Economics 206 Exam 1 Vincent Crawford

This take-home mini-exam covers the first of the course, and consists of three questions taken verbatim from Problem Set 1. This exam was posted on the course website at approximately 4 p.m. Wednesday, February 7, and is due by email or in the course mailbox in Economics Student Services by 4 p.m. Friday, February 9. You must now work on these questions individually, without consulting anyone but me (and me only for clarification). The 48-hour time limit should not be binding.

1. There are three possible states of the world, with known, objective probabilities $q_1 = 2/6$, $q_2 = 3/6$, and $q_3 = 1/6$. There is one consumption good, which can be purchased contingent on which state occurs. Consumption in state i is denoted c_i , and the price of this contingent commodity is denoted p_i . (That is, a contract to deliver c_i units of the consumption good if and only if state i happens costs p_ic_i .) There are two persons, Mr. A and Ms. B, both risk-averse, with state-independent, differentiable von Neumann—Morgenstern utility functions denoted U(c) and V(c) respectively. Mr. A and Ms. B have the same income, denoted I.

(a) Derive and interpret the first-order conditions for an interior solution of Mr. A's problem of allocating his income optimally among consumption in the three states. (Do not assume that the p_i are proportional to the q_i .)

(b) Can Mr. A's second-order conditions fail to be satisfied under the stated assumptions?

(c) Do the stated assumptions rule out the possibility of a corner solution?

(d) When Mr. A faces the prices $p_{1A} = 8/25$, $p_{2A} = 10/25$, and $p_{3A} = 3/25$, he purchases $c_{1A} = 7$, $c_{2A} = 8$, and $c_{3A} = 9$; and when Ms. B faces the prices $p_{1B} = 8/25$, $p_{2B} = 6/25$, and $p_{3B} = 8/25$, she purchases $c_{1B} = 8$, $c_{2B} = 9$, and $c_{3B} = 7$. What can you conclude about their comparative levels of risk aversion?

(e) Now suppose that Mr. A faces the prices $p_{1A} = 9/25$, $p_{2A} = 10/25$, and $p_{3A} = 2/25$, and purchases $c_{1A} = 7$, $c_{2A} = 8$, and $c_{3A} = 10$. Why does this additional information demonstrate that Mr. A. cannot in fact be an expected-utility maximizer?

2. A risk-averse, state-independent expected utility-maximizing investor must decide how to divide his portfolio between two assets. (These are the only two possible investments, and he cannot borrow or lend.) There are two states of the world, such that \$1 invested in asset 1 yields (1+r) > 1 in state 1 and \$1 in state 2, and \$1 invested in asset 2 yields (1+s) > 1 in state 2 and \$1 in state 1.

(a) Letting x_i denote the investor's final wealth if state i occurs, graph his opportunity set, given initial wealth I, in (x_1, x_2) -space, (putting x_1 on the horizontal axis). Label your graph clearly to show how I, r, and s, determine the opportunity set.

(b) Compute the portfolio that makes the investor's final wealth independent of the state. Is it always optimal for a risk-averse investor to choose this portfolio? Explain.

(c) Suppose that the investor has constant relative risk aversion. Draw the path in (x_1, x_2) -space that shows how the state-contingent final wealths generated by his optimal portfolios vary with I. What theorem justifies your answer, and why? (Please explain carefully.) Can you tell whether the investor's coefficient of absolute risk aversion is increasing or decreasing and wealth in this case?

3. The random variable x has a trinomial distribution, with $Pr{x = A} = a$, $Pr{x = B} = b$, and $Pr{x = C} = 1 - a - b$, with a, b, c > 0 and A < B < C.

(a) What kinds of change in a, b, and c (with A, B, and C constant) induce a first-order stochastically dominating increase in the distribution of x? (A carefully drawn graph may help.) Does such a change necessarily increase the mean of x? Does a change that increases the mean necessarily induce a first-order stochastically dominating increase? Explain.

(b) What kinds of change in a, b, and c (with A, B, and C constant) induce a meanpreserving spread in the distribution of x? Does such a change necessarily increase the variance of x? Does a change that increases the variance necessarily induce a mean preserving spread? Explain.

(c) How would your answers to parts (a) and (b) change if C < B < A?