Economics 200C, Second Half, Practice Problems **Spring 2009**
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A. Adverse Selection (see also problems 13.B.1-9 at MWG 473-474 and 1 at Kreps 654)

A1. Person 1 owns an indivisible financial asset that is potentially more valuable to Person 2. Person i, i = 1,2, has a privately observed signal—a noisy but unbiased estimate—of the asset's value, y_i, and cares only about the expected value of her/his financial worth (the asset value net of its price for the buyer, the price net of the asset value for the seller). Persons 1 and 2 can trade the asset only at price p, and will trade only if both think they are strictly better off trading. Trade is governed by the following rules: Each player i simultaneously observes his y_i, and then simultaneously says Trade or No trade. Trade takes place, at price p, only if both say Trade; otherwise there is no trade.

(a) Prove that in any equilibrium of this trading game, the probability of trade is zero.

A2. Bill owns a car whose value to him is v, where v is uniformly distributed on [0,1]. Sam values the car at 1.5 v. Bill and Sam are both risk neutral, maximizing their expected values of the car and whatever money they receive or pay for it. Bill always knows the actual value of v, but Sam may or may not know v, as indicated below. Except for this, the structure of the game (including the rules discusssed below and the fact that Sam's value is 1.5 times Bill's) is common knowledge.

Suppose first that Sam knows v, and Sam can make Bill a continuously variable all-or-nothing offer to buy the car at a price p of Sam's choosing, which Bill must either accept or reject. If Bill accepts, then Sam gets the car and pays Bill p; and if Bill rejects, the game ends with no trade.

(a) Identify the subgame-perfect Nash equilibrium or equilibria of this game.

Now suppose that Sam does not know v, and Sam can make Bill a continuously variable all-or-nothing offer to buy the car at a price q of Sam's choosing, which Bill must either accept or reject.

(b) If Bill accepts an offer of q, what can Sam infer about v from the assumption that Bill is sequentially rational, and thus avoids strategies that do not maximize his expected payoff in every subgame? What is Sam's conditional expected value of v, given q, this inference, and his prior?

(c) What prices q, if any, is it consistent with expected payoff maximization for Sam to offer, if Bill has a positive probability of accepting Sam's offer?

(d) Identify the subgame-perfect Nash equilibrium or equilibria of this game, and indicate when the car is sold in your equilibrium or equilibria.

Finally, suppose that Sam does not know v, and Bill can make Sam a continuously variable all-or-nothing offer to sell the car at a price r of Bill's choosing, which Sam must either accept or reject.

(e) If Bill offers to sell at price r, what can Sam infer about v from the assumption that Bill avoids strategies that are weakly dominated? What is Sam's conditional expected value of v, given his prior and this inference?

(f) What prices r, if any, is it consistent with expected payoff maximization for Sam to accept?

(g) Identify a weak perfect Bayesian equilibrium of this game, and indicate when the car is sold in your equilibrium.

A3. Suppose that there are two states of the world, s_1 and s_2, and that an individual who knows the probabilities, p_1 and p_2 respectively, of the two states chooses among state-contingent consumption bundles to maximize the expectation of a state-independent von Neumann-Morgenstern utility function u(x) = ln x (where ln x is the natural logarithm of x, so that u'(x) =1/x). Suppose that the individual's income is $7 if state 1 occurs and $2 if state 2 occurs, and that an insurance company offers him a contract whereby for each $1 he pays (independent of the state, before observing it), he receives $2 if state 2 occurs and nothing if state 1 occurs.

(a) Write the problem that determines the individual's demand for insurance, at this price, as a function of his estimate of the probability, p_2, that state 2 will occur.

(b) For what value of p_2 is expected profit zero for the 2 insurance company at this price?

(c) Using the first-order condition, show that if p_2 is higher than this value, the individual will buy so much insurance that he is actually better off if state 2 occurs than if state 1 occurs. Is this consistent with the insurance company staying in business? Explain.

A4. Consider the following market game. First, Nature draws a worker's productivity type from a discrete distribution whose support ranges from a to b, where a < b. After the worker observes her/his type, s/he can choose whether to submit to a costless test that reveals her/his ability perfectly. Finally, after observing whether the worker has taken the test and its outcome if s/he has, two risk-neutral firms bid simultaneously for the worker's services. The worker then chooses between the firms' offers.

(a) Prove that in any subgame-perfect equilibrium, all worker types except possibly type a submit to the test, and all firms offer at most a to any worker who does not submit to the test.

B. Signaling and Screening (see also problems 8.E.1-3 at MWG 265; 9.C.1, 3-4, and 7 at MWG 304-305; 17-18 at Kreps 457; 2-4 at Kreps 498-501; 13.C.1-6 and 13.D.1-4 at MWG 474-476; and 2-5, 7-10 at Kreps 654-660)

B1. Consider a Spence signaling model of a competitive job market, in which firms can observe workers' education levels, but not their productivities, when they decide which workers to hire, and workers, who know their own productivities, choose their education levels with rational expectations about how education will influence firms' beliefs about their productivities. Assume that the output of a worker of type n who has had y years of education is $S(n,y) = ny^a$, where $0 < a < 1$, and that the cost of y years of education to a worker of type n is $C(n,y) = y/n$. Let w(y) be the wage offered in equilibrium to a worker with y years of education. Assume that $S(\cdot)$, $C(\cdot)$, and $w(\cdot)$ are differentiable.

(a) Assuming that each worker selects his education level to maximize the difference between his wage and the cost of his education, write the first-order condition that determines how much education a worker of type n chooses to acquire.

(b) Briefly explain why, in a separating equilibrium (that is, an equilibrium in which workers of different types always choose different education levels), the wage offered a worker with y years of education equals the output of the type of worker who chooses that level of education.

(c) Using your answers to (a) and (b), derive an equation that describes the equilibrium relationship between a worker's wage and his chosen level of education. (Hint: Eliminating n should leave you with an equation that relates w'(y) to w(y) and y. Verify that $w(y, K) = [2(y^{1+a} + K)/(1 + a)]^{1/2}$ satisfies your equation for any value of the parameter K.)

(d) Assuming that a = 0 (so education is not productive), determine the level of education chosen by a worker of type n in an equilibrium with parameter K. If n is continuously distributed on [1,2], for what values of K will your solution from (c) satisfy the nonnegativity constraint on education?

(e) Indicate which workers would gain, when a = 0, if education were abolished.

B2. Consider a two-person game with payoff matrix as shown. Before choosing simultaneously between T and B, or L and R, Column must send Row a costless, nonbinding message announcing her/his intention to play either L or R. (Because the message is costless and nonbinding, it has no direct effect on players' payoffs, but there could be equilibria or subgame-perfect equilibria in which players use strategies in which their decisions depend on Column's message.) Assume that both players know the rules of the game, including the values of *x* and y, as common knowledge.

	L	R
T	**2**, **2**	**0**, **x**
B	**1**, **1**	**1**, **Y**

(a) For what values of x and y are the choices T for Row and L for Column (each with probability one) consistent with subgame-perfect equilibrium in the entire game?

(b) For what values of x and y are the choices T for Row and L for Column (each with probability one) each part of some rationalizable strategy (in the entire game)?

(c) Do either subgame-perfect equilibrium or rationalizability assure the credibility of an announcement by Column that s/he intends to play L when $2 \geq x > y$, $2 \geq x = y$, or $2 \geq y > x$? (Background: It has been argued that in games like this with $x \geq y$, an announcement by Column that he intends to play L should not alter Row's belief that Column will actually play L, because Column does as well or better when Row plays T without regard to whether Column plays L or R; see Robert Aumann, "Nash Equilibria are not Self-Enforcing," in *Economic Decision Making: Games, Econometrics and Optimisation*, ed. Gabszewicz, Richard, and Wolsey, Elsevier 1990.)

B3. Consider the following market game. First, Nature draws a worker's productivity type from a discrete distribution whose support ranges from a to b, where $a < b$. After the worker observes her/his type, s/he can choose whether to submit to a costless test that reveals her/his ability perfectly. Finally, after observing whether the worker has taken the test and its outcome if s/he has, two risk-neutral firms bid simultaneously for the worker's services. The worker then chooses between the firms' offers.

(a) Prove that in any subgame-perfect equilibrium, all worker types except possibly type a submit to the test, and all firms offer at most a to any worker who does not submit to the test.

B4. An econometrician ("E") and a macroeconomist ("M") who do not know each other plan to meet at Black's Beach on Time Series Tee Shirt Day ("If you think life outside the spectral domain lacks savor, wear a C.W.J. Granger tee shirt on June 7"). Before they meet E observes his type, Time Series ("T") with probability $\lambda \; \varepsilon \; (0,1)$ or Cross Section ("C") with probability $1-\lambda$. M knows λ, but never observes E's type. After E observes his type he chooses between two actions: wear a C.W.J. Granger tee shirt ("W") or not ("N"). M then observes whether E is wearing the tee shirt and chooses between treating E deferentially ("D") or undeferentially ("U"). Both E and M are expected-payoff maximizers. M prefers to treat E in a way appropriate to E's status, with payoff function $v(\cdot)$, where $v(T,D) = 5$, $v(T,U) = -5$, $v(C,D) = -5$, and $v(C,U) = 5$. Both types of E prefer to be treated with deference, getting payoff 5 from D and –5 from U, other things equal. However, type T likes wearing the tee shirt and gets an additional payoff of 1 if he chooses W, while type C dislikes wearing the tee shirt and loses 3 units of payoff if he chooses W. Thus E's payoff function is $u(\cdot)$, where $u(T,W,D) = 6$, $u(T,W,U) = -4$, $u(T,N,D) = 5$, $u(T,N,U) = -5$, $u(C,W,D) = 2$, $u(C,W,U) = -8$, $u(C,N,D) = 5$, and $u(C,N,U) = -5$. Except for E's type, the structure is common knowledge.

(a) Assuming $\lambda \geq ½$, identify a pure-strategy weak perfect Bayesian *pooling* equilibrium, describing E's and M's strategies and M's beliefs.

Can you find another pure-strategy weak perfect Bayesian pooling equilibrium with a different equilibrium outcome? Explain.

(b) When $\lambda < ½$, is there a pure-strategy weak perfect Bayesian pooling equilibrium? Explain.

(c) Is there a pure-strategy weak perfect Bayesian *separating* equilibrium for any value of $\lambda \varepsilon (0,1)$? Explain.

Now suppose the cost of W is increased from 3 to 12 for type C of E, so E's type-C payoffs become u(C,W,D) = -7, u(C,W,U) = -17, u(C,N,D) = 5, and u(C,N,U) = -5. Everything else is unchanged.

(d) Is there a pure-strategy weak perfect Bayesian pooling equilibrium when $\lambda \geq ½$?

(e) Identify a pure-strategy weak perfect Bayesian separating equilibrium, describing E's and M's strategies and M's beliefs. Does the existence of this equilibrium depend on where λ falls in (0,1)?

C. Agency (see also problems 14.B.1-8 at MWG 507-508; 14.C.1-9 at MWG 508-510; and problems 1-2 and 4-7 at Kreps 616-623)

C1. A worker is an expected-utility maximizer, with von Neumann-Morgenstern utility function $u(s,z) = \ln(1+2s) - z$, where s is his salary and z is his level of effort. (ln x is the natural logarithm of x; so ln(1) = 0 and ln(e) = 1, where $e \approx 2.7$.) The worker's salary is specified by his contract with the firm that employs him, and must be nonnegative; but he can choose any level of effort in the interval [0,1] unless his contract rules it out. He will accept any contract that yields him an expected utility of at least 0, given his effort. His output is jointly determined by his effort and luck, and output takes the values (to the firm, which is risk-neutral) \$1 and \$0 with probabilities z (= the worker's chosen level of effort) and 1-z respectively.

First suppose that the firm can observe the worker's effort and output, so that it can use contracts that specify both the worker's salary and his effort.

(a) Compute the firm's optimal contract, given that the worker will accept any contract that yields him expected utility of at least 0, given his effort choice.

Now suppose that the firm can observe realized output but not effort, so that the contract can relate the worker's salary to realized output but not effort.

(b) If the worker is paid $s_1 \geq 0$ if his output is worth \$1 and $s_0 \geq 0$ if it is worth \$0, compute his expected utility-maximizing level of effort as a function of s_0 and s_1.

(c) Compute the firm's expected profit-maximizing salary schedule (s_0, s_3), given that the worker's salary is nonnegative and can depend only on his realized output, and the worker responds as in (b).

Now suppose that the firm can observe neither realized output nor effort, so that the contract must specify a constant salary for the worker, independent of effort or output.

(d) Compute the worker's optimal level of effort as a function of the salary.

(e) Compute the firm's expected profit-maximizing salary, given that it must be constant and that the worker will respond to a constant salary as in (d).

C2. A worker is an expected-utility maximizer, with von Neumann-Morgenstern utility function $u(s,z) = \ln(1+s) - z$, where s is his salary and z is his level of effort. ($\ln x$ is the natural logarithm of x; so $\ln 1 = 0$ and $\ln e = 1$, where $e \approx 2.7$.) The worker's salary is specified by his contract with the firm that employs him, and must be nonnegative; but he is free to choose any level of effort in the interval [0,1]. He will accept any contract that yields him an expected utility of at least 0, given his optimal effort choice. His output is jointly determined by his effort and luck, taking the values (to the firm) \$3 and \$0 with probabilities z (= the worker's chosen level of effort) and 1-z, respectively.

(a) Suppose first that the firm must offer the worker a constant salary, independent of his effort and his output. Compute the worker's optimal level of effort as a function of the salary.

(b) Compute the firm's expected profit-maximizing salary, given that it must be constant and that the worker will respond to a constant salary as in (a).

(c) Now suppose that the firm can offer the worker a contract that makes his salary contingent on his realized output, so that the worker is paid $s_3 \geq 0$ if his output is worth \$3 and $s_0 \geq 0$ if it is worth \$0. Compute the worker's expected utility-maximizing level of effort as a function of s_0 and s_3.

(d) Compute the firm's expected profit-maximizing salary schedule (s_0,s_3), given that the worker's salary is nonnegative and can depend only on his realized output, and the worker responds as in (c).

C3. Consider the interaction between a manager and a worker. The worker has private information about his preferences; the manager has no private information. In this relationship, the worker expends effort to produce $y \geq 0$ units of output which the manager sells at a price of 1, for revenue of y. The manager pays the worker a wage w. The worker's private information is described by his type t in $\{1, 2\}$. (That is, $t = 1$ or 2.) The worker's utility is $u(w, y|t) = w - g(y|t)$, where $g(y|t)$ is type t's disutility associated with producing y. The manager's utility is $v(w, y) = y - w$. Assume that the function $g(\cdot)$ is twice continuously differentiable in y and that $g(0|t) = 0$ for $t = 1, 2$. Also assume that $dg(y|t)/dy > 0$ and that $d^2g(y|t)/dy^2 > 0$ for $t = 1, 2$. Finally, assume that $dg(y|2)/dy > dg(y|1)/dy$.

The manager and worker can write a contract that specifies the wage and the quantity that the worker is supposed to produce. That is, a contract is given by (w, y).

(a) Suppose that, in the market equilibrium, the type t worker accepts the contract (w_t, y_t), for $t = 1$, 2. (It may be that $w_1 = w_2$ and/or $y_1 = y_2$.) Prove that $w_1 \geq w_2$. Also demonstrate the result graphically, by showing the appropriate indifference curves in (w, y) space.

(b) Now suppose that the worker's type is common knowledge between the worker and the manager. Consider two ultimatum bargaining games. In the first game, the manager offers a contract to the worker; the worker then accepts or rejects the contract. If the contract is accepted, then it determines the outcome; if it is rejected, both parties receive payoff zero. In the second game, the worker offers the contract and the manager accepts or rejects it. Let y^M be the subgame-perfect equilibrium production in the first game and let y^W be the subgame-perfect equilibrium production in the second game. What is the relation between y^M and y^W? Explain, verbally or graphically.

C4. There are two states of the world, s_1 and s_2, with known probabilities p_1 and p_2. A risk-neutral, expected-profit maximizing insurance firm has an opportunity to make a contract with a risk-averse consumer, before it is known which state will occur. After the uncertainty is resolved, the state will be common knowledge, so the contract can make the outcome depend on the state. The consumer has income y_1 in state 1 and y_2 in state 2, and maximizes the expectation of a state-*in*dependent von Neumann-Morgenstern utility function $u(x) = \ln (1+x)$ (where ln is the natural logarithm, so that $u'(x) = 1/(1+x)$.

Suppose the insurance company offers the consumer a contract whereby s/he pays x, with x (> or =) 0), and receives \$0x in state 1 and \$2x in state 2.

(a) Write the problem that determines the consumer's optimal demand for insurance, x, as a function of the probability $p_2 = 1 - p_1$.

(b) For what value of p_2 is the insurance company's expected profit zero? Does your answer depend on the consumer's demand for insurance? Explain.

(c) Use the consumer's first-order condition to show that if p_2 is higher than the value you identified in (b), the consumer will buy so much insurance that s/he is better off if state 2 occurs than if state 1 occurs. Is this consistent with the insurance company staying in business? Explain.

(d) Characterize, as far as you can, the Pareto-efficient contract that gives the consumer the same expected utility as no insurance (that is, as her/his state-contingent endowment). Characterize the contract in terms of the firm's *net* payment from the consumer in states 1 and 2, x_1 and x_2. How do the firm and the consumer share the risk in this Pareto-efficient contract?

Now suppose everything is as described above except that the state and the consumer's endowment will be observed only by the consumer, whose report of the state cannot be verified.

(e) Write the incentive constraints that restrict x_1 and x_2 if the firm must rely on the consumer's unverifiable report of the state (and the consumer will lie if it yields him a better outcome).

(f) Characterize, as far as you can, the contract for the firm that maximizes its expected profit subject to the constraint that the consumer has at least the same expected utility as with no insurance (that is, with her/his endowment).

C5. Consider contracting between a single firm and a worker who may have one of two productivity "types," H and L, where $0 < \text{Prob}\{\theta = \theta_H\} = \lambda < 1$. Productivity is measured in dollars, so a firm that hires a worker of type θ at wage w realizes profits $\theta - w$ from that worker. The worker's utility of not working for the firm is R_H for H and R_L for L, where $\theta_H > R_H > 0$ and $\theta_L > R_L > 0$. Only the worker observes θ before contracting, but λ is common knowledge.

In contracting, the firm makes an all-or-nothing contract offer. The worker then observes his type, and accepts or rejects the contract. If the worker rejects, he gets R_H if H and R_L if L, and the firm gets 0. If he accepts, the contract governs the relationship.

Suppose first that θ will become common knowledge before the terms of the contract are enforced, and this fact is common knowledge.

(a) Being careful to explain your notation and what contracts can be enforced, write the problem that determines the firm's optimal contract, including all relevant constraints, and solve it as far as you can.

Now suppose that the firm will never know θ, and this fact is common knowledge.

(b) Being careful to explain your notation and what contracts can be enforced, write the problem that determines the firm's optimal contract, including all relevant constraints, and solve it, graphically or algebraically, as far as you can. (Hint: What are the only possible optimal values of the wage in the firm's contract? How can you tell which is optimal?)

(c) If an H worker is employed when λ is near 0, would he be employed if λ were near 1? Explain.

C6. Reconsider the model of problem C5 when the job has a task level, $t \geq 0$, which the firm can set along with the wage. A type-θ worker who accepts a job with wage w and task level t has utility $u(w,t \mid \theta) = w - c(t, \theta)$, where $c(0, \theta) = 0$, $c_t(t, \theta) > 0$, and $c_{tt}(t, \theta) > 0$ for all θ; and $c_\theta(t, \theta) < 0$ and $c_{t\theta}(t,\theta) > 0$ for all $t > 0$. The task level t has no effect on the worker's productivity, which is still either θ_H or θ_L. Everything else about the job is just as in problem 2. (This version of the model is close to the analysis we did in class, except that $c_{t\theta}(t,\theta) > 0$ for $t > 0$, so H workers dislike jobs with high t *more* than L workers; the firm is a monopsonist rather than a competitor; and $R_H > R_L > 0$.)

Suppose first that, although θ is unknown to the firm at the time of contracting, θ will become common knowledge before the terms of the contract are enforced, and this fact is common knowledge.

(a) Being careful to explain your notation and what contracts can be enforced, write the problem that determines the firm's optimal contract, including all relevant constraints, and solve it as far as you can.

Now suppose that the firm will never know θ, and this fact is common knowledge.

(b) Being careful to explain your notation and what contracts can be enforced, write the problem that determines the firm's optimal contract, including all relevant constraints, and solve it, graphically or algebraically, as far as you can. Be sure to take the acceptance decision of each worker type into account. (Hint: The firm would like to find a way to pay an L worker less than an H worker, because the L worker has a lower reservation wage ($R_H > R_L > 0$), but the firm can't *enforce* different contracts for H and L because it will never know which is which. Try to figure out if, when $c_{t\theta}(t,\theta) > 0$ for $t > 0$, there is any way the firm could benefit from offering two different contracts that screen H and L workers.)

C7. Suppose the government wishes to contract with a contractor to do a project, and seeks to minimize the monetary compensation it must pay to the contractor to complete the project. The contractor's production cost for the project depends on the contractor's effort level, $z \geq 0$, and a random factor, t. Specifically, production cost $C \equiv c(z,t) \equiv 2t/(z+1)$, where $t = 1$ with probability ½ and $t = 2$ with probability ½. The contractor seeks to maximize his expected earnings net of production cost and effort cost: monetary compensation minus production cost minus effort cost z (effort costs 1 per unit). Assume that first the government proposes a contract, which the contractor must accept; and then the contractor chooses an effort level z before the uncertainty is resolved (that is, not knowing t). When the project is complete, the contractor is then paid as specified in the contract.

First suppose that the government cannot directly observe the contractor's effort level, and that it uses a "cost plus" contract, so the contractor's compensation is $(1+s)C$, where $s > 0$ is a constant.

(a) If the government can observe the actual production cost, then how hard does the contractor work?

(b) If $s = 3$, then what is the contractor's expected utility?

Now suppose that the government cannot directly observe the contractor's effort level, and that it pays the contractor a fixed fee, K.

(c) Show that the contractor supplies the same level of effort for all K. Find this effort level.

(d) Compute the value of K that makes the contractor indifferent between a fixed fee and a cost-plus contract with $s = 3$. Show that with this value of K, the government prefers the fixed-fee to the cost-plus contract.

Now imagine that the government *can* directly observe the contractor's effort level.

(e) What effort level would the government want the contractor to choose in order to minimize the production and effort cost of the project, subject to the constraint that the expected earnings of the contractor (net of effort and production cost) are nonnegative?

C8. There are two states of the world, 1 and 2. The probability of state 1 is known to be p. A risk-neutral, expected-profit maximizing insurance firm has an opportunity to make a contract with a risk-averse consumer, before it is known which state will occur. After the uncertainty is resolved, the state will be common knowledge, so the contract can make the outcome depend on the state. There is one consumption good, whose price is known to be one in each state. The consumer's endowment of the consumption good is w_i if state i occurs, so in general he bears uncertainty. An insurance contract can be thought of as specifying his consumption in each state, which in general will be higher than w_i in one state and lower in the other. Write the consumer's consumption of the good if state i occurs as c_i, and his von Neumann-Morgenstern utility function as u(c). Thus, given his opportunities, he seeks to maximize expected utility $pu(c_1) + (1-p)u(c_2)$. The firm, given its opportunities, seeks to maximize $p(w_1 - c_1) + (1-p)(w_2 - c_2)$.

(a) Characterize, as far as you can, the Pareto-efficient contract that gives the insurance firm zero expected profits. How do the firm and the consumer share the risk in this contract?

(b) Characterize, as far as you can, the Pareto-efficient contract that gives the consumer the same expected utility as no insurance (that is, as his endowment). How do the firm and the consumer share the risk in this contract?

Now suppose that everything is as described above, but that the state and the consumer's endowment will be observed only by the consumer, and the consumer's report of the state cannot be verified by the insurance firm.

(c) Write the incentive constraints that must hold if the firm must rely on the consumer's report of the state (and the consumer will lie if it yields him a better outcome).

(d) Characterize, as far as you can, the best contract for the firm that gives the consumer the same expected utility as no insurance (that is, as his endowment).

C9. A firm must make a wage contract with a worker. The worker expends effort $e \geq 0$ on the job. Effort e costs the worker e^2. The firm's profit, before paying the worker, is πe, where $\pi > 0$. Suppose the firm first offers a contract to the worker, who observes it and then accepts or rejects it. If the worker rejects, both parties get 0. If the worker accepts, then he selects an effort level e, the firm's profits πe are realized, and the firm's and the worker's payoffs are determined by the contract.

First suppose that the contract must be a linear "profit sharing" contract, so that firm chooses a fixed fraction, t, in the contract, and the worker gets $t\pi e$, that fraction of the firm's profit, and the firm gets the remaining $(1-t)\pi e$.

(a) Find the subgame-perfect equilibrium of this game by backward induction, being sure to specify the strategies in full, and determine the subgame-perfect equilibrium levels of t and e.

(b) Compute the subgame-perfect equilibrium payoffs. Is this outcome efficient?

Now suppose that the contract must take the form $w(\pi e) = r\pi e + s$, so that the worker receives base salary s and, in addition, a fraction r of the firm's profit.

(c) Find the subgame-perfect equilibrium of this game by backward induction, being sure to specify the strategies in full, and determine the subgame-perfect equilibrium levels of r, s, and e.

(d) Compute the efficient level of effort, e. Can you design a contract that depends only on gross profit πe (that is, a function of the form $v(\pi e)$) that induces the worker to choose the efficient level of effort? Explain why or why not.

D. Incentives and Mechanism Design (see also problems at MWG 918-925; and 1-5 at Kreps 715-717)

D1. Consider a firm with two owners, Alice and Bill. Suppose that owner Alice is the sole operating partner, who makes all decisions in the firm, while owner Bill makes no decisions. Each tries to maximize his/her money income. Suppose that the firm's demand function is given by $Q = 342 – 2p$ and the firm's total variable costs of production are given by $TC(Q) = 20Q$. (Assume for the purposes of this question that there are no fixed costs.)

Consider two alternative contracts between Alice and Bill that specify how they are to split the earnings of the firm:

Contract 1: Alice gets 66.7% (2/3rds) of all revenues, but also pays all costs. Bill gets 33.3% (1/3rd) of all revenues (and doesn't pay any costs).

Contract 2: Alice and Bill split the firm's net profits 50%-50%, but, in addition, Alice gets a salary of $1700 for operating the firm. (In other words, Alice's salary comes out of gross profit (= total revenues less total production costs) *before* they split net profits (= total revenues less total production costs less the $1700 payment to Alice).)

(a) Which contract does each owner prefer? Explain why by showing the amount of income each gets under each contract. (Hint: The calculations here and below are simpler if you start by inverting the demand curve to get the average revenue curve.)

(b) Is there a contract that would be better than Contract 1 for both owners? If so, describe such a contract and explain how the firm's optimal decisions (quantity and price) would differ under this contract from those under Contract 2 (Note: Contract 2, not Contract 1).

(c) Is there a contract that would be better than Contract 2 for both owners? If so, describe such a contract and explain how the firm's optimal decisions (quantity and price) would differ under this contract from those under Contract 2. (Note: Contract 2, not Contract 1).

(d) Explain why Bill could ever prefer to share a smaller fraction of *revenues* to sharing a larger fraction of *profits*.

D2. If an individual works he produces L units of output and has utility function u(c), where c is his consumption. If he doesn't work he has utility function v(c). Both $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave. With probability θ a worker is well and can work, and with probability $(1 - \theta)$ he is sick, and cannot work. Sickness does not *directly* affect the worker's utility; so, for example, when he cannot work his utility function is still v(c). The worker always knows whether he is sick or well, but the government may or may not know this, as indicated below. All this is common knowledge. The government deals with many ex ante identical workers, and sickness is independent across workers, so the government's budget constraint reduces to equating the expected value of output and the expected value of consumption.

First suppose that the government can observe whether each worker is sick or well.

(a) Identify the set of rules for deciding who will work and determining each worker's consumption that are feasible for the government.

(b) Which of the rules from part a maximizes the ex ante (before he knows if he is sick or well) expected utility of a typical worker? Assume that the allocation with no work and zero consumption is not optimal, say who will work, and write the conditions that determine the optimal levels of consumption when the worker is sick or well, c_s and c_w (but don't try to solve them).

Now suppose that the government can only observe whether a worker works or not, not if he is sick or well. c_w and c_s now denote a worker's consumption when he does (c_w) and does not (c_s) work.

(c) For what combinations of c_s and c_w will a worker work when he can? Identify the set of rules for deciding who will work and determining workers' consumptions that are feasible for the government.

(d) Show that if $v'(c_s) = u'(c_w)$ implies $v(c_s) < u(c_w)$, then the optimal rule from part b is still optimal.

(e) If $v'(c_s) = u'(c_w)$ implies $v(c_s) > u(c_w)$ and the government tries to implement the optimal rule identified in part b, what will happen? Write the problem that determines the optimal rule when $v'(c_s) = u'(c_w)$ implies $v(c_s) > u(c_w)$, and write the conditions that determine the optimal c_s and c_w.

E. Miscellany (see also problems 8.B.1-7, 8.C.1-4, and 8.D.1-9, 8.E.1-3 at MWG 262-266; 9.B.1-14, 9.B.10, 9.B.11; 9.C.1, 3-4, and 7 at MWG 304-305; 12.C.1-17 at MWG 430, 12.C.5, 12.C.8; 12.D.1-5, 12.E.1-7, 12.F.1-4, 12.AA2 at MWG 434-435; 13.B.1-9 at MWG 473-474; 13.C.1-6 and 13.D.1-4 at MWG 474-476; 14.B.1-8 at MWG 507-508; 14.C.1-9 at MWG 508-510; 3-4 at Kreps 385-386; 2-4 at Kreps 498-501; 1- 4, 14-18, and 20-21 at Kreps 451-462; 1 at Kreps 546-547, 1-2 and 4-7 at Kreps 616-623; 1-5 and 7-10 at Kreps 654)

E1. Consider a duopoly game in which firm A's and firm B's goods are *im*perfect substitutes, with demand functions $q^A(p^A, p^B)$ and $q^B(p^A, p^B)$ and inverse demand functions $p^A(q^A, q^B)$ and $p^B(q^A, q^B)$, where p^i is firm i's price and q^i is firm i's quantity. Each firm's unit cost is 0, and each firm maximizes profit. Price is the strategic variable, as in a Bertrand model, but firm A chooses its price first, irreversibly, and firm B observes firm A's price before choosing its own price. The structure of the game is common knowledge.

(a) Write the problem that determines firm B's optimal price, given firm A's price. Write and interpret the first-order conditions for this problem. (You are not asked to solve the problem.)

(b) Which is the more natural assumption for Bertrand duopolists whose goods are *im*perfect substitutes, that p^A and p^B are strategic substitutes or complements? Explain, and use your first-order conditions from (a) to derive conditions on the demand function that imply your "natural assumption."

(c) Assuming that firm B has a unique optimal price for every value of p^A, and writing this optimal price $p^{B*}(p^A)$, write the problem that determines firm A's subgame-perfect equilibrium price. Assuming that the function $p^{B*}(p^A)$ is differentiable, write and interpret the first-order conditions for this problem. (You are not asked to solve the problem.)

(d) Under your "natural assumption" from (b), will firm A's price in subgame-perfect equilibrium be higher or lower than it would be in equilibrium in a standard Bertrand model with the same demand and cost functions, in which the firms choose their prices simultaneously? Explain, using the idea of strategic substitutes or complements, natural assumptions about demand, and your first-order condition from (c).

E2. Firm A and Firm B are duopolists whose goods are *im*perfect substitutes, with demand functions $q^A(p^A, p^B; a)$ and $q^B(p^A, p^B; b)$ and inverse demand functions $p^A(q^A, q^B; a)$ and $p^B(q^A, q^B; b)$, where p^i is firm i's price, q^i is firm i's quantity, and i (= a, b) is firm i's level of advertising. Each firm has unit cost 1 for the good and constant unit cost for advertising. The firms play a two-stage game, choosing a and b simultaneously and observably in the first stage and then choosing prices or quantities (as explained below) in the second stage.

For the first part of the question, suppose that the duopolists are Bertrand competitors in the second stage, choosing p^A and p^B simultaneously.

(a) Write the first-order conditions that determine the equilibrium p^A and p^B, given a and b.

Now suppose that the cost of advertising goes down for firm A (but not Firm B), so that Firm A chooses a higher level of a in the first stage. Suppose further that advertising and price are complements for each firm, so that Firm A's higher a would make it optimal for Firm A to choose a higher p^A in the second stage, for any level of b and p^B.

(b) Which is the more natural assumption for Bertrand duopolists whose goods are *im*perfect substitutes, that p^A and p^B are strategic substitutes or complements? Explain.

(c) Given your answer to (b), analyze the effect of Firm A's reduction in advertising cost on the equilibrium values of b, p^A, and p^B. (Correct verbal answers here are as acceptable as algebra.)

For the rest of the question, suppose that the firms are Cournot competitors in the second stage, choosing q^A and q^B simultaneously.

(d) Write the first-order conditions that determine the equilibrium q^A and q^B, given a and b.

Suppose again that the cost of advertising goes down for firm A (but not Firm B), so that Firm A chooses a higher level of a in the first stage. Suppose further that advertising and quantity are complements for each firm, so that Firm A's higher level of a would make it optimal for Firm A to choose a higher q^A in the second stage, for any given level of b and q^B.

(e) Which is the more natural assumption for Cournot duopolists whose goods are *im*perfect substitutes, that q^A and q^B are strategic substitutes or complements? Explain.

(f) Given your answer to (e), analyze the effect of Firm A's reduction in advertising cost on the equilibrium values of b, q^A, and q^B. (Correct verbal answers here are as acceptable as algebra.)

E3. Consider an ultimatum bargaining game with two players, R and C, and three possible contracts, A, B, and Z. The rules of bargaining allow R to propose one of these contracts, which C must then either accept or reject. If C accepts R's proposal, it determines the outcome; if he rejects it, the outcome is N (for "No Deal"). R's payoffs for the outcomes A, B, Z, and N are 1, 2, 3, and 0 respectively, and C's payoffs are 2, 1, -1, and 0 respectively. Clearly identifying each player's feasible pure strategies and making R the Row player, write the game tree and payoff matrix and identify the pure-strategy subgame-perfect equilibrium or equilibria, the pure-strategy equilibrium or equilibria (subgame-perfect or not), and the weak perfect Bayesian equilibrium or equilibria (including their beliefs, when they don't follow immediately from the strategies), when:

(a) C can observe, before deciding whether to accept or not, exactly which contract R has proposed

(b) C can observe, before deciding whether to accept or not, whether R has proposed Z or [either A or B] (but if R has proposed [either A or B], then C cannot tell which one). (For the extensive form, you need not re-draw the tree; just say how it must be changed from your tree for (a).)

(c) C can observe, before deciding whether to accept or not, whether R has proposed A or [either B or Z] (but if R has proposed [either B or Z], then C cannot tell which one). (For the extensive form, you need not re-draw the tree; just say how it must be changed from your tree for (a).)

(d) Assuming that players will play a weak perfect Bayesian equilibrium, say in which if any of the environments described in parts (a), (b), and (c) both players would benefit if contracts like Z were made legally unenforceable, so that if Z were proposed and accepted, the outcome would be N rather than Z. What if players could play any subgame-perfect equilibrium?

E4. Consider a two-firm Cournot model in which the firms have constant unit costs but the costs differ across firms. Let c_j be firm j's unit cost, j=1,2, and assume that $c_1 > c_2$. The firms' products are perfect substitutes, and if $q = q_1 + q_2$ is total output in the market, the inverse demand function is $p(q) = a – bq$, with $a > c_1 > c_2$ and $b > 0$. The structure is common knowledge.

(a) Derive the Nash equilibrium of the Cournot game in which firms choose their quantities simultaneously. For what values of c_1, c_2, a, and b does this equilibrium involve only one firm producing? Which firm will this be?

(b) When the equilibrium in (a) involves both firms producing, how do their equilibrium outputs and profits vary when c_1 increases? Explain your answer for firm 2, using the notion of strategic substitutes.

E5. Two people, Rhoda ("R" for short) and Colin ("C") must decide independently whether to try to meet at the fights ("F") or the ballet ("B"). Before R and C decide where to try to meet, R (but not C) must announce her intentions, f or b, about where she plans to go. Then R and C choose, simultaneously and independently, between F and B. R is free to choose either F or B independent of her announcement f or b, but if she announces f but chooses B, or announces b but chooses F, she incurs a cost $c \geq 0$. If R and C both choose F, R's payoff is 2 less whatever cost she incurs, and C's payoff is 1. If R and C both choose B, R's payoff is 1 less whatever cost she incurs, and C's payoff is 2. If R chooses F but C chooses B, or vice versa, R's payoff is 0 less whatever cost she incurs, and C's payoff is 0. For example, if R announces f, chooses B, and C chooses B, R's payoff is 1-c. The structure of the game, including the announcement stage, is common knowledge.

(a) Clearly identifying players' decisions and information sets, draw the extensive form (game tree) for this game. (It's easier to draw the information sets if you put both of R's decisions first.)

(b) Identify the pure-strategy subgame-perfect equilibrium outcome(s) and payoffs when $c > 2$.

(c) Identify the pure-strategy subgame-perfect equilibrium outcome(s) and payoffs when $0 \leq c < 1$.

E6. Consider a Bertrand duopoly game with two profit-maximizing firms, 1 and 2, whose goods are perfect substitutes. Total market demand is $Q(p) = a–bp$, where $a > 0$ and $b > 0$; so that if p_1 and p_2 are the firms' prices, and $p_i < p_j$, firm i's demand is $Q(p_i)$ and firm j's demand is 0; and if $p_i = p_j$, firm i's and firm j's demands are each $Q(p_i)/2 = Q(p_j)/2$. Each firm's unit cost is $c > 0$, but firm 1 (only) has the option of irreversibly and observably paying a fixed fee $F > 0$ to lower its unit cost to w, where $0 < w < c$, before firms 1 and 2 simultaneously choose their prices, (F, w, and c are exogenous parameters.) The structure is common knowledge.

(a) Clearly identifying each firm's possible pure strategies, solve for the subgame-perfect Nash equilibrium strategy profile or profiles (all of them, if there is more than one), as a function of the parameters a, b, F, w, and c; that is, say how the parameters determine the equilibrium. (Hint: You may have to model the effect of one firm undercutting the other's price by a very small amount. You can either make this amount a small number $e > 0$ and pass to the limit as e approaches 0, or assume that prices must be set in pennies, which are small relative to the other magnitudes, and that c, w, and F are in even numbers of pennies.)

E7. Consider a duopoly game in which each firm's unit cost is 0. The firms' goods are perfect substitutes and total market demand is $Q(p) = 10 – p$. Thus, if $p_i < p_j$, firm i's demand is $Q(p_i)$ and firm j's demand is 0; and if $p_i = p_j$ the firms share the market equally, so that firm i's and firm j's demands are each $Q(p_i)/2 = Q(p_j)/2$. Each firm seeks to maximize profits. Price is the strategic variable, but firm 1 chooses its price first, then firm 2 decides whether or not to enter at a cost of 9, which is irreversibly sunk if firm 2 enters; and finally firm 2, if it enters, chooses its own price. The structure of the game is common knowledge, including what firms observe and when below.

Suppose first that firm 1 first chooses p_1, irreversibly, and then firm 2 observes p_1 before deciding whether or not to enter and choosing p_2, also irreversibly.

(a) Approximately what price p_2 will firm 2 choose if it decides to enter after observing p_1?

(b) For what values of p_1 will firm 2 find it profitable to enter, given the sunk entry cost of 9?

(c) Write the problem that determines firm 1's optimal pricing policy, taking the possibility of deterring entry by firm 2 into account, and use it to identify an approximate subgame-perfect equilibrium, being sure to fully describe both firms' strategies.

Now suppose that firm 1 first chooses p_1, then firm 2 observes p_1 before deciding whether or not to enter and choosing p_2, irreversibly. However, if firm 2 enters, firm 1 can observe p_2 and then costlessly change p_1 to any desired value. (If firm 2 stays out, firm 1 must keep its initial p_1.)

(d) Identify a subgame-perfect equilibrium, being sure to fully describe both firms' strategies.

Now suppose that firm 1 first chooses p_1, irreversibly, but that firm 2 *does not* observe p_1 before deciding whether or not to enter and choosing p_2 (so that firm 1's and firm 2's pricing decisions and firm 2's entry decision are all strategically simultaneous).

(e) Is there a pure-strategy Nash equilibrium? If so, identify one. If not, briefly explain why not.

E8. Consider a duopoly game in which each firm's unit cost is 0. Price is the strategic variable, and the firms maximize profits. The firms' goods are perfect substitutes and total market demand is $Q(p) = 10 – p$. Thus, if $p_i < p_j$, firm i's demand is $Q(p_i)$ and firm j's demand is 0; and if $p_i = p_j$ the firms share the market equally, so that firm i's and firm j's demands are each $Q(p_i)/2 = Q(p_j)/2$. The structure of the game is common knowledge, including the parts described below.
Suppose first that firm 1 first chooses p_1 irreversibly, and then firm 2 observes p_1 before choosing p_2, also irreversibly.

(a) Approximately what price p_2 will firm 2 choose if it decides to enter after observing p_1? (I say "approximately" because exact best responses do not exist in the continuous version of this model. Answer here and below by assuming prices must be in multiples of a very small number $e > 0$.)

(b) Identify an approximate subgame-perfect equilibrium (one is enough), fully describing both firms' strategies.

Now suppose that firm 1 first chooses p_1, but that this choice is reversible at a cost as explained below. Firm 2 then observes p_1 and chooses p_2, irreversibly. Finally, firm 1 observes p_2 and decides whether to change its price at cost c, where $0 < c < 25$. (That is, firm 1 can stick with its initial value of p_1, incurring no additional costs, or switch to any desired different value p_1', at a cost of c. If it switches, p_1' then determines profits just as p_1 would have, except for the cost c.)

(c) For what values of p_2 will firm 1 decide (for very small e) to pay c to change p_1, and what value of p_1' will it choose then? (Just write the condition and give the answer; don't simplify the algebra.)

(d) Taking your answer to (c) into account, what value of p_2 is optimal for firm 2?

(e) What initial value or values of p_1 is or are optimal for firm 1?

(f) Identify an approximate subgame-perfect equilibrium, fully describing both firms' strategies.

E9. This question concerns games in which two firms, Top Dog and Hot Dog, decide simultaneously whether to enter a new product market. The payoffs are as follows:

		Hot Dog	
		In	Out
Top Dog	In	1 , 1	5 , 2
	Out	2 , 4	3 , 3

First suppose that before the firms make their entry decisions, Top Dog (only) must announce its intentions to choose In (announce "i") or Out (announce "o"). Top Dog is not required to abide by its announcement, but it incurs a cost of c if its action differs from its announcement.

(a) Write the extensive form.

(b) Identify all of the pure-strategy subgame-perfect equilibria when $0 < c < 1$. Be sure to describe players' strategies completely.

E10. There are I firms in an industry. Each can try to convince Congress to give the industry a subsidy. Let h_i denote the number of hours of effort put in by firm i, and let $c_i(h_i) = w_i(h_i)^2$, where w_i is a positive constant, be the cost of this effort to firm i. When the effort levels of the firms are $(h_1,\ldots,h_I)$, the value of the subsidy that gets approved to firm i is $\alpha\, \Sigma_i\, h_i + \beta\, (\Pi_i\, h_i)$, where $\alpha > 0$ and $\beta \geq 0$ are constants. Consider a game in which the I firms decide simultaneously and independently how many hours they will each devote to this effort.

(a) Prove (using second- as well as first-order conditions!) that each firm has a strictly dominant strategy if and only if $\beta = 0$, and derive each firm's dominant strategy in this case.

(b) When $\beta > 0$, are firms' efforts strategic substitutes or strategic complements? Explain.

(c) When $\beta > 0$, how does the symmetric equilibrium level of effort relate to the symmetric, Pareto-efficient level of effort (where efficiency is defined taking only the firms' benefits into account)—that is, the level of effort that would be best for all firms, if all firms chose it? (Hint: consider the nature of the externalities that firm i's effort creates for the other firms when $\beta > 0$.)

E11. Consider a Cournot duopoly game in which two firms, 1 and 2, simultaneously choose the quantities they will sell, q_1 and q_2, with the goal of maximizing their profits. Each firm's cost is c per unit sold, and the price each firm receives per unit sold, given q_1 and q_2, is $P(q_1, q_2) = a – b\,(q_1+ q_2)$, where $a > 0$ and $b > 0$. The structure of the game is common knowledge.

(a) Write firm i's profit-maximization problem and use the first- and second-order conditions to derive its best-response function, expressing its optimal output q_i as a function of firm j's output q_j.

(b) Identify the Nash equilibrium strategy profile(s) in this game.

(c) Which values of q_i are rational for firm i, for some feasible value of q_j?

(d) Which values of q_j are rational for firm j, for some value of q_i that is consistent with firm i being rational?

(e) Identify each firm's rationalizable strategies in this game. (Hint: This can be done using algebra, but I think it's easier to use your answer to (a) and a graphical argument.)
(f) Answer parts (a), (b), and (e) (yes, (e)) again for the analogous game with three firms, 1, 2, and 3, with unit cost c, outputs q_1, q_2, and q_3, and price per unit sold $P(q_1, q_2, q_3) = a - b\,(q_1 + q_2 + q_3)$.

E12. Imagine a market setting with three firms. Firms 2 and 3 are already operating as monopolists in two different industries (they are not competitors). Firm 1 must decide whether to enter Firm 2's industry and compete with Firm 2, or enter Firm 3's industry and thus compete with Firm 3. Production in Firm 2's industry occurs at zero cost, while the cost of production in Firm 3's industry is 2 per unit. Demand in Firm 2's industry is given by $p = 9 - Q$, while demand in Firm 3's industry is given by $p' = 14 - Q'$, where p and Q denote price and total quantity in Firm 2's industry and p' and Q' denote price and total quantity in Firm 3's industry.

The firms interact as follows. First, Firm 1 chooses between E^2 and E^3, where E^2 means "enter Firm 2's industry" and E^3 means "enter Firm 3's industry." This choice is observed by Firms 2 and 3. Then, if Firm 1 chose E^2, Firms 1 and 2 compete as Cournot duopolists, where they select quantities q_1 and q_2. In this case, Firm 3 automatically gets the monopoly profit of 36 in its own industry. On the other hand, if Firm 1 chose E^3, then Firms 1 and 3 compete as Cournot duopolists, where they select quantities q_1' and q_3'. In this case, Firm 2 automatically gets the monopoly profit of 20¼ in its own industry.

(a) Calculate the subgame-perfect Nash equilibrium of this game and report the subgame-perfect equilibrium quantities. In the equilibrium, does Firm 1 enter Firm 2's industry or Firm 3's industry?

(b) Is there a Nash equilibrium (not necessarily subgame-perfect) in which Firm 1 selects E^2? If so, describe it. If not, briefly explain why.

E13. Two players play a game in which they must agree how to divide a prize. Baker decides how large the total prize will be, either \$10 or \$100 (the only choices). Cutler decides how to divide the prize chosen by Baker, either a 50%-50% split or a 90%-10% split (the only choices), where in the latter case Cutler gets 90%. The payoffs, in dollars (with Cutler's listed first), are:

		Baker: \$10 prize	Baker: \$100 prize
Cutler	50:50	5, 5	50, 50
	90:10	9, 1	90, 10

First suppose that the players must make the above decisions simultaneously, and cannot change the game.

(a) Find each player's rationalizable strategies and the Nash equilibrium or equilibria.

Now suppose that the players' basic decisions are the same, but that some kind of *strategic move* may be possible. A strategic move might involve one or the other player observably committing to his decision before the other decides, or one or the other player observably committing to a contingent rule that makes his decision depend on the other's decision in a particular way. An example of the latter kind of commitment might be for Cutler to commit himself to decide on a 50-50 split if Baker decides on the $100 prize, but to decide on a 90-10 split if Baker decides on the $10 prize. Note that now we are analyzing a different game, in which Cutler moves first, irreversibly choosing a contingent rule, Baker observes the rule and decides on the size of the prize, and Cutler then makes the decision dictated by his chosen rule. In the rest of the question you are asked to consider all games with basic decisions as above, but in which one or the other player can make a strategic move, either a simple commitment or a commitment to a contingent rule.

(b) Which, if either, player could benefit from making a strategic move in this game, and by which kind of strategic move? Explain how the strategic move works and its effect on the outcome, using the idea of subgame-perfect equilibrium.

E14. Consider the Battle of the Sexes game with payoffs as indicated below. Assume, here and below, that the structure is common knowledge. For each of the variations of timing and information described below, write the game tree and payoff matrix, and then find the game's subgame-perfect equilibrium or equilibria, and its equilibria (subgame-perfect or not):

	Fights	Ballet
Fights	1 3	0 0
Ballet	0 0	3 1

(a) The original simultaneous-move game is a complete model of the players' situation.

(b) The game is modified so that Row chooses her/his strategy first and Column gets to observe her/his choice (including the realization of randomization) before choosing her/his own strategy.

(c) The game is modified so that Row chooses her/his strategy first but Column does NOT get to observe her/his choice before choosing her/his own strategy.

(d) The game is modified so that Row chooses her/his strategy first and Column gets to observe her/his choice (including the realization of randomization) before choosing her/his own strategy, but then Row gets to observe Column's choice and (costlessly) revise her/his own choice if s/he wishes, and this decision ends the game (so Row cannot revise her/his choice). In this part, you are not asked to completely describe players' strategies or the entire set of equilibria, just describe the subgame-perfect equilibrium or equilibria. Hint: Does Row's initial choice have any effect on the subgames that follow?

E15. Consider a duopoly game in which each firm's unit cost is 0. Price is the strategic variable, and the firms maximize profits. The firms' goods are perfect substitutes and total market demand is $Q(p) = 10 - p$. Thus, if $p_i < p_j$, firm i's demand is $Q(p_i)$ and firm j's demand is 0; and if $p_i = p_j$ the

firms share the market equally, so that firm i's and firm j's demands are each $Q(p_i)/2 = Q(p_j)/2$. The structure of the game is common knowledge, including the parts described below.
Suppose first that firm 1 first chooses p_1 irreversibly, and then firm 2 observes p_1 before choosing p_2, also irreversibly.

(a) Approximately what price p_2 will firm 2 choose if it decides to enter after observing p_1? (I say "approximately" because exact best responses do not exist in the continuous version of this model. Answer here and below by assuming prices must be in multiples of a very small number $e > 0$.)
(b) Identify an approximate subgame-perfect equilibrium (one is enough), fully describing both firms' strategies.

Now suppose that firm 1 first chooses p_1, but that this choice is reversible at a cost as explained below. Firm 2 then observes p_1 and chooses p_2, irreversibly. Finally, firm 1 observes p_2 and decides whether to change its price at cost c, where $0 < c < 25$. (That is, firm 1 can stick with its initial value of p_1, incurring no additional costs, or switch to any desired different value p_1', at a cost of c. If it switches, p_1' then determines profits just as p_1 would have, except for the cost c.)

(c) For what values of p_2 will firm 1 decide (for very small e) to pay c to change p_1, and what value of p_1' will it choose then? (Just write the condition and give the answer; don't simplify the algebra.)

(d) Taking your answer to (c) into account, what value of p_2 is optimal for firm 2?
(e) What initial value or values of p_1 is or are optimal for firm 1?
(f) Identify an approximate subgame-perfect equilibrium, fully describing both firms' strategies.