Economics 206 Problem Set 2 **Winter 2007**
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This problem set, which is optional, covers the material in the second half of the course. At the end is the flexible essay question that will be part of the second mini-exam. The problems are meant to help you think about some of the issues we discuss in lectures, and also as practice for the second take-home mini-exam, which will include at least one of the problems in addition to the essay question. Although once the second mini-exam is posted (by 4 p.m. Wednesday, March 14), you must work on it individually, without consulting anyone but me, before it is posted you are encouraged to work together and/or consult me as much as you wish.

Choice under uncertainty (or certainty)

1. Suppose that there are two states of the world, s_1 and s_2, and that an individual who knows their probabilities, p_1 and p_2, chooses among state-contingent consumption bundles (x_1, x_2), facing budget sets with varying prices as in Figure 1 (the dotted line is the 45-degree certainty line). Figure 2 graphs $x_1/(x_1+x_2)$ against ln (p_1/p_2) for experimental subjects who faced many such choices, with varying prices. (The paper is at http://socrates.berkeley.edu/~kariv/CFGK_III.pdf, but it won't help you answer this question.) Subject A: ID 304 always chooses $x_1 = x_2$ without regard to the price ratio, while Subject C: ID 307 chooses $x_1 = x_2$ for price ratios very near one, but otherwise puts "all his eggs" in the basket for which eggs are cheaper. (The other three subjects have demands that vary smoothly with prices.) This question will focus entirely on Subjects A and C.

(a) Can A's and C's choice behavior be rationalized by maximizing a preference function over (x_1, x_2) bundles?

(b) Graph an indifference map for A, and one for C, that generates his observed choice pattern.

(c) Can your indifference maps for A and C from part (b) be generated by maximizing differentiable, state-independent von Neumann-Morgentern utility functions? Why or why not? (Allowing non-differentiable utility functions doesn't change the answer to this, but you are not asked to show that.)

(Hint: Recall problem 15 from Problem set 1:
15. There are two states of the world, 1 and 2, and a single consumption good; the state-contingent consumption vector e ≡ (e_1, e_2) represents consumption of e_i units of the consumption good if state i occurs. The probability of state i is p_i. Suppose that an individual chooses among state-contingent consumption vectors to maximize the expectation of the state-independent von Neumann—Morgenstern utility function u(·).
(a) Write the equation of a typical indifference curve for the individual.
(b) Derive an expression for $MRS_{12}(e_1, e_2)$, the individual's marginal rate of substitution between consumption in states 1 and 2 at consumption vector (e_1, e_2).
(c) Suppose that $e^a \equiv (\underline{e}_1, \underline{e}_2)$, $e^b \equiv (\underline{e}_1, \bar{e}_2)$, $e^c \equiv (\bar{e}_1, \underline{e}_2)$, and $e^d \equiv (\bar{e}_1, \bar{e}_2)$, so that these four consumption vectors form a rectangle in (e_1, e_2)-space. Show that $MRS_{12}(e^a)/MRS_{12}(e^b) = MRS_{12}(e^c)/MRS_{12}(e^d)$.
(d) Does the result of part (c) remain valid when the utility function is state-dependent? Explain.

(e) Does the result of part (c) remain valid when the individual is not an expected-utility maximizer? Explain.

Figure 1: An example of a budget constraint with two states and two assets.

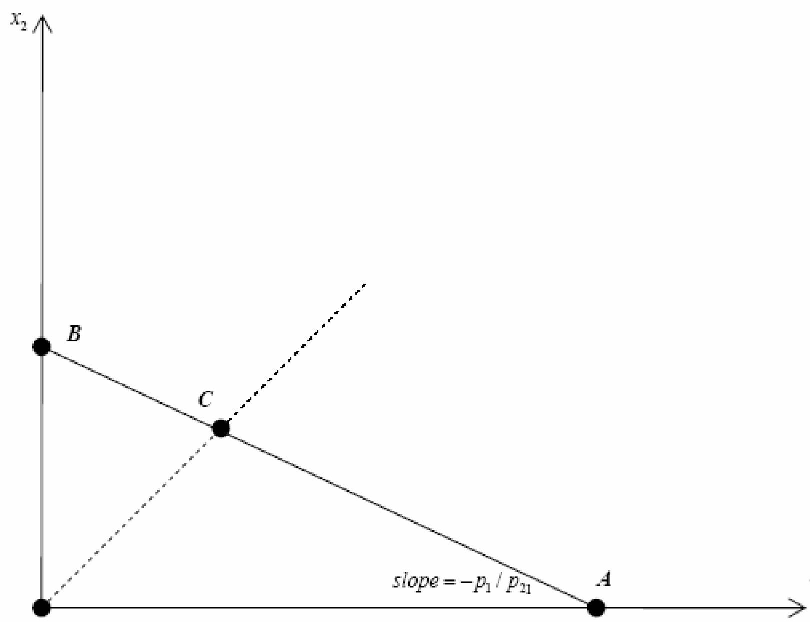
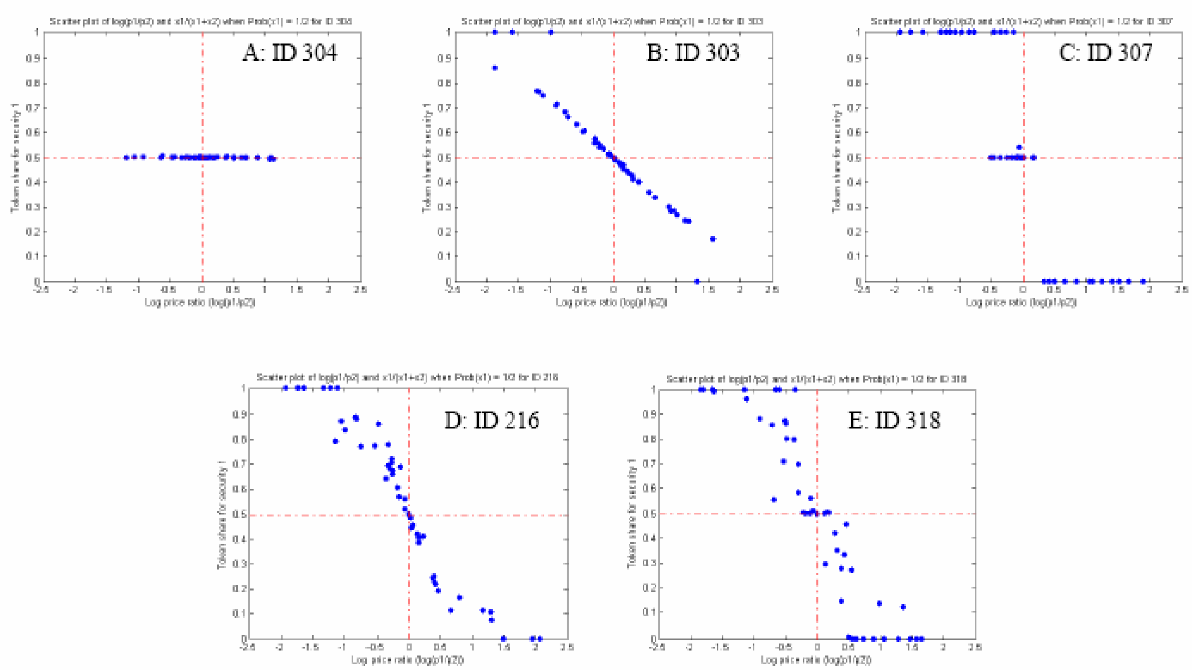


Figure 2: The relationship between the log-price ratio $\ln(p_1 / p_2)$ and the token share $x_1 /(x_1 + x_2)$ for selected subjects.



2. (expected utility theory and risk-taking) Consider a gamble to win \$200 with probability 0.50 and lose \$100 with probability 0.50. Consider an expected utility-maximizing consumer with initial wealth of \$10,000 and a constant relative risk aversion (CRRA) utility of wealth function $U(w) = w^{(1-\rho)}/(1-\rho)$, where $\rho>0$.

(a) Show that for $\rho = 45$, the consumer would accept this gamble but that for $\rho = 55$, the consumer would reject it. Argue that ρ would have to be much, much larger for the consumer to reject the gamble if initial wealth were \$1 million.

(b) Now suppose the consumer has utility of wealth function $U(w) = w^{(1-\rho*)}/(1-\rho^*)$, where $\rho^* = 55$. You offer the consumer a gamble to win \$*z* with probability 0.50 and lose \$1,000 with probability 0.50. What is the smallest value of *z* such that the consumer will accept your gamble? (Hint: This is a trick question, but try to answer it!) Does this seem like reasonable behavior?

(c) Explain intuitively why rejecting a small-stakes gamble that has positive expected value is qualitatively consistent with Expected Utility Theory but *quantitatively* inconsistent with it.

3. (Samuelson's gamble, narrow bracketing, and loss aversion) Paul Samuelson's colleague Robert Bishop once offered E. Cary Brown the opportunity to play a win \$200/lose \$100 gamble by flipping a coin. Brown replied, "I won't bet because I would feel the \$100 loss more than the \$200 gain." He went on to explain that he *would* accept many such bets. Samuelson then wrote a paper (1963 *Scientia*) in which he argued that Brown's reply was inconsistent with expected utility theory. (The characters are identified by name in Samuelson's 1952 *Econometrica* note, although they are not identified in his 1963 *Scientia* paper.)

(a) Suppose the win \$200/lose \$100 gamble is repeated three times. What are the possible outcomes, and with what probabilities would they occur? Now consider that sequence of three coin flips as a single gamble. Show that a consumer with utility of wealth function $U(w) = w^{(1-\rho*)}/(1-\rho^*)$ would reject this composite gamble. Is the consumer's utility higher or lower from the three-flip gamble or the one-flip gamble?

(b) Samuelson proved the following theorem: If an expected utility-maximizing consumer would reject a gamble for any initial wealth level in some range, then that consumer would reject any independent sequence of such gambles (that leaves the consumer within that range of wealth levels). Explain this result intuitively and/or prove the theorem.

(c) Show that loss aversion can explain *both* why Brown rejected the gamble *and* why he would accept a sequence of such gambles. To focus on the role of loss aversion, ignore the other features of prospect theory such as concavity of the value function in gains, convexity of the value function in losses, and nonlinear probability weighting. Suppose that Brown maximizes $\Sigma_{i=1,\dots,n}\, p_i\, V(x_i)$, where each of *n* gain or loss outcomes x_i occurs with probability p_i. Assume that the value function is given by:

$$V(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2.5x & \text{for } x < 0. \end{cases}$$

Show that Brown would reject a single coin flip gamble but would accept a three-flip gamble. Explain this result intuitively.

(d) Throughout life, we face many positive-expected-value small-scale risks. What are some examples? Normally, we consider each risk in isolation – this is called *narrow bracketing*. When we narrowly bracket the risks we face, loss aversion may lead us to turn down positive-expected-value gambles. Explain why this is a mistake. Argue that it is good advice even to a loss-averse person to accept positive-expected-value gambles. Do you think it is generally a good idea to pay extra for a one-year warranty on a CD player?

4. (prospect theory) G is a 50-50 win $1000 lose $550 gamble. Consider an agent with a non-decreasing probability weighting function $\pi(p)$ and with the following prospect theory value function:

$$V(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2.5x & \text{for } x < 0. \end{cases}$$

(a) What will this agent choose among:
(i) do not participate,
(ii) play G one time,
(iii) play G two times with a single payment done at the end by adding up the two results.

(b) What will he do if he has also the extra option:
(iv) play G one time, see the result and have the option of playing it a second time. A single payment is done at the end.

(c). Give an example of a situation:
(i) where people will aggregate the risks and take their decision based on the final outcome,
(ii) where they will do the opposite

5. (The Allais Paradox) In 1953, Maurice Allais proposed the following thought-experiment. You must make a choice between Gamble A and Gamble B (you can interpret these dollar amounts as final wealth levels):

Gamble A: $1 million for sure

Gamble B: $1 million with probability 0.89
$5 million with probability 0.10
$0 with probability 0.01

Which would you choose?

Next, you must make a choice between Gamble C and Gamble D:

Gamble C: $1 million with probability 0.11
$0 with probability 0.89

Gamble D: $5 million with probability 0.10
$0 with probability 0.90

Which would you choose?

(a) Most people choose Gambles A and D. Explain why (no matter what utility function a person has!) this pattern of choices violates expected utility theory.

(b) Explain intuitively how the probability weighting feature of prospect theory can resolve the Allais Paradox. Consider a value function of the form:

$$V(x) = \begin{cases} x^{(1-\sigma)}/(1-\sigma) & \text{for } x \geq 0 \\ -2\,(-x)^{(1-\sigma)}/(1-\sigma) & \text{for } x < 0, \qquad \sigma > 0, \end{cases}$$

and a probability weighting function of the form:

$$\pi(p) = \exp\{-(-\ln p)^{\alpha}\}, \qquad 0 < \alpha < 1.$$

Can you find values for σ and α such that you would choose Gamble A over Gamble B *and* you would choose Gamble D over Gamble C? Try several reasonable values and then draw a conclusion about how well prospect theory resolves the Allais Paradox quantitatively.

6. (The Ellsberg Paradox) Daniel Ellsberg (long before the Pentagon papers) proposed the following thought-experiment. An urn contains 90 balls, 30 of which are red. The other 60 are black or yellow, in unknown proportions. One ball will be drawn randomly from the urn. You must make a choice between Gamble A and Gamble B:

Gamble A: You win $100 if the ball is red.
Gamble B: You win $100 if the ball is black.

Which would you choose?

Next, you must make a choice between Gamble C and Gamble D:

Gamble C: You win $100 if the ball is either red or yellow.
Gamble D: You win $100 if the ball is either black or yellow.

Which would you choose?

Most people strongly prefer Gambles A and D.

(a) Explain why this pattern of choices violates expected utility theory.
(b) Explain why this pattern of choices is also inconsistent with prospect theory.

(c) Ellsberg argued that people treat uncertainty (situations without known probabilities) differently than they treat risk (situations with known probabilities). Can you think of real-world examples where people go out of their way to avoid uncertainty?

7. (Applications of prospect theory) For each of the following anecdotes, briefly explain (i) why the person's behavior is *prima facie* inconsistent with expected utility theory, (ii) why it is consistent with prospect theory, and (iii) how the behavior might be reconciled with expected utility theory.

(a) Some students who were about to buy season tickets to a campus theater group were randomly selected and given a discount. During the first part of the season, those who paid full price attended significantly more plays than those who received discounts.

(b) Cab drivers in New York City work longer hours on warm, sunny days when their per-hour wage is low.

(c) People purchase insurance against damage to their telephone wires at 45 cents a month even though the probability that they'd incur the $60 repair cost in any month is 0.4%.

(d) Bettors tend to shift their bets toward longshots, and away from racetrack favorites, later in the racing day.

(e) Unionized workers have their wages set 1 year in advance and they receive some bad news that their wages will be cut next year, but they do not cut their spending. However, the previous year when they learned that their wages would increase, they increased their spending.

8. (optimism and pessimism of prospect theory maximizers) Tim owns a house. His company has offered him a job elsewhere, which he has accepted, and he has therefore decided to sell the house. He does not have much time, thus he just plan to post a take-it-or-leave-it offer with price x. For any price x from $1 million to $2 million, Tim assesses the probability q of selling as $q = 2 - x$. If he doesn't find a buyer, he can always sell the house to a friend for $1 million.

Tim is a prospect theory maximizer, and he integrates over different accounts (house and money). In particular, he values any two-outcome distribution of changes to his reference point, say s with probability p and t with probability $1 - p$, at $V = v(t) + (v(s) - v(t))p$ whenever $s > t \geq 0$ or $s < t \leq 0$. Here $v(z) = |z|^{1/2}$ if $z \geq 0$ and $v(z) = -2|z|^{1/2}$ if $z < 0$. Tim's reference point already includes all the changes required by the move to Europe other than the sale of the house.

(a) Assume that Tim is a pessimist and his reference point is based on presumption that he will sell the house for $1 million. Thus, he will see it as a gain of $x - 1$ if he obtains a price x higher than $1 million. What price x would Tim ask for?

(b) Now assume that Tim is an optimist and his reference point is based on presumption that he will sell the house for $2 million. Thus, he will see any price x below $2 million as a loss of $2 - x$. What price x would Tim ask for?

(c) Is there a difference between the optimal prices in questions 1 and 2? If not, try to explain why not. If yes, tell which one is higher and explain intuitively why the prices are different.

9. It is sometimes suggested that risk-reducing improvements in products, for example increases in the quality of bicycle safety helmets, can cause consumers to take so many more risks that the rate of injuries actually increases (see for example, "A Bicycling Mystery: Head Injuries Piling Up," by Julian E. Barnes, 29 July 2001 New York *Times*). Some (but of course not the *Times*) go even further and suggest that the improvement can end up making consumers worse off.

Consider a consumer who has only one choice to make, the speed, s, at which he rides his bicycle, which is continuously variable. He prefers higher speed, other things equal, but higher speed increases the probability of an accident, which he doesn't like. (Simplify by assuming that there is only one kind of accident, and that the consumer can have at most one accident, so that the accident outcome can be represented as a zero-one variable.) Further assume that the consumer faces an exogenous increase in quality, a, which lowers the probability of having an accident for any given level of speed, at no cost.

(a) Describe the consumer's preferences over speed-accident probability combinations mathematically, and formulate his decision problem as an optimization problem.

(b) Making reasonable assumptions about the consumer's preferences and the "accident technology" as needed, say in what, if any, circumstances a quality improvement can make the consumer increase his speed so much that he is more likely to have an accident after the quality improvement than before. (Answer graphically or algebraically, whichever you prefer, but be precise.)

(c) Say in what, if any, circumstances a quality improvement can actually make the consumer worse off after the quality improvement than before. (Answer graphically or algebraically, whichever you prefer, but be precise.)

(d) How might your answer to (c) change if the consumer was not an expected-utility maximizer? Explain. (There are many right ways to answer this part.)

10. Comment, using ideas from this course:

Entrees Reach $40, and, Sorry, the Sides Are Extra
By JODI KANTOR

A new dish is appearing on menus across the nation. Restaurateurs say they have little choice other than offer it, though it horrifies many customers.

That item is the $40 entree.

Until recently, such prices were the stuff of four-star, white-tablecloth meals, the kind that ended with a diamond ring on the petit four tray. But now entrees over $40 can be found in restaurants that are merely upscale, where diners wear jeans and tote children. In geographic terms, New York and Las Vegas have led the charge, and in culinary ones, luxury items like steak and lobster were first and are still most prevalent.

But the $40 entree is migrating: to restaurants in Philadelphia, Fort Lauderdale and Denver, and to ingredients like fish and even pasta. Several national chains serve entrees priced above $40.

“Forty is the new 30,” said Richard Coraine, the chief operating officer of Union Square Hospitality Group, which recently began charging $42 for a 1¾-ounce appetizer portion of lobster at lunchtime at the Modern in New York. Ten percent of its lunch patrons order the dish, it says.

Hovering just below the $40 mark is an even vaster group of $38 and $39 entrees, waiting to cross the line like thirtysomethings approaching a zero-ended birthday. The arctic char at the Indianapolis branch of the Oceanaire Seafood Room chain is $38.50. Metropolitan Grill in Seattle serves shrimp scampi for $39.95. At Mike’s, a new steakhouse in Brooklyn Heights, $9.95 chicken nuggets share the menu with $38.95 veal chops.

Like the $100 Broadway ticket, $200 jeans and the $20 museum admission, the $40 entree is provoking a righteous burst of populist outrage, especially among those who pay their own way. When Angela Dansby, a Chicago diner, sees a 4 in front of a price, she thinks: “Either this must be out of this world, or it’s totally overpriced and I’m not going to order it. It’s usually the latter.” When she does pay, she compensates by skimping on appetizers and wine.

Restaurateurs say rising rents, ever more elaborate interior-decoration schemes and the increasing cost of premium ingredients — especially beef and fish — leave them little choice. Chefs, so fond of listing purveyors on menus, do not want those names to be Tyson and Del Monte. They “take pride in getting carrots or beets that no one has,” Mr. Coraine said.

Bobby Flay acknowledges that “the needle has moved very fast.” Mr. Flay recently crossed the $40 mark in his Las Vegas and Atlantic City outposts, though he says he intentionally loses money on many other entrees in order to keep prices reasonable. His entrees at Mesa Grill in New York top out at $34. (When it opened in 1991, the steepest entree was $19, or $28.30 when adjusted for inflation.)

But what makes the rise of the $40 entree so significant is not just the price creep, it’s the sophisticated calculation behind it. A new breed of menu “engineers” have proved that highly priced entrees increase revenue even if no one orders them. A $43 entree makes a $36 one look like a deal.

“Just putting one high price on the menu will take your average check up,” said Gregg Rapp, one such consultant. “My mom taught me to never order the most expensive thing on the menu, but you’ll order the second.”

With just a few keystrokes, restaurateurs can now digitally view the entire history of a dish: how the lamb sold around this time last year, whether it did better when paired with squash or risotto, and how orders rose or fell when the price went from $39 to $41.

With a few more clicks and a new stack of paper in the office printer, the menu can be revised to test new prices.

“In the old days, restaurateurs printed up menus and they were stuck with them for six months or a year; now they can do it daily, experimenting with price or placement,” said Tim Ryan, president of the Culinary Institute of America, which teaches menu engineering to all its chefs in training.

The towering prices at wildly luxurious restaurants like Per Se and Masa in New York and Alinea in Chicago have set a new price in the collective dining consciousness for a truly top meal, nudging up what diners will pay for far more modest dinners. In Las Vegas, the current talk is about Guy Savoy at Caesars Palace, where desserts alone are $22 each and a meal for two can easily run $500.

“I love when I hear about that stuff, because then Craft becomes inexpensive,” said Tom Colicchio, chef of the quickly multiplying restaurants, including a steakhouse in Las Vegas.

Oddly, as entrees rise in price, they seem to be shedding their traditional accompaniments. Today a $40 main dish is often now just that. Order a side dish, and the entree price climbs dizzyingly close to the 50’s. At the highly influential Craft, Mr. Colicchio serves pricey, naked hunks of protein and charges extra for vegetables. (He says the portions are enough for two.) Porter House, a new steakhouse at the Time Warner Center in New York, even charges diners separately for sauce.

“I blame Tom Colicchio for this,” said Barry Okun, a New York lawyer who has established a personal price limit of “between $50 and $60” per entree. “It’s not that I’m happy about it,” he added.

Mr. Colicchio acknowledged the influence of his pricing, adding that restaurants like those of the Bistro Laurent Tourondel group in New York “completely ripped off the concept” of focusing on individual elements.

To which Mr. Tourondel replied, “He should look back at the old-time steakhouse menus that were around way before Craft ever existed.”

Liz Johannesen, senior manager of restaurant marketing at OpenTable, which takes online reservations for 6,000 restaurants nationwide, said that in the last year diners had started occupying tables for longer periods, mimicking the leisurely pace at the very top establishments and forcing restaurants to raise entree prices because they were turning fewer tables.

“Just like in other cultural pursuits, trends filter downwards,” Ms. Johannesen said.

That applies to a taste for splurging as well. Kobe and Wagyu beef, from pampered Japanese cattle renowned for their tender meat, is cropping up at restaurants around the nation, according to Technomic, an industry research concern. Steakhouses and sushi restaurants, now so ubiquitous, have trained diners to pay large sums for specific ingredients, leading some to fall for the old “if it’s expensive it must be great” trick.

Indeed, no chef needs a menu engineer to explain a time-honored truth of the restaurant industry: many business diners look to spend money, not save it.

“If I’m entertaining clients, it’s all about making sure my clients are having the best time,” said Andrew Passeri, a private banker in New York who last week dined at davidburke & donatella, where he chose the $44 lobster over the $46 Dover sole and the $44 ostrich scramble.

At those prices, dinner is garnished with a large dusting of skepticism. Underattentive service or an overcooked piece of fish is not merely a minor annoyance but an unjustifiable offense.

“I’m happy to pay good money for something I can’t replicate at home,” said Ms. Dansby, of Chicago, “but when you get charged these prices for bad service, or quality, or visual presentation that isn’t so great, it’s really irritating.”

Two years ago, when Gray Kunz opened Cafe Gray in the Time Warner Center, he said he hoped it would become a destination for secretaries who work in the surrounding office buildings. The priciest entree then was the short ribs at $34. Now they are $38, the chicken is $37, and Mr. Kunz just introduced a lobster ravioli at $41.

“The biggest gasp I ever had at menu prices was the first time I went to Cafe Gray,” Mr. Okun said. “It looks like it should be a casual ‘stop in here for a bite’ place. For the amount of money you’re spending, you really want a special experience.”

Mr. Kunz now calls his restaurant “right in between the high end and low end” and said he provided “good product for very good value.”

According to Zagat, which measures what diners estimate paying, not actual prices, the average check at the most expensive 200 restaurants in San Francisco has risen 14 percent in the last two years, after remaining fairly stable earlier in the decade. At the 200 priciest restaurants in New York, Zagat users say, checks have followed the same pattern.

For his part, Tim Zagat, publisher of the guides that bear his name, said he was almost over the shock of entree prices. But now, he said, he finds himself startled by another development.

“Your $40 plate?” Mr. Zagat said. “It comes with a $20 first course.”

Probabilistic judgment

11. (Representativeness) Suppose that, in the course of a regular check-up, a doctor discovers that the patient has a potentially cancerous lesion. Most lesions are benign (non-cancerous), say 99%. The doctor orders an x-ray just in case. In laboratory tests on malignant (cancerous) lesions, the x-ray returns positive (cancer-affirming) results 79.2% of the time and negative results 20.8% of the time. In laboratory tests on benign lesions, the x-ray returns positive results only 9.6% of the time and negative results 90.4% of the time.

(a) The patient’s x-ray comes back positive. What is the probability that the patient has cancer?

(b) Suppose that the doctor calculates the probability that the patient has cancer without regard to the base rate of cancer in the population – that is, the doctor uses Bayes’ Rule but assumes that cancerous and non-cancerous lesions are equally likely. What mistaken conclusion will the doctor draw from the test? How is this mistake an example of representativeness? Explain why it is important in these situations to have hospital procedures that require additional tests to be performed before a patient undergoes treatment.

12. Consider the Kahneman & Tversky base-rate neglect experiment. In Problem A, subjects are told that Jack has been drawn from a population of 30% engineers and 70% lawyers and that Jack wears a pocket protector.

(a) Let p_1 denote the probability that Jack is an engineer, given that he wears a pocket protector. Using Bayes' Rule, show that the odds that Jack is an engineer as opposed to a lawyer is given by:

$$p_1/(1-p_1) = [0.30 \Pr(\text{pocket protector} \mid \text{Jack is engineer})] / [0.70 \Pr(\text{pocket protector} \mid \text{Jack is lawyer})].$$

In Problem B, subjects are told that Jack has been drawn from a population of 70% engineers and 30% lawyers and that Jack wears a pocket protector.

(b) Let p_2 denote the probability that Jack is an engineer, given that he wears a pocket protector. Show that:

$$p_2/(1-p_2) = [0.70 \Pr(\text{pocket protector} \mid \text{Jack is engineer})] / [0.30 \Pr(\text{pocket protector} \mid \text{Jack is lawyer})].$$

Conclude that, if subjects form beliefs according to the laws of probability, it must be the case that:

$$[p_1/(1-p_1)] / [p_2/(1-p_2)] = (3/7)^2.$$

(c) Explain intuitively why this ratio of odds does *not* depend on Pr(pocket protector | Jack is engineer).

(d) Explain why Kahneman & Tversky set up the experiment in this way.

(e) What values for $[p_1/(1-p_1)] / [p_2/(1-p_2)]$ imply that subjects exhibit base-rate neglect?

(f) Kahneman & Tversky ran this experiment as a *between-subjects* design – different groups of subjects responded to Problems A and B. How might their results have changed if they had run a *within-subjects* design – where each subject responded both problems? Why do you think Kahneman & Tversky chose a between-subjects design?

13. Consider the following hypothetical facts: "One percent of people in the world are rational. We have a test for rationality. If someone is rational, they have a 60% chance of passing. If someone is irrational, they have a 40% chance of passing. Adam was just given the test, and he passed."
(a) Assume that Adam was drawn randomly from the world population. What is the probability that he is truly rational? Don't bother with the long division. Expressing your answer as a ratio is fine.
(b) Predict the responses of a population of naive subjects who are asked to estimate the probability of Adam's rationality, given the information above. Justify your answer. Describe the kinds of errors that they are likely to make.

14. Another manifestation of the representativeness heuristic is that people believe in the "Law of Small Numbers."

(a) Define the Law of Small Numbers, and explain why it is an error. Describe some evidence for the "Law of Small Numbers."

(b) Suppose that basketball players are, during any given game, in one of three states: Hot (they make 75% of their shots), Normal (they make 50% of their shots), or Cold (they make only 25% of their shots). Suppose Paul Pierce is Hot. What is the probability that he will make 3 baskets in a row? What if he is Normal? Cold? If you have no idea what state he'll be in before the game (that is, each state is equally likely), what would you believe about the likelihood that he is Hot after he makes his first 3 baskets in a row?

(c) One of the fans at this game believes in the Law of Small Numbers. She has the wrong model of how likely Paul Pierce is to make a basket. Here's how her model works. The fan imagines that there is a deck of 4 cards. When Paul is Hot, 3 of these cards say "hit" on them, and only 1 says "miss." Every time Paul takes a shot, one of these cards is drawn randomly *without replacement* from the deck, and the outcome is whatever the card says. Therefore, when Paul is Hot, he *always* makes 3 out of every 4 shots he takes. (When the deck is used up, the 4 cards are replaced, the deck is shuffled, and the process begins again – but that isn't important for this problem.) Similarly, when Paul is Normal or Cold, the outcome of every shot is determined by the draw of a card without replacement from a deck of 4 cards. When Paul is Normal, the deck has 2 "hit" cards and 2 "miss" cards. When Paul is Cold, the deck has 1 "hit" card and 3 "miss" cards. Explain how her model corresponds to the Law of Small Numbers.

Suppose Paul Pierce is Hot. According to the fan, what is the probability that Paul will make his first basket? After Paul makes his first basket, what does the fan believe about the probability that Paul will make his next basket? Explain why it is lower than the fan's belief about the probability that Paul will make his first basket.

According to the fan, what is the probability that Paul will make 3 baskets in a row? What if he is Normal? Cold? If the fan has no idea what state Paul will be in before the game (that is, each state is equally likely), what would she believe about the likelihood that he is Hot after he makes his first 3 baskets in a row? Explain intuitively why the fan's beliefs differ from the normatively correct probability that you calculated in part (ii).

(d) Suppose that, in reality, there is no such thing as being "Hot" or "Cold." Paul Pierce is, in fact, always Normal. Over many games, with what frequencies will Paul score 0, 1, 2, and 3 baskets in his first 3 attempts? Suppose the fan attends many games and observes these frequencies. Explain why the fan would not believe you if you tried to convince her that there is no such thing as being "Hot" or "Cold."

In this example, the fan's misunderstanding of probability leads her to believe (falsely) in "hot hands" and "cold hands." Something similar may be going on in the mutual fund industry. Even if mutual fund returns are almost entirely due to luck, there will be some mutual funds that have done exceptionally well and others that have done exceptionally poorly in recent years due entirely to chance. Explain how the Law of Small Numbers would lead some investors to conclude (falsely) that mutual fund managers differ widely in skill.

15. In Monte Hall's game show "Let's Make a Deal," contestants chose one of three doors. Behind one of the doors was the grand prize (a car), and behind the other two doors were booby prizes (goats). Each door was equally likely to contain the grand prize. After the contestant chose a door

and *before* Monte Hall opened the chosen door, Monte Hall opened a *different* door – and he *always* intentionally opened a door that contained a booby prize. Then Monte Hall gave the contestant a choice between sticking with the original choice or switching to the third, unopened door. Before reading on – would you switch or stick with your original choice?

(a) In fact, the best strategy is to switch. Explain why. (Many actual contestants did not switch, and many eminent scholars have been confused about this problem.)

(b) Explain how this problem is similar to the winner's curse and why people get so confused about these problems. (Think about how conditional probabilities differ from unconditional probabilities. In particular, did the fact that Monte Hall opened one of the doors give you information about whether the door you originally chose has the grand prize? Did the fact that Monte Hall *didn't* open the third door give you information about whether *that* door has the grand prize?)

16. Suppose that you are making a take-it-or-leave-it offer to Chair Watson, CEO of the UCSD Economics Department. The value of the Department to Watson is distributed uniformly between $0 and $10 million. He knows its true value, but you don't (and he knows that you don't, etc.). However, you do know that whatever the Department is worth, it is worth 2.5 times more to you than it is to Watson. For example, if the Department is worth $10 million to Watson, then it is worth $25 million to you. If your take-it-or-leave-it offer is above Watson's value, he will sell you the Department. If not, he won't. (Ties have zero probability here, so don't worry about them.)

(a) What is your optimal offer? Explain.

(b) Predict what a population of naive people would offer in this situation. Explain by describing the kinds of errors they are likely to make.

17. First suppose that you are bidding for a single, indivisible object in an independent-private-value sealed-bid auction in which each bidder's value is an i.i.d. draw from a common, continuous distribution. It is a second-price ("Vickrey") auction, in which the highest bidder wins the object, but pays only the second-highest bid. (The other bidders win nothing and pay nothing; ignore ties.)

(a) What is your optimal bidding strategy (relating your optimal bid to the realization of your own value)? Explain why it is optimal, no matter what bidding strategies other bidders adopt. (That is, explain why it is a dominant strategy.)

(b) Would you expect naïve experimental subjects to bid optimally in this situation? If not, what kind of errors do you think they are most likely to make?

(Optional.) Now suppose that all is as in the first part, but the auction is a common-value one. That is, the value is commonly known to be the same to all bidders ex post, but ex ante they do not know what the value will be. Instead they observe only noisy, i.i.d. signals of its value. (In a common-value auction, there is a well-known phenomenon known as the winner's curse, in that in equilibrium, bidders' bidding strategies are increasing in their own values, so that the winner is normally the bidders whose value signal was highest among all bidders' signals. This means that his signal, taken by itself, overestimates the value, so that to bid optimally he must correct for this.)

(c) What is your optimal bidding strategy? Explain why it is optimal. Is it still a dominant strategy? Explain why or why not.

(d) Would you expect naïve experimental subjects to bid optimally in this situation? If not, what kind of errors do you think they are most likely to make (over and above those in (b))?

Intertemporal Choice

17. (hyperbolic discounting) Consider a consumer faced with a "vice" good like potato chips, which they are tempted to consume rapidly. The consumer can buy a large (2-serving) or small (1-serving) pack at period 0. In period 1, she must decide how much to consume. If she bought only the small pack, she consumes one serving. If she bought the large pack, she can consume two servings right away, or one serving and save another serving for the future (which is automatically consumed in period 2).

Assume there is positive utility in period 1 from consumption, and negative utility in period 2 (a reduced-form expression for poor health, say). Because the large size has some production economies, it is cheaper, which is reflected in higher immediate consumption utility. The Table below shows numerical utilities. (If she chooses to eat 1 serving from the large pack in period 1, then she gets utility of +3 in period 2, and –2 in period 3, from the second pack.)

Consider a β-δ quasi-hyperbolic framework. For simplicity assume δ=1 to focus attention on the β term. Analyze the optimal consumption decisions of three types of agents: Exponential (β=1, β'=1); naïve hyperbolic (β<1, β'=1); and sophisticated hyperbolic (β<1, β'=β).

Purchase Decision **Consumption Decision**	**Instantaneous Utility in Period 1**	**Instantaneous Utility in Period 2**
Small		
1 serving	2.5	-2
Large		
1 serving	3	-2
2 servings	6	-7

For each agent, figure out:

(i) What will they expect to do, at time 0, if they buy either the large and small packages?

(ii) Given your answer in (i), which package will the period-0 "self" purchase, for each of the three types?

(iii) After they buy their optimal package, how much will they consume in period 1?

(iv) Which of the type's (if any) plans embedded in (i) are actually violated in (iii)

(v) Suppose agents could purchase external commitment, in which they could only consume 1 of the 2 servings in the large pack in period 1, at a price of P>0 (think of this as buying pre-packaged dietary portions of food). Which agents would commit at time zero to pay P, and how much would they pay?

17. As question 16 indicates, an important empirical demarcation between naïve and sophisticated hyperbolic agents is whether they will pay in advance for planned self-control (a la Ulysses and the Sirens). Give an example of external self-control that is *voluntarily* chosen by agents (other than those discussed in class). Try to think of the biggest examples in the economy that you can think of.

18.

2 Intertemporal choice

Consider a consumer with temporaneous utility of consumption

$$u(c) = \frac{c^{1-\rho} - 1}{1 - \rho}$$

for some parameter $\rho \in (0, 1)$ who discounts future temporanous utilities that are one period ahead by δ_1 and utilities that are two period ahead by δ_2. The consumer has positive wealth W_0 and is going to consume it over three periods $0, 1, 2$. He has access to bank deposits which pays the interest rate $r > 0$. Thus we can formulate his problem as:

$$\max_{\{c_0, c_1, c_2\}} u(c_0) + \delta_1 u(c_1) + \delta_2 u(c_2)$$

subject to the constraint:

$$W_0 = c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2}.$$

1. [15 points] Compute consumptions c_0, c_1, c_2.
2. [10 points] What condition on δ_1 and δ_2 ensures time consistency of the consumer?

19.

4. (Option Value: When you should stay in a "just-okay" relationship.) Suppose you are in a romantic relationship. Every night (after you go out on a date), there is some probability $1-\mu$ that the relationship ends permanently for reasons outside of your control – perhaps because your beau moves out of town. With probability μ, on the other hand, the relationship continues and you can go out on a date the next night. Of course, you can choose to break up at any time, which permanently ends your relationship.

As is well-known, relationships come in three varieties: great (G), bad (B), and just-okay (J). At date t, let's call the state of your relationship S_t. If you are in a great relationship ($S_t = G$), then you get 1 util from every date. If you are in a bad relationship ($S_t = B$), then you get -2 utils from every date. If you are in a just-okay relationship ($S_t = J$), then you get -1 util from every date (not because it's all that bad, just because of the opportunity cost in lost studying for Ec 1030). When your relationship ends – either for reasons outside your control or because you choose to break up – you get 0 utils.

If you are in a great relationship at date t, then (as long as your beau stays in town) you remain in a great relationship for sure at date t+1: that is, $\Pr(S_{t+1} = G \mid S_t = G) = 1$. Unfortunately, if you are in a bad relationship at date t, then you remain in a bad relationship for sure at date t+1: that is, $\Pr(S_{t+1} = B \mid S_t = B) = 1$. Finally, if your relationship is just-okay at date t, then anything could happen at date t+1. With probability Δ, it could turn good; with probability Δ, it could turn bad; and with probability $1-2\Delta$, it remains just-okay: $\Pr(S_{t+1} = G \mid S_t = J) = \Pr(S_{t+1} = B \mid S_t = J) = \Delta$, and $\Pr(S_{t+1} = J \mid S_t = J) = 1-2\Delta$.

(a) Explain why the value of a great relationship is $1/(1-\mu)$ utils. Explain why the value of a bad relationship is 0 utils.

b) Suppose you are currently in a just-okay relationship. Let's call V the value of staying in a just-okay relationship at time t (why doesn't V depend on t?). Show that $V = [\Delta\mu/(1-\mu) - 1] / [2\Delta\mu + 1 - \mu]$. (Hint: Write V as a recursive function of itself using your answers from part (a), and then solve the equation for V.)

(c) Suppose μ=0.99. For what values of Δ should you stay in a just-okay relationship? Explain why Δ can be so small and you would still want to be in the relationship? (What if Δ is smaller than that?) What mistake might people make when deciding whether to stay in relationships and why? Based on the logic of this problem, what prescriptive advice might you give to a non-fully-rational friend about whether to stay in a just-okay relationship. Do you think people ever make the opposite mistake of remaining too long in a bad/just-okay relationship? If so, what psychological phenomena might explain that mistake?

(d) Suppose you couldn't break up. Then what is the value of a bad relationship? What is the value of a just-okay relationship? If you had to make a take-it-or-leave-it choice about entering a just-okay relationship, would you enter? Explain why breaking up if things turn sour is the "option" that gave "value" to the just-okay relationship in the earlier parts of this problem. Calculate the difference in value between a just-okay relationship now and a just-okay relationship in part b. This difference is the option value of being able to break up.

21. Essay question

This essay question will be part of the second mini-exam. It is meant to get you thinking about how to use behavioral decision theory to do economics; the choice gives you some freedom to make the question about the kind of economics you are interested in.

Write a brief (one-page or less) essay on how research on how behavioral decision theory should change how we think about a non-trivial economic application of your choice. Full credit will be given for any answer that sketches a coherent and empirically plausible analysis of a non-trivial application. If you are in doubt about any of this, please discuss your idea with me in advance.