Economics 201C Segment Practice Problems **Spring 2009**
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In some of the empirical-judgment parts of questions more than one answer is defensible.

1. In each of the following games, identify the rationalizable strategies for each player, and then identify the equilibrium strategy combination or combinations. Then pick one rationalizable strategy for the Row player—a nonequilibrium strategy if this is possible in the game you are considering—construct beliefs that are consistent with common knowledge of rationality that support it as a best response, and explain your answer. What distinguishes the beliefs that support rationalizable strategies that are in equilibrium from those that support rationalizable strategies that are not in equilibrium?

(a)

	L	C	R
T	0 7	5 0	3 0
M	0 5	2 2	0 5
B	7 0	5 0	3 7

(b)

	L	C	R
T	0 7	5 0	7 0
M	0 5	2 2	0 5
B	7 0	5 0	0 7

2. Consider the following two-person guessing game. Each player has her/his own target, lower limit, and upper limit. These are possibly different across players, and they influence players' payoffs as follows. Players make simultaneous guesses, which are required to be within their limits. Each player then earns 1000 points minus the distance between her/his guess and the product of her/his target times the other's guess.

(a-d) Find the Nash equilibrium or equilibria for the following targets and limits:

a)

	Lower Limit	Target	Upper Limit
Player 1	200	0.7	600
Player 2	400	1.5	700

b)

	Lower Limit	Target	Upper Limit
Player 1	300	0.7	500
Player 2	400	1.3	900

c)

	Lower Limit	Target	Upper Limit
Player 1	400	0.5	900
Player 2	300	0.7	900

d)

	Lower Limit	Target	Upper Limit
Player 1	300	1.3	500
Player 2	200	1.5	900

(e) State and prove a general result that determines the equilibrium as a function of the targets and limits for these guessing games.

(f) Would you expect people randomly paired from students who have not studied game theory to play their equilibrium strategies in these guessing games? Explain why or why not. If not, explain what you think they might do instead.

3 (harder). Now consider an *N*-person generalization of the two-person guessing game from problem 1. The only differences are (i) players' lower and upper limits are all the same, with the lower limit > 0; and (ii) players are arrayed around a circle, and each player earns 1000 points minus the distance between her/his guess and the product of her/his target times the guess of the person immediately on her/his left (who earns 1000 points minus the distance between her/his guess and the product of her/his target times the guess of the person immediately on her/his left).

(a) State and prove a general result that determines the equilibrium as a function of the targets and limits for these *N*-person guessing games. Specialize to $N = 3$ if you wish.

(b) Would you expect intelligent people randomly paired from students who have not taken this class to play their equilibrium strategies in these guessing games? Explain why or why not. If not, explain what you think they might do instead.

4. Two risk-neutral, expected money-maximizing bargainers, U and V, must agree on how to share $1. They bargain by making simultaneous demands; if their demands add up to more than $1, they each get nothing; if they add up to less than or equal to $1, each bargainer gets exactly his demand. Assume that any real number is a possible demand, and is also a possible division of the money.

(a) Find an infinite number of mixed-strategy Nash equilibria in this game. Explain why, in your equilibria, neither bargainer can do better by switching to any other strategy, pure or mixed.

(b) Show how to compute the equilibrium probability of disagreement, and show that it is always strictly positive in the mixed-strategy equilibria you identified in part (a).

(c) Are there any Pareto-efficient equilibria in this game?

(d) Now suppose that there are two plausible, but rival, notions of what it means to divide the dollar fairly. Redo your analysis from part (a), assuming that bargainers can put positive probability only on demands that are consistent with one or both of these notions of fairness. Is the equilibrium identified here also an equilibrium in the original game?

(e) Give a fairly detailed real-world (but not experimental) example in which common ideas of fairness appear to determine bargaining outcomes (and the likelihood of impasse) as in your answer to (d).

5 (harder). Suppose three identical, risk-neutral firms must decide simultaneously and irreversibly whether to enter a new market which can accommodate only two of them. If all three firms enter, all get payoff 0; otherwise, entrants get 9 and firms that stay out get 8.

(a) Identify the unique mixed-strategy equilibrium and describe the resulting probability distribution of the total ex post number of entrants. (You are not asked to show this, but the game also has three pure-strategy equilibria, in each of which exactly two firms enter; but these equilibria are arguably unattainable in a one-shot game in the absence of prior agreement or precedent. The mixed-strategy equilibrium is symmetric, hence attainable.)

Now suppose that each firm follows a behavioral rule that is an independent and identically distributed draw from a distribution that assigns equal probabilities to two types: either *L1* (best response assuming the other firms are each equally likely to enter or stay out, and probabilistically independent), or *L2* (best response to *L1*).

(b) Describe the decisions of types *L1* and *L2* and the resulting *actual* (as opposed to what *L1* or *L2* expect) probability distribution of the total ex post number of entrants when each firm's type is drawn as explained above. Show that the expected number of entrants is closer to the ex post optimal number (2) than in your equilibrium from part (a), and that that the probability of exactly 2 entrants is higher than in (a). (In experiments subjects' initial responses come systematically closer to ex post optimality than the symmetric mixed-strategy equilibrium predicts, a result Kahneman has described as "magic." This analysis shows that bounded strategic rationality works like fairy dust.)

Now suppose that each firm follows a rule that is an independent and identically distributed draw from a distribution that assigns probability ½ to type *L1*, ¼ to *L2*, and ¼ to a type called *Sophisticated*, which plays an equilibrium in the game in which the prior probabilities of *L1*, *L2*, and *Sophisticated* players are common knowledge.

(c) Plugging in the behaviors of *L1* and *L2* players (which do not depend on the prior type probabilities), characterize equilibrium in the game played by *Sophisticated* players.

(d) How does your answer to (c) change, if at all, if the prior probability of *Sophisticated* players is $\varepsilon \approx 0$, and the prior probability of *L2* players is $½ - \varepsilon$ (with the prior probability of *L1* players held constant at ½)?

6. (a) Imagine that you are playing the following game with one other person, randomly selected from people who have not studied game theory. What is your choice: Stag or Rabbit? Explain your argument clearly, using whatever concepts you find helpful.

	Stag	Rabbit
Stag	2 2	1 0
Rabbit	0 1	1 1

(b) Now imagine you are playing the following game with twenty other people, randomly selected from people who have not studied game theory. What is your choice: Stag or Rabbit? Explain your argument clearly, using whatever concepts you find helpful.

7. (a) Imagine that you are playing the following game as Row player, with one other person, randomly selected from people who have not studied game theory. What is your choice: H or T? Explain your argument clearly, using whatever concepts you find helpful.

	H	T
H	-2 2	1 -1
T	1 -1	-1 1

(b) Now imagine that you are playing the game as Column player, with one other person, randomly selected from people who have not studied game theory. What is your choice: H or T? Explain your argument clearly, using whatever concepts you find helpful.

8. Consider a single, large population of people randomly and anonymously paired to play a two-person game with payoff matrix as shown, once only, and with no common history of previous play, communication, etc. The game is presented to them as a story, without a matrix: "Each player chooses either X or Y. If you both choose X then you each get $5. If you both choose Y then you each get $5. If you choose differently, then neither one of you receives any money." But it is publicly known that all the subjects are told the same story, so the common labeling of the actions is public knowledge.

	X	Y
X	5, 5	0, 0
Y	0, 0	5, 5

76% of the Row and Column players (whose choices can be pooled because the people are indistinguishable and the game is symmetric) choose X.

Now suppose the setting is exactly as before, except that payoffs are changed as follows.

	X	Y
X	5, 5.1	0, 0
Y	0, 0	5.1, 5

This game is asymmetric from Row and Column players' viewpoints, so even though the people are indistinguishable, their choices cannot be pooled across player roles. Now, 78% of the Row players choose X, but only 28% of the Column players choose X.

The hypothesis that for most subjects X is "more salient" than Y seems to directly explain the results from the first treatment (with some noise), without considering the subtleties of strategic decision-making. In the second treatment the increased payoff of 5.1 for coordinating on X for Column players seems to make X even more attractive for them, but Column players choose X much less frequently than in the first treatment. The increased payoff of 5.1 for coordinating on Y for Row players seems to make Y more attractive for them, yet they play X slightly more than in the first treatment. Thus the simple explanation suggested above for the first treatment doesn't work for the second.

(a) Outline a model that has the potential to explain the role-asymmetric patterns in both treatments, using behavioral assumptions that are the same for Row and Column players, and the same in both treatments. (Hint: In the second treatment, the more salient label X bears a different relation to Row players' payoffs than to Column player's payoffs.)

9 (harder). Consider a two-person game with payoff matrix as shown. Before choosing simultaneously between T and B, or L and R, Column must send R a costless, nonbinding (“cheap talk”) message announcing her/his intention to play either L or R. Both players know the rules of the game, including the values of *x* and y, as common knowledge.

	L	R
T	2 2	x 0
B	1 1	y 1

(a) For what values of x and y are the choices T for Row and L for Column (each with probability one) consistent with subgame-perfect equilibrium in the entire game?

(b) For what values of x and y are the choices T for Row and L for Column (each with probability one) each part of some rationalizable strategy (in the entire game)?

In “Nash Equilibria are not Self-Enforcing” (in *Economic Decision Making: Games, Econometrics and Optimisation*, edited by Gabszewicz, Richard, and Wolsey, Elsevier 1990) Aumann argues that in games like this with $x \geq y$, an announcement by Column that s/he intends to play L should not (or will not, in a positive theory) alter Row’s belief that Column will actually play L, because Column does as well or better when Row plays T without regard to whether Column plays L or R.

(c) Do either subgame-perfect equilibrium or rationalizability distinguish between the credibility of an announcement by Column that s/he intends to play L when $2 \geq x > y$, $2 \geq x = y$, or $2 \geq y > x$?

(d) What assumptions about strategic behavior suffice to justify Aumann’s argument against the credibility of such an announcement.

(e) Evaluate the credibility of an announcement by Column that s/he intends to play L behaviorally, making whatever assumptions and using whatever arguments and evidence you find useful. What, if any, meanings might such an announcement convey beyond those it conveys in arguments based on subgame-perfect equilibrium or rationalizability? Make clear how and why your evaluation of the credibility of the announcement distinguishes between games where $2 \geq x > y$, $2 \geq x = y$, or $2 \geq y > x$.

10 (harder). Consider the Battle of the Sexes game. Assume, here and below, that the structure is common knowledge, that both players are self-interested, and that there are no observable differences between the players or their roles in the game. In each of the variations described below, say whether you would expect the players to be able to coordinate on one of the efficient pure-strategy equilibria, and what strategies you would expect the players to use, on average. Briefly but clearly explain your answers.

	Fights	Ballet
Fights	1 3	0 0
Ballet	0 0	3 1

Battle of the Sexes

(a) The original simultaneous-move game is a complete model of the players' situation.

(b) The game is modified so that Row chooses her/his strategy first and Column gets to observe her/his choice before choosing her/his own strategy.

(c) Row chooses her/his strategy first and Column does NOT get to observe her/his choice before choosing her/his own strategy.

(d) Row chooses her/his strategy first, Column observes her/his choice before choosing her/his own strategy, but Row then gets to observe Column’s choice and costlessly revise her/his own choice, and this decision ends the game (so that Column cannot revise her/his choice).

(e) The original simultaneous-move game is a complete model of the players' situation, except that Row (only) can make a non-binding suggestion about the strategies players should use before they choose them.

(f) The original simultaneous-move game is a complete model of the players' situation, except that both players can make simultaneous, non-binding suggestions about the strategies players should use before they choose them.

(g) The original simultaneous-move game is a complete model of the players' situation, except that players can make sequential, non-binding suggestions about the strategies players should use before they choose them, say with Row making the first suggestion.

11. (In memory of Bob Rosenthal; see his paper with Dale and Morgan, "Coordination through Reputations: A Laboratory Experiment," *Games and Economic Behavior* 38 (2002), 52-88.) Suppose a large group of people are repeatedly, randomly, and anonymously paired to play the Hawk-Dove game below. The game is symmetric, there is nothing to distinguish player's roles, and the people are indistinguishable, with one exception: each person's past realized history of play (an ordered sequence of pure actions, such as H,H,D,D,D,H,D,…) is made public within each pair before they choose their actions in the current play. Imagine for simplicity that each person is randomly assigned an initial one-period history, either H or D, before play begins.

	Hawk	Dove
Hawk	0, 0	2, 1
Dove	1, 2	0, 0

(a) Describe informally but clearly at least three qualitatively different kinds of (pure or mixed) repeated-game strategies that are consistent with symmetric subgame-perfect equilibrium in the game played by the entire group, with the payoffs of their strategies evaluated before the uncertainty of pairing is resolved.

(b) Identify a symmetric subgame-perfect equilibrium that is as efficient as possible. (Note that the restriction to symmetric equilibria makes this equivalent to maximizing the expected lifetime payoff of a representative player.)

(c) How would expect this game to be played by intelligent, well-motivated, self-interested real people?

12. Write a one-page (or less) essay on how research on the parts of behavioral game theory studied in this segment should change how we think about your choice of one of the following kinds of application. For some or perhaps all of them, more than one answer is defensible. Full credit will be given for any answer that includes a coherent and empirically plausible rationale. In some cases, there are readings on the syllabus beyond those discussed in class that may be helpful.

(a) the standard use of the revelation principle in designing auctions or incentive schemes
(b) the standard use of the Folk Theorem to characterize outcomes sustainable as implicit contracts in complete-information repeated games
(c) the use of subgame-perfect equilibrium to predict outcomes in infinite-horizon alternating-offers bargaining with complete information, as in Rubinstein (*Econometrica* 1982)
(d) the use of sequential or perfect Bayesian equilibrium in models with "crazy types" to characterize reputation building, as in Kreps and Wilson, Milgrom and Roberts, or all of the above (*Journal of Economic Theory* 1982)
(e) the use of refinements such as the "intuitive criterion," as in Cho and Kreps (*Quarterly Journal of Economics* 1987), to derive unique predictions despite multiple equilibria in signaling games
(f) the use of refinements such as risk-dominance to derive unique predictions despite multiple equilibria in macroeconomic models based on coordination failure like those discussed in Cooper and John (*Quarterly Journal of Economics* 1988)
(g) the use of iterated dominance in incomplete-information games with small idiosyncratic payoff trembles ("global games") to select among multiple Pareto-ranked equilibria in coordination games, as in Carlsson and Van Damme, "Global Games and Equilibrium Selection" (*Econometrica* 1993) and recent applications to bank runs and other problems.

13. Write a one-page summary and critique (a referee's report) of one of these papers from the reading list (most but not all are linked on the html version of the syllabus):

Andres Aradillas-Lopez and Elie Tamer, "The Identification Power of Equilibrium in Simple Games," *Journal of Business & Economic Statistics* 26 (2008), 261-283

Jeffrey Banks, Colin Camerer, and David Porter, "An Experimental Analysis of Nash Refinements in Signaling Games," *Games and Economic Behavior* 6 (1994), 1-31

Ray Battalio, F. Rankin, and John Van Huyck, "Strategic Similarity and Emergent Conventions Evidence from Similar Stag Hunt Games," *Games and Economic Behavior*, 32 (2000), 315-337

Ken Binmore, John McCarthy, Giovanni Ponti, Larry Samuelson, and Avner Shaked, "A Backward Induction Experiment," *Journal of Economic Theory* 104 (2002), 48-88

Jordi Brandts, and Charles Holt, "An Experimental Test of Equilibrium Dominance in Signaling Games," *American Economic Review* 82 (1992), 1350-1365

Alexander Brown, Colin Camerer, and Dan Lovallo, "To Review or Not To Review? Limited Strategic Thinking at the Movie Box Office," 2008; http://econweb.tamu.edu/abrown/cold.pdf

Hans Carlsson and Mattias Ganslandt, "Noisy Equilibrium Selection in Coordination Games," *Economics Letters* 60 (1998), 23–34

Russell Cooper, Douglas DeJong, Robert Forsythe, and Thomas Ross, "Alternative Institutions for Resolving Coordination Problems: Experimental Evidence on Forward Induction and Preplay Communication," 129-146 in James Friedman (ed.), *Problems of Coordination in Economic Activity*, Boston: Kluwer, 1994

David Cooper and John Van Huyck, "Evidence on the Equivalence of the Strategic and Extensive Form Representation of Games," *Journal of Economic Theory* 110 (2003), 290-308

Miguel Costa-Gomes and Georg Weizsäcker, "Stated Beliefs and Play in Normal-Form Games," *Review of Economic Studies*, 75 (2008), 729-762

Tore Ellingsen and Robert Östling, "Communication and Coordination: The Case of Boundedly Rational Players," 2007; http://www2.hhs.se/personal/Ellingsen/pdf/BRC271107b.pdf

Erik Eyster and Matthew Rabin, "Naive Herding," LSE and UC Berkeley, 2008; http://else.econ.ucl.ac.uk/conferences/bbw08/talks/eyster.pdf

Joseph Farrell, "Cheap Talk, Coordination, and Entry," *RAND Journal of Economics* 18 (1987), 34-39

Ernst Fehr and Jean-Robert Tyran, "Limited Rationality and Strategic Interaction. The Impact of the Strategic Environment on Nominal Inertia," Econometrica 76 (2008), 353-394

Jacob Goeree and Charles Holt (2004), "A Model of Noisy Introspection," *Games and Economic Behavior* 46, 365–382

Avi Goldfarb and Botao Yang, "Are All Managers Created Equal?," *Journal of Marketing Research* XLVI (2009), in press; http://www.marketingpower.com/ResourceLibrary/Documents/JMRForthcoming/Are%20All%20Managers.pdf

Philip Haile, Ali Hortaçsu, and Grigory Kosenok, "On the Empirical Content of Quantal Response Equilibrium," *American Economic Review* 98 (2008), 180-200

Teck-Hua Ho, Colin Camerer, and Keith Weigelt, "Iterated Dominance and Iterated Best Response in Experimental '*p*-Beauty Contests'," *American Economic Review* 88 (1998), 947-969

Teck Hua Ho and Keith Weigelt, "Task Complexity, Equilibrium Selection, and Learning: An Experimental Study," *Management Science* 42 (1996), 659-679

Eric Johnson, Colin Camerer, Sankar Sen, and Talia Rymon, "Detecting Failures of Backward Induction: Monitoring Information Search in Sequential Bargaining," *Journal of Economic Theory* 104 (2002), 16-47

Navin Kartik, Marco Ottaviani, and Francesco Squintani, "Credulity, lies, and costly talk," *Journal of Economic Theory* 134 (2007), 93-116

Richard McKelvey and Thomas Palfrey, "An Experimental Study of the Centipede Game," *Econometrica* 60 (1992), 803-836

Richard McKelvey and Thomas Palfrey, "Quantal Response Equilibria for Normal-Form Games," *Games and Economic Behavior* 10 (1995), 6-38

Judith Mehta, Chris Starmer, and Robert Sugden, "The Nature of Salience: An Experimental Investigation of Pure Coordination Games," *American Economic Review* 84 (1994), 658-674

Stephen Morris and Hyun Song Shin, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review* 88 (1998), 587-97

Matthew Rabin, "A Model of Pre-game Communication," *Journal of Economic Theory* 63 (1994), 370-391

Alvin Roth, "Bargaining Phenomena and Bargaining Theory," Chapter 2 in Roth (ed.), *Laboratory Experimentation in Economics: Six Points of View*, Cambridge, 1987

Alvin Roth, "Toward a Focal-Point Theory of Bargaining," Chapter 12 in Roth, (ed.), *Game-Theoretic Models of Bargaining*, Cambridge, 1985

Joseph Wang, Michael Spezio, and Colin Camerer, "Pinocchio's Pupil: Using Eyetracking and Pupil Dilation To Understand Truth–telling and Deception in Games," 2006; http://www.hss.caltech.edu/~camerer/pinocchio2.pdf