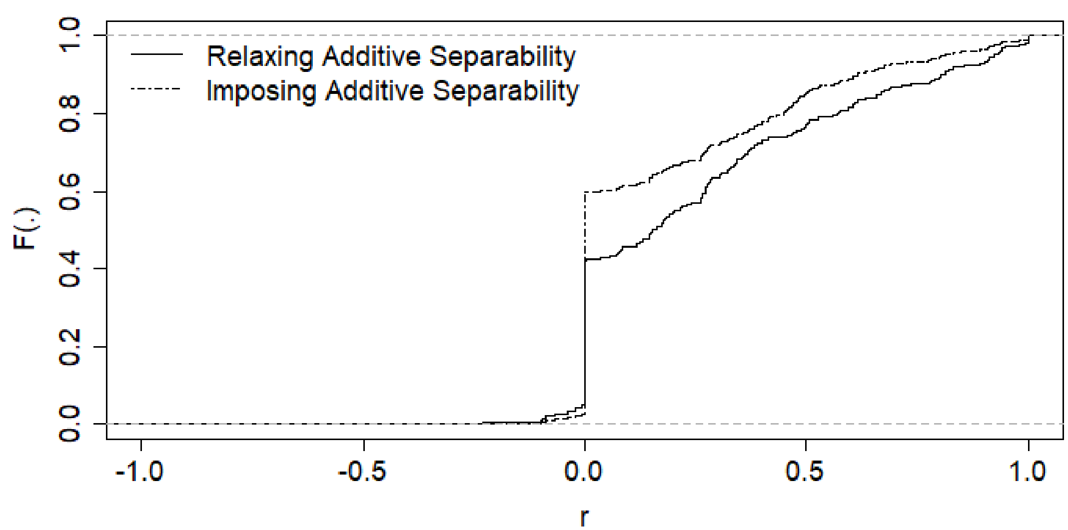


Figure 9: Empirical CDFs of Selten Measures for Reference-dependent Models



REFERENCE-POINT MODELS

Figures C.1 and C.2 compare the empirical CDFs for different kinds of reference-point model (expectations- or experience-based, with various conditionings).

Figure C.1: Empirical CDFs of Proximities for Different Kinds of Reference-dependence

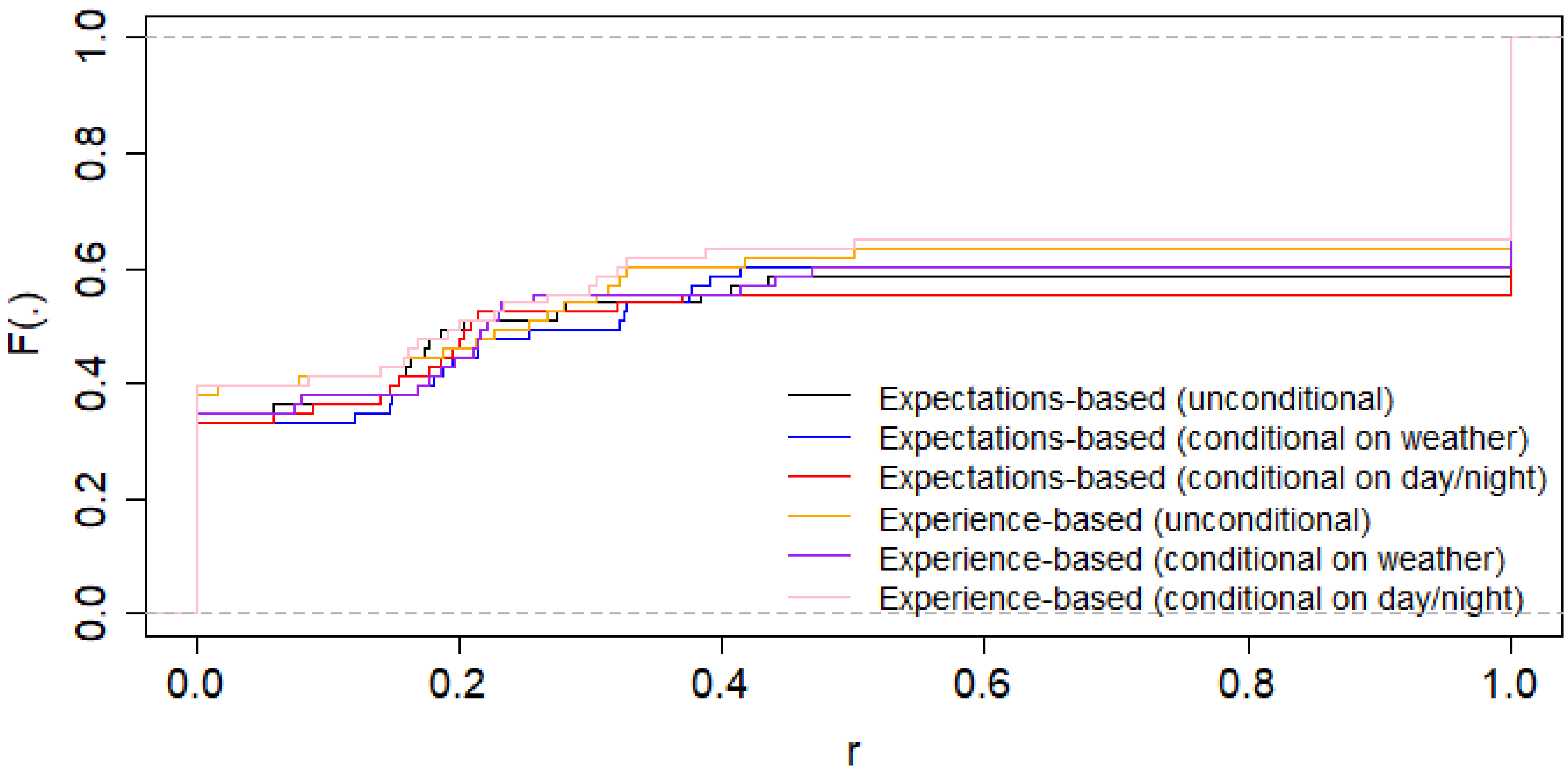
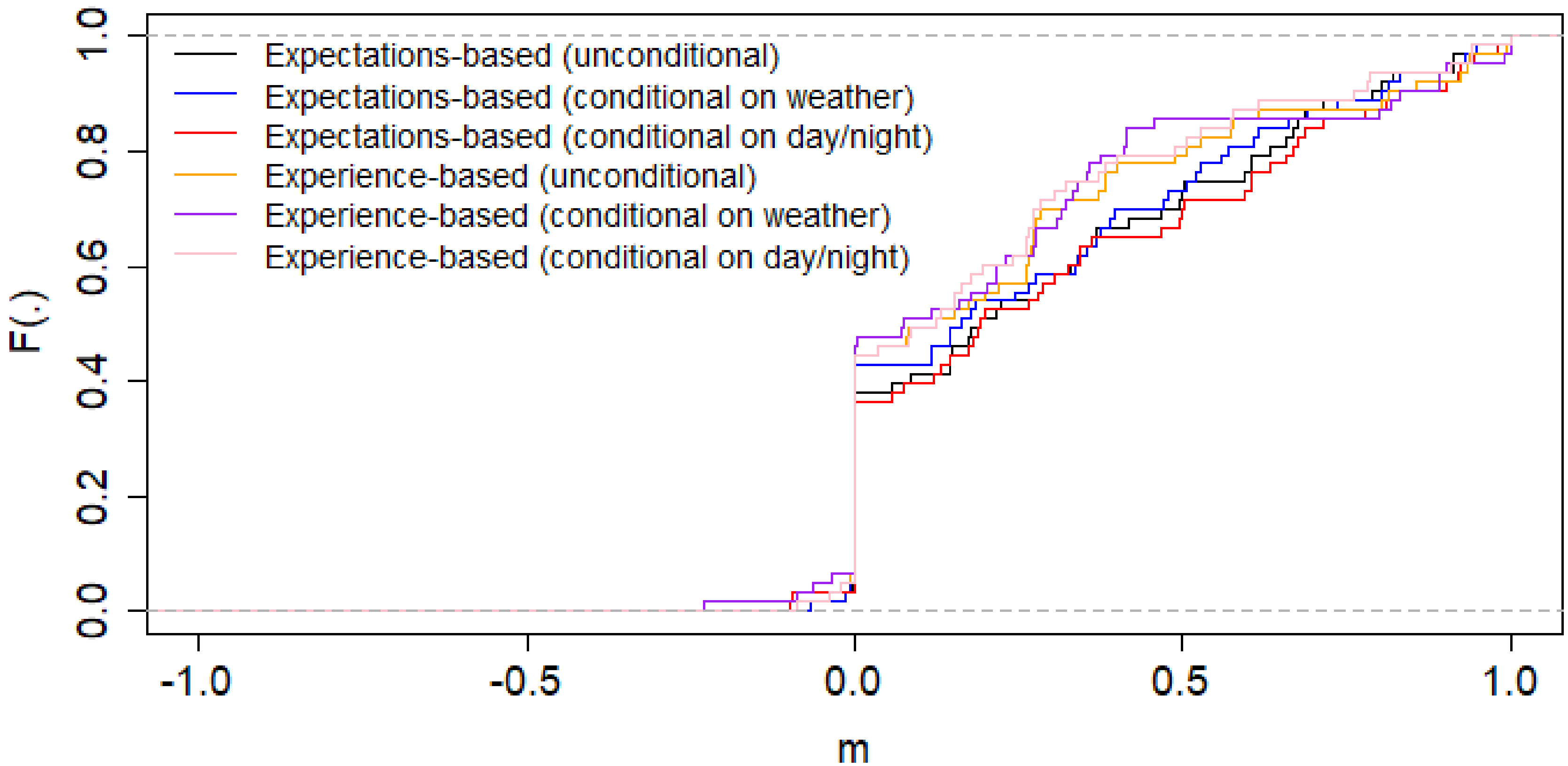


Figure C.2: Empirical CDFs of Selten Measures for Different Kinds of Reference-dependence



Figures C.3 and C.4 compare the empirical CDFs for different forms of reference-dependent model (hours only, earnings only, or both hours and earnings).

Figure C.3: Empirical CDFs of Proximities
for Different Forms of Reference-dependence

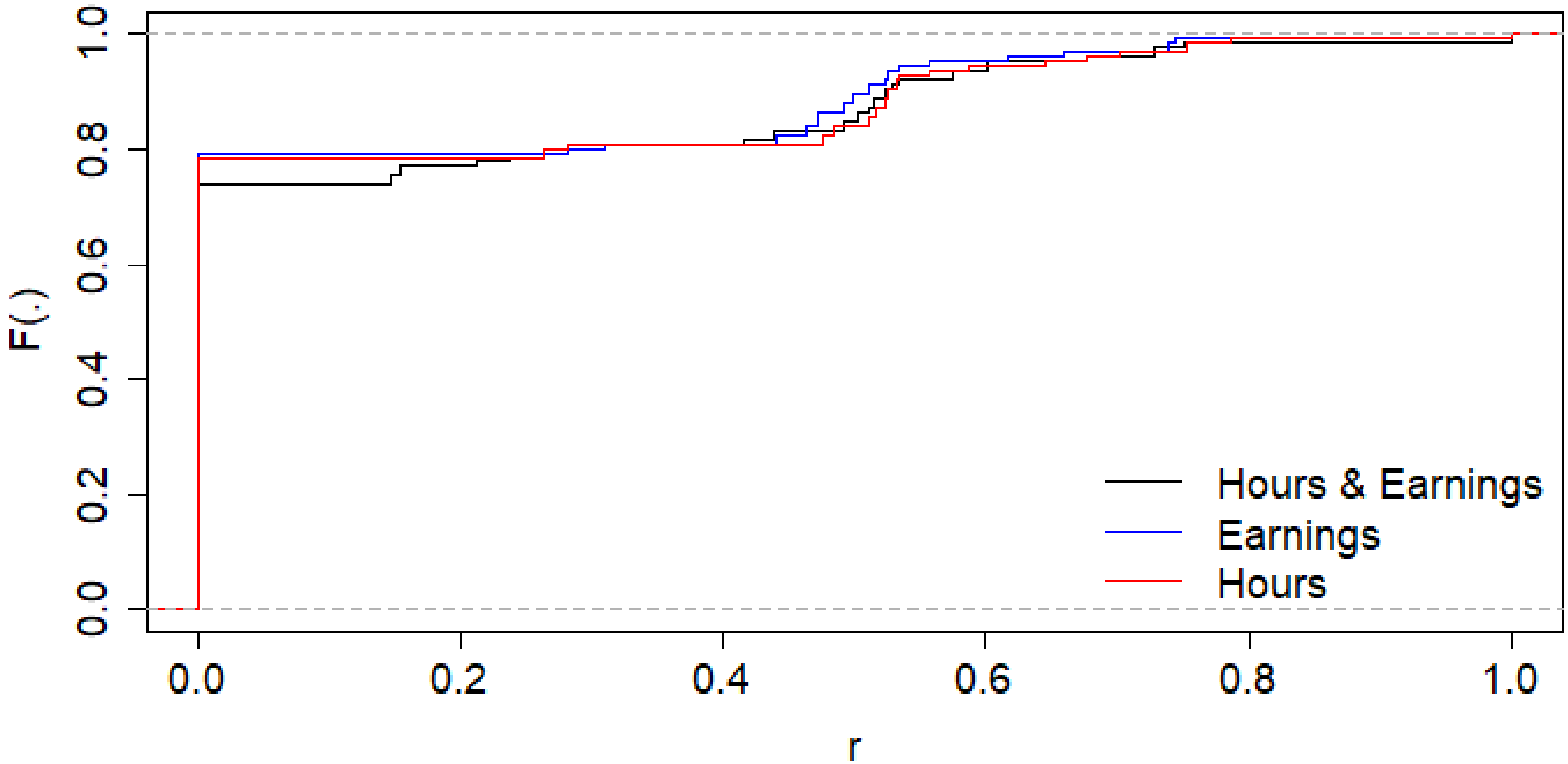
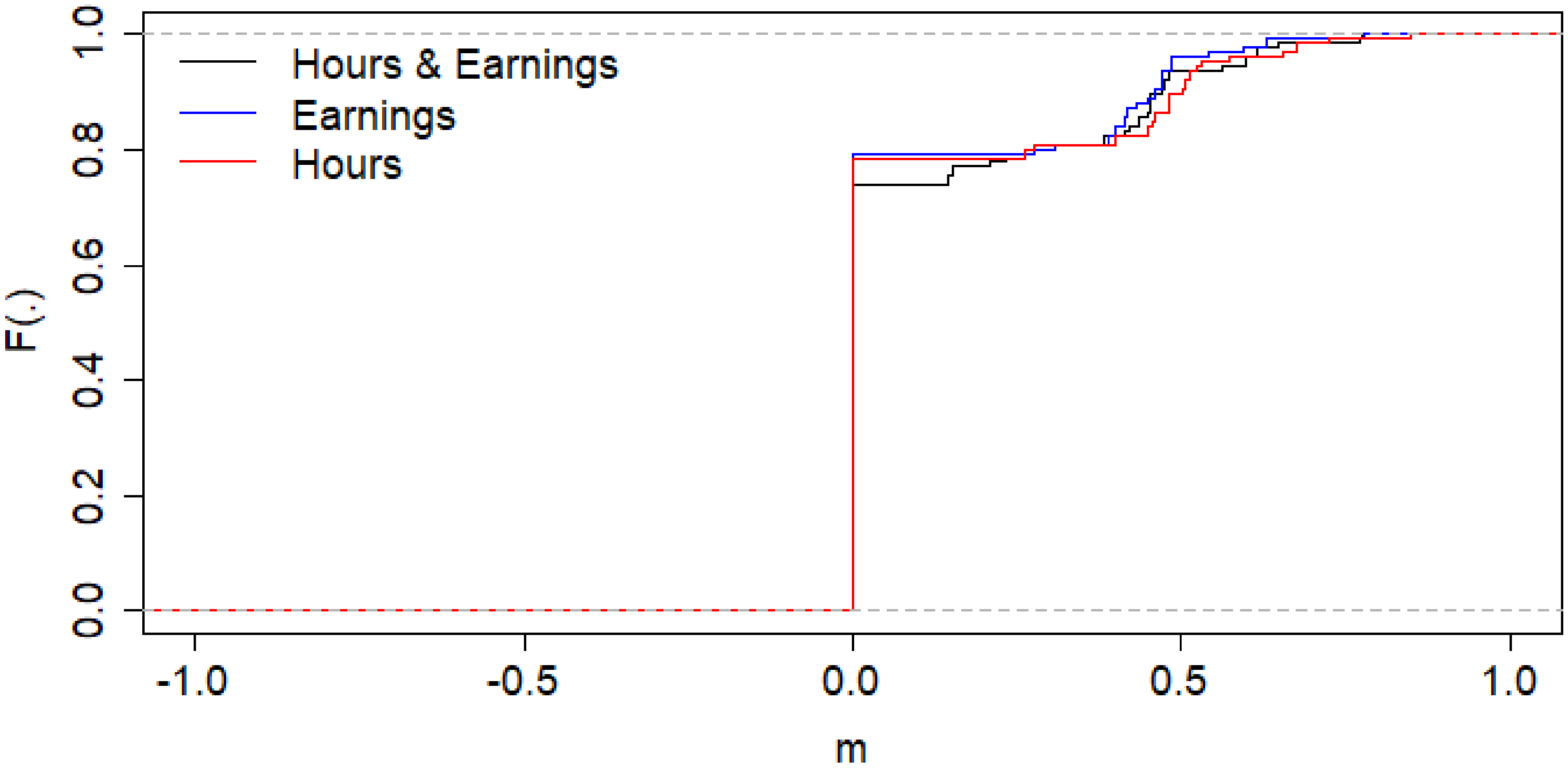


Figure C.4: Empirical CDFs of Selten Measures
for Different Forms of Reference-dependence



Figures C.2’s and C.4’s Selten measures show that in these data there are comparatively small differences among models’ kinds and forms of reference-dependence.

Expectations-based models usually have higher Selten measures than experience-based models, and unconditioned expectations-based models have measures almost as high as conditioned ones, though expectations-based models that are conditioned on day/night usually have even higher Selten measures.

Expectations-based models with hours- and earnings-targeting have measures approximately as high as such models with only hours-targeting and somewhat higher measures than such models with only earnings-targeting

NEOCLASSICAL VERSUS REFERENCE-DEPENDENT MODELS

We close by comparing neoclassical and reference-dependent models.

As the previous comparisons suggest, we focus on models that relax additive separability across goods and on models that do not condition on weather or day/night.

We report driver-by-driver results for expectations- and experience-based models with reference-dependence in both hours and earnings, earnings only, and hours only.

We focus on expectations-based models with reference-dependence in both hours and earnings, but the results differ comparatively little for other models.

First, Figures 10 and 11 compare empirical cumulative distribution functions for all drivers together, of pass rates and Selten measures for neoclassical and reference-dependent models. In the aggregate, neither kind of model is clearly superior.

Figure 10: Empirical CDFs of Proximities for Neoclassical and Reference-dependent Models

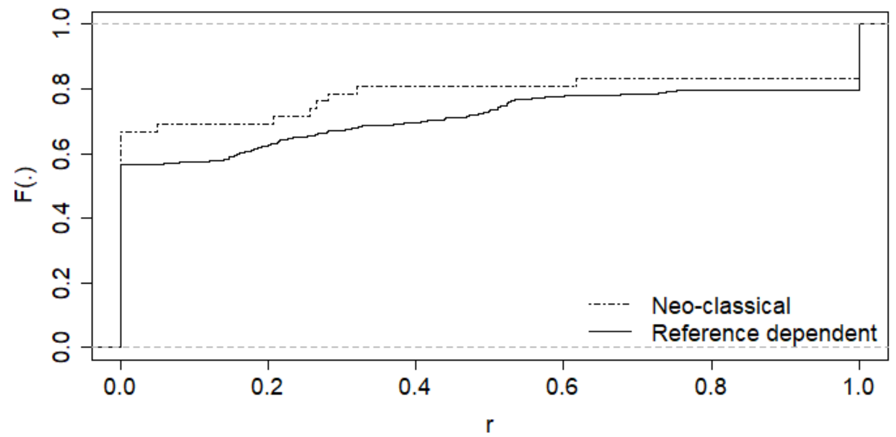
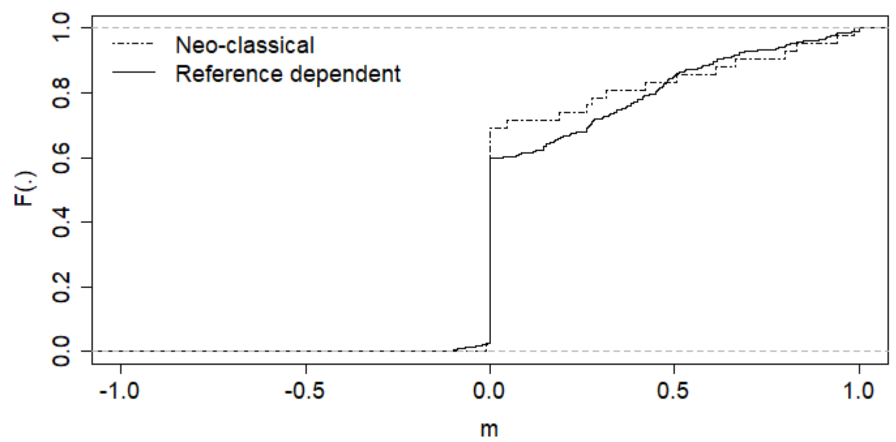


Figure 11: Empirical CDFs of Selten Measures for Neoclassical and Reference-dependent Models



Figures 12-15 give driver by driver plots for neoclassical and expectations-based and experience-based reference-dependent models' proximities and Selten measures.

Each figure has separate plots for different forms of reference-dependence, with a separate "spoke" for each driver.

Figures 12's and 14's proximity plots are centered at -0.25, for clarity a tick below the lowest possible value of 0; with outer rims at the highest possible value of 1.

The solid lines trace proximities for the neoclassical model.

The shaded areas depict Section III.C's approximate bounds on the proximities for the reference-dependent models.

Figures 13's and 15's Selten measure plots are centered at the lowest possible value of -1, with outer rims at the highest possible value of 1.

The solid lines trace measures for the neoclassical model.

Figure 12. Proximities for Alternative Forms of Expectations-based Models

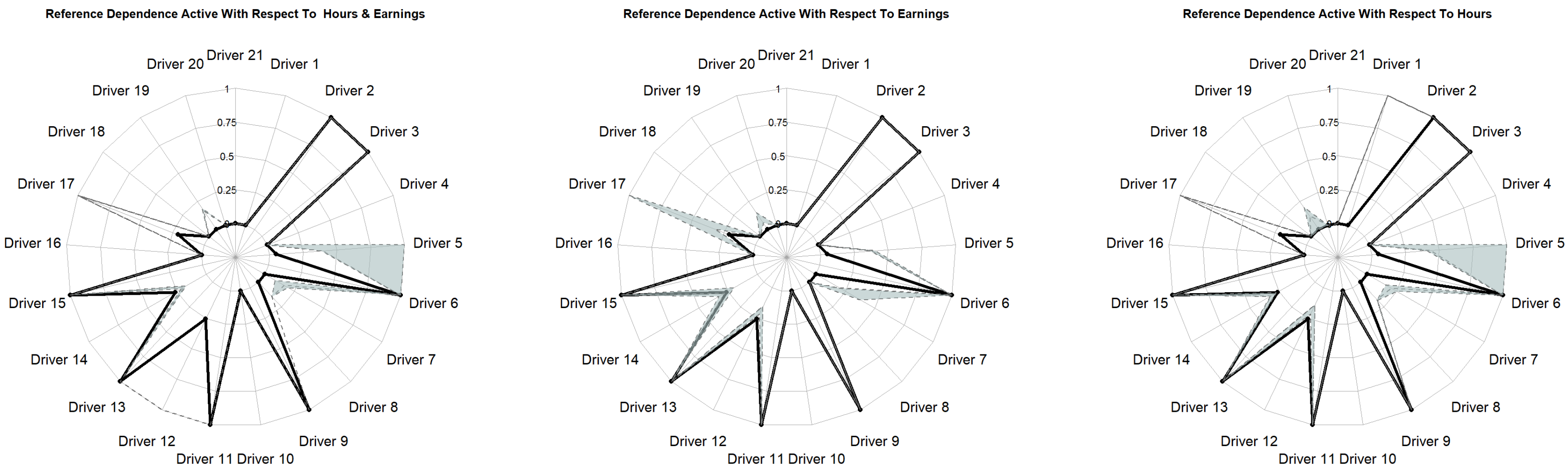


Figure 13. Selten Measures for Alternative Forms of Expectations-based Models

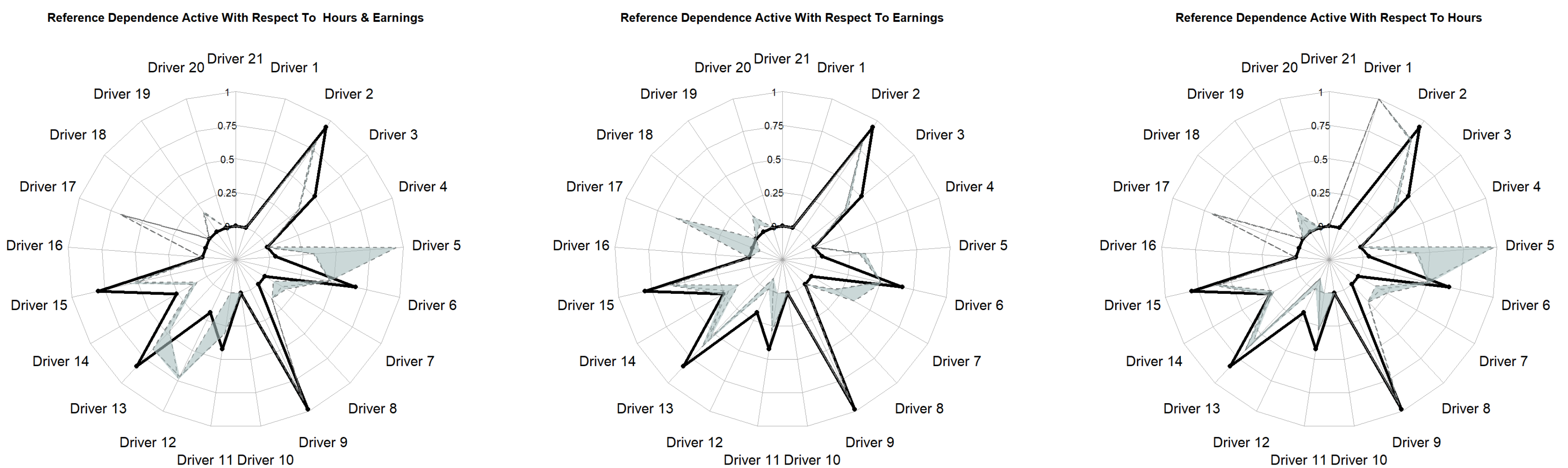


Figure 14. Proximities for Alternative Forms of Experience-based Models

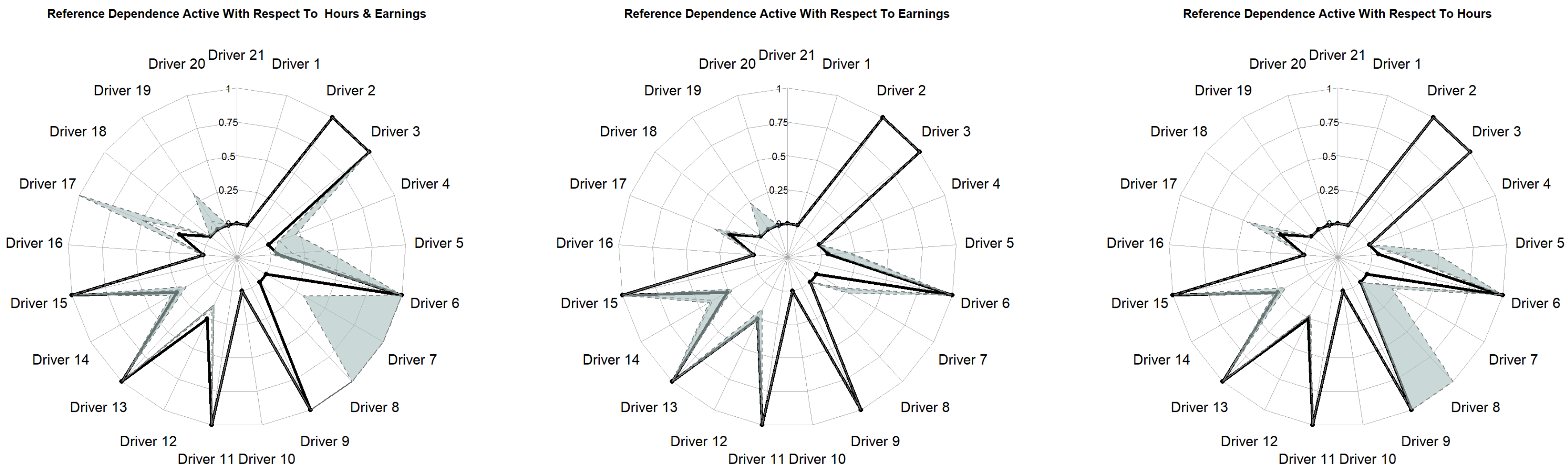
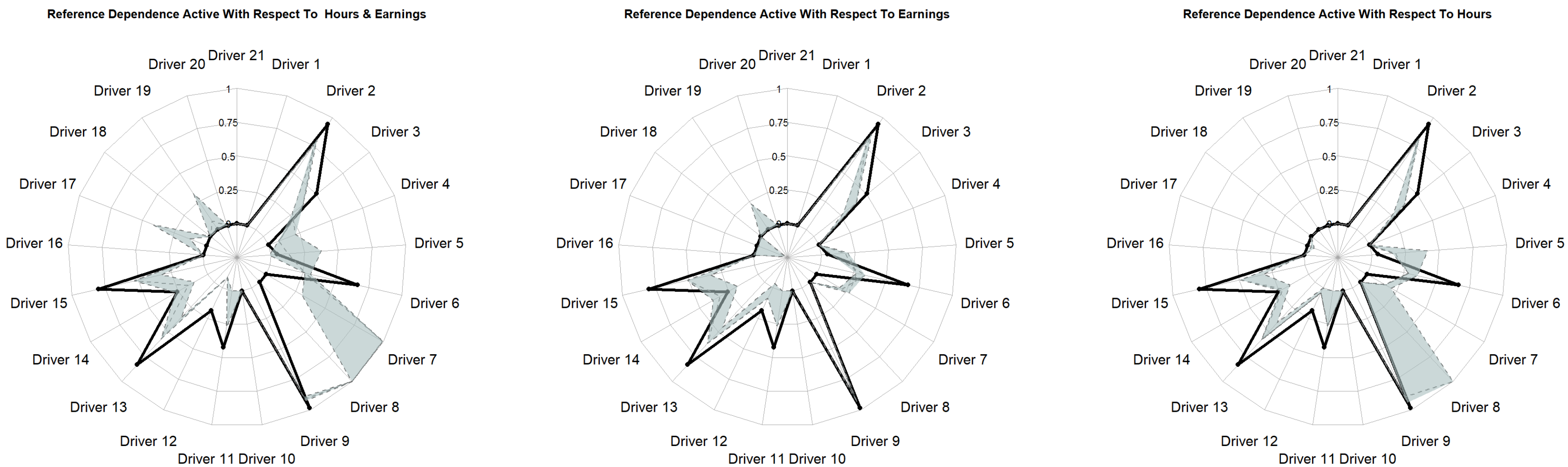


Figure 15. Selten Measures for Alternative Forms of Experience-based Models



Focus on (left-most) models with reference-dependence in both hours and earnings.

In Figure 13, among the full 21 drivers, the expectations-based reference-dependent model has the same bounded Selten measure as the neoclassical model (thus possibly higher, Section III.C) for seven drivers: 1, 4, 10, 16, 18, 20, and 21; an unambiguously higher measure for six drivers: 5, 7, 8, 12, 17, and 19; and an unambiguously lower measure for eight drivers: 2, 3, 6, 9, 11, 13, 14, and 15.

Similarly, in Figure 15, the experience-based reference-dependent model has the same (possibly higher) bounded Selten measure as the neoclassical model for six drivers: 1, 10, 16, 18, 20, and 21; a higher measure for four: 4, 8, 17, and 19; a lower measure for nine: 2, 3, 6, 9, 11, 12, 13, 14, and 15; and ambiguous bounds for two: 5 and 7.

However, not all drivers' comparisons are equally informative.

Consider the expectations-based model with reference-dependence in both hours and earnings.

With our CPI adjustment, all but one of the six drivers Farber and CM excluded due to small (≤ 10) sample sizes (3, 6, 11, 13, 15, and 17) has an exact neoclassical fit, and the neoclassical model has a higher Selten measure than its more flexible reference-dependent counterpart.

This is good news for the neoclassical model, but might only reflect overfitting.

For seven other drivers (1, 4, 10, 16, 18, 20, and 21) the sample sizes were too large for us to estimate the set of sets of observations that fit exactly. So for them the proximities are set to 0 for both models and the neoclassical model again has a higher measure; but that does not truly favor the neoclassical over the reference-dependent model.

For the eight remaining drivers (2, 5, 7, 8, 9, 12, 14, and 19), the expectations-based model with reference-dependence in hours and earnings has a higher measure for five (5, 7, 8, 12, and 19) and the neoclassical model has a higher measure for three (2, 9, 14).

Similarly, the experience-based model with reference-dependence in hours and earnings has a higher Selten measure for four drivers (7, 8, 14, and 19) and the neoclassical model has a higher measure for four (2, 5, 9, and 12).

Thus, for many of Farber's drivers who violate rationality for a neoclassical model, a reference-dependent model gives a coherent rationality-based account of their choices.

Judging by Selten measures, for many of those drivers the reference-dependent model is more parsimonious, despite its greater flexibility.