ONLINE APPENDICES A TO E FOR

MEANINGFUL THEOREMS: NONPARAMETRIC ANALYSIS OF REFERENCE-DEPENDENT PREFERENCES

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ONLINE APPENDIX A. Generalizing Tversky and Kahneman’s notion of loss aversion with constant sensitivity and using it to simplify Proposition 4’s sufficient conditions for a rationalization.

There is strong experimental and empirical support for loss aversion, whereby reference-dependent preferences are more sensitive to changes below a reference point than to equal changes above it (Kahneman and Tversky 1979; Tversky and Kahneman 1991; Goette, Graeber, Kellogg, and Sprenger 2020). We begin with a nonparametric generalization of Tversky and Kahneman’s (1991, pp. 1047-1048) definition for the two-good case to the multi-good case. Like Tversky and Kahneman we assume constant sensitivity, but we relax their assumption of additive separability across goods. (The idea of loss aversion is still well defined with variable sensitivity, but formalizing it then is more complex, and Propositions 2 and 3 show that it is then nonparametrically irrefutable anyway.)

DEFINITION A1: [Preferences with constant sensitivity and loss aversion.]
Assume that reference-dependent preferences and an associated utility function $u(q, r)$ have constant sensitivity. A collection of regime preferences over consumption bundles satisfies loss aversion if and only if, for any observation $\{p_t, q_t, r_t\}$, the preference ordering’s global better-than set is weakly contained in each regime preference ordering’s local better-than set at the same observation.
Figure A1. Loss aversion with one active reference point (solid curves for the loss indifference map, dashed curves for the gain map)

Figure A1 illustrates loss aversion with one active reference point and two gain-loss regimes. Loss aversion is a property of the relationship between regimes’ preferences over consumption bundles, hence independent of reference points. Because Definition A1’s nesting of local and global better-than sets must hold throughout commodity space, global loss aversion is equivalent to requiring that the regime indifference maps satisfy a global single-crossing property: For any observation, across regimes that differ only in the gain-loss status of good $i$, the loss-side marginal rates of substitution between good $i$ and any other good (generalized as needed for non-differentiable preferences) must be weakly more favorable to good $i$ than the gain-side marginal rates of substitution. (Thus, neoclassical preferences are weakly loss averse.) It is this single-crossing property, not the kinks in global indifference maps that it creates, that shapes loss aversion’s nonparametric implications, which may be testable with finite data. Loss aversion precludes nonconvex kinks, so if all regime maps have convex better-than sets, then so do the associated global maps.

Corollary A1 shows that GARP for each regime’s observations plus a condition weaker than loss aversion are sufficient for a rationalization. The literature views loss aversion as an empirically well-supported assumption with important behavioral implications, but not as one that is linked to the existence of a reference-dependent rationalization, but Corollary A1 makes it part of one plausible set of sufficient conditions for such a rationalization.

**COROLLARY A1:** [Rationalization with modelable reference points via preferences with constant sensitivity that satisfy a condition weaker than loss aversion.] Suppose that reference-dependent preferences are defined over $K \geq 2$ goods, that reference-dependence is active for all $K$ goods, that the preferences, satisfy constant sensitivity and are jointly continuous, and that they satisfy Proposition 1’s equation (2).

Consider data $\{p_t, q_t, r_t\}_{t=1,...,T}$ with modelable reference points. If each regime’s
observations satisfy GARP, then any combination of rationalizing regime preferences such that there are no observations for which \( q_t \) is not on the boundary of the convex hull of \( q_t \)'s upper contour set for the associated candidate for a global preference ordering for \( r_t \), yields a rationalization with associated utility function as in (2).

**PROOF:** Any combination of rationalizing regime preferences ensures that each observation’s consumption bundle is optimal within its own regime. Consider a defection from \( q_t \in G(g; r_t) \) to some \( q \in G(g'; r_t) \) with \( g' \neq g \) and \( p_t \cdot q \leq p_t \cdot q_t \). If such a \( q \) were in regime \( g \) for \( r_t \), we would have:

\[
(A1) \quad u(q, r_t) \equiv V(q) + \sum_k [v_g^k(q^k) - v_g^k(r_t^k)] \leq V(q_t) + \sum_k [v_g^k(r_t^k) - v_g^k(r_t^k)] \equiv u(q_t, r_t).
\]

If the combination of rationalizing regime preferences is such that there are no observations \( t \) for which \( q_t \) is not on the boundary of the convex hull of \( q_t \)'s upper contour set for the candidate global preference ordering for \( r_t \), assuming global loss aversion is without loss of generality, because the global ordering can then be replaced by a convexified ordering whose upper contour sets are the convex hulls of the original global ordering without changing any observation’s optimal bundle. With \( q \in G(g'; r_t) \) with \( g' \neq g \), loss aversion implies, cancelling terms for which good \( i \)'s gain-loss status with \( r_t \) is the same for \( g \) and \( g' \):

\[
(A2) \quad \sum_k [v_{g'}^k(q^k) - v_{g'}^k(r_t^k)] \leq \sum_k [v_g^k(q^k) - v_g^k(r_t^k)].
\]

Combining (A1) and (A2) shows that defections to bundles in the budget set in other regimes is not beneficial either. ■

Corollary A1’s final “no observations not on the boundary” condition rules out bunching of observations in regions of commodity space where the rationalizing regime preferences violate loss aversion, and is therefore vacuously satisfied for regime preferences that satisfy loss aversion. This restriction on bunching brings the analysis closer to tangible features of the data, and appears to be unusual in a nonparametric analysis.
Figure A2. Rationalizing data that violate GARP when the Afriat regime preferences violate loss aversion but satisfy Corollary A1’s conditions (solid lines for the loss map, dashed lines for the gain map)

In Figure A2 the entire dataset violates GARP, the Afriat regime preferences violate loss aversion, but the data satisfy Corollary A1’s final conditions, allowing a rationalization. Only reference point \( r_1 \) is shown and observation 1 is in the good-2 loss regime. Assume that \( r_2 = [0, 0] \), so that observation 2’s budget set is entirely in the good-2 gain regime; and that \( r_3 = [0, m] \), where \( m \) is large enough that observation 3’s budget set is entirely in the good-2 loss regime. The Afriat regime preferences yield a candidate for global preferences that make all three observations’ consumption bundles optimal: Observations 2’s and 3’s budget sets are entirely in their regimes (good-2 gain and good-2 loss, respectively), so their bundles’ optimality in their regimes suffices for global optimality. Observation 1’s bundle is optimal for its good-2 loss regime preferences and Corollary 1 ensures that its bundle’s optimality extends to its entire budget set.
ONLINE APPENDIX B. Proof that Figure 4’s example cannot satisfy Proposition 5’s condition (10) for a rationalization using the Afriat rationalizing regime utilities.

Recall that \( G(g; r) \equiv \{ q \in \text{regime } g \text{ for } r \} \) and \( H(\{ q_t, r_t \}_{t=1,...,T}; g) \equiv \{ t \in \{ 1, ..., T \} \mid q_t \in G(g; r_t) \} \). In general, Proposition 5’s condition (11) is

\[
(11) \quad u(q, r) - V(r) \equiv \min_{p \in H(\{ q_t, r_t \}_{t=1,...,T}; g')} \{ U_t^{g'} + \lambda_p^{g'} p \cdot (q - q_p) \}
- \min_{p \in H(\{ q_t, r_t \}_{t=1,...,T}; g')} \{ U_t^{g'} + \lambda_p^{g'} p \cdot (r - q_p) \}
\leq \min_{p \in H(\{ q_t, r_t \}_{t=1,...,T}; g')} \{ U_t^{g} + \lambda_p^{g} p \cdot (q - q_p) \}
- \min_{p \in H(\{ q_t, r_t \}_{t=1,...,T}; g')} \{ U_t^{g} + \lambda_p^{g} p \cdot (r - q_p) \} \equiv u(q_t, r_t) - V(r_t).
\]

To show that Figure 4’s example cannot satisfy (11), specialize it observation by observation, with \( r_1 = (0, r_{21}) \) and \( r_2 = (0, r_{22}) \), so \( r_{11} = r_{12} = 0 \) and for all such \( r_1 \) and \( r_2 \) all bundles are in the gain-loss regimes with gains for good 1. With one observation per regime, observation 1 in \( g \) with gain for good 1 and loss for good 2 and observation 2 in \( g' \) with gain for goods 1 and 2, subscripts on terms like \( \lambda_p^g \) are redundant, but we keep them for comparability with (11). With one observation per regime we can eliminate the \( \min \) operators. (11) precluding advantageous defections from observation 1 in regime \( g \) to some affordable \( q \) in \( g' \) becomes

\[
(B.1) \quad u(q, r_1) - V(r_1) \equiv \{ U_1^{g'} + \lambda_2^{g'} p_2 \cdot (q - q_2) \} - \{ U_2^{g'} + \lambda_2^{g'} p_2 \cdot (r_1 - q_2) \}
\leq \{ U_1^{g} + \lambda_1^{g} p_1 \cdot (q_1 - q_1) \} - \{ U_2^{g} + \lambda_1^{g} p_1 \cdot (r_1 - q_1) \} \equiv u(q_1, r_1) - V(r_1) \text{ for } q \in G(r_1; g'),
\]

That is, for \( q \) with \( q_2 \geq r_{21} \).

Simplifying,

\[
(B.2) \quad \lambda_2^{g'} p_2 \cdot (q - r_1) \leq \lambda_1^{g} p_1 \cdot (q - r_1) \text{ for all } q \in G(r_1; g') \text{ i.e. for all } q \text{ with } q_2 \geq r_{21}.
\]

Expanding the vector products, denoting goods as scalars indexed by subscripts, so that \( q \equiv (q_1, ..., q_K) \) and \( q_t \equiv (q_{1t}, ..., q_{Kt}) \), with analogous notation for \( p, p_t, r, \) and \( r_t \), and using \( r_{11} = r_{12} = 0 \)

\[
(B.3) \quad \lambda_2^{g'} (p_{12} q_1 + p_{22} q_2 - p_{22} r_{21}) \leq \lambda_1^{g} (p_{11} q_{11} + p_{21} q_{21} - p_{21} r_{21}) \text{ for all } q \text{ with } q_2 \geq r_{21}.
\]

For a rationalization (B.3) must hold for any \( q \) with \( q_2 \geq r_{21} \) in observation 1’s budget set, or without loss of generality on its budget line \( p_{11} q_1 + p_{21} q_2 = \text{constant} \). Along that budget line (B.3)
is hardest to satisfy when \( p_{12}q_1 + p_{22}q_2 \) is maximized. Given the qualitative relationship of Figure 4’s example’s budget lines, that maximum occurs when \( q_1 = 0 \) and \( q_2 = (p_{11}q_{11} + p_{21}q_{21})/p_{21} \), which satisfies \( q_2 \geq r_{21} \). Further, \( \lambda_1^g \) and \( \lambda_2^g \)’ are their (interior) observations’ marginal utilities of income. If utility at observation 1 is \( p_{11}q_1 + p_{21}q_2 \), a dollar spent on good 1 yields \( 1/p_{11} \) units of good 1 and \( p_{21}/p_{11} = 1 \) util. Ditto for a dollar spent on good 2 at observation 1’s prices, or on good 1 or 2 at observation 2’s prices. So \( \lambda_1^g = \lambda_2^g = 1 \). Plugging in \( \lambda_1^g = \lambda_2^g = 1 \) and \( q_1 = 0 \) and \( q_2 = (p_{11}q_{11} + p_{21}q_{21})/p_{21} \) reduces the infinity of inequalities in (B.3) to the inequality

\[
(B.4) \quad p_{22}(p_{11}q_{11} + p_{21}q_{21})/p_{21} - p_{22}r_{21} \leq p_{11}q_{11} + p_{21}q_{21} - p_{21}r_{21} \\
or \quad p_{11}p_{22}q_{11} + p_{21}p_{22}q_{21} - p_{21}p_{22}r_{21} \leq p_{11}p_{21}q_{11} + p_{21}p_{21}q_{21} - p_{23}p_{21}r_{21}.
\]

Given the qualitative relationship of the example’s budget lines, if you spend all your money on good 2, observation 1’s budget set yields more of good 2 than observation 2’s. That is, for good 2,

\[
(p_{11}q_{11} + p_{21}q_{21})/p_{21} > (p_{12}q_{12} + p_{22}q_{22})/p_{22}
\]

Combining that inequality with the first line of (B.4) yields

\[
(B.5) \quad p_{12}q_{12} + p_{22}q_{22} - p_{22}r_{21} < p_{22}(p_{11}q_{11} + p_{21}q_{21})/p_{21} - p_{22}r_{21} \leq p_{11}q_{11} + p_{21}q_{21} - p_{21}r_{21} \\
or \quad p_{12}q_{12} + p_{22}q_{22} - p_{22}r_{21} < p_{11}q_{11} + p_{21}q_{21} - p_{21}r_{21}.
\]

That is, the utility of defecting from observation 1 to the best defection in regime \( g' \) (whose utility is constant along observation 2’s budget line), taking loss costs into account (with \( r_{11} = 0 \), is \( < 0 \).

Similarly, precluding advantageous defections from observation 2 in regime \( g' \) to some affordable \( q \) in regime \( g \), (11) becomes

\[
(B.6) \quad u(q, r_2) - V(r_2) \equiv \{U_1^g + \lambda_1^g p_1 \cdot (q - q_1)\} - \{U_1^g + \lambda_1^g p_1 \cdot (r_2 - q_1)\} \\
\leq \{U_2^{g'} + \lambda_2^{g'} p_2 \cdot (q_2 - q_2)\} - \{U_2^{g'} + \lambda_2^{g'} p_2 \cdot (r_2 - q_2)\} \equiv u(q_2, r_2) - V(r_2) \quad \text{for} \quad q \in G(r_2; g),
\]

that is, for \( q_2 \leq r_{22} \).

Simplifying,

\[
(B.7) \quad \lambda_1^g p_1 \cdot (q - r_2) \leq \lambda_2^{g'} p_2 \cdot (q_2 - r_2) \quad \text{for} \quad q \in G(r_2; g) \quad \text{i.e. for all} \quad q \quad \text{with} \quad q_2 \leq r_{22}.
\]

Expanding the vector products, with \( r_{11} = r_{12} = 0 \)
(B.8) \( \lambda_1^g(p_{11}q_1 + p_{21}q_2 - p_{21}r_{22}) \leq \lambda_2^g(p_{12}q_{12} + p_{22}q_{22} - p_{22}r_{22}) \) for all \( q \) with \( q_2 \leq r_{22} \).

For a rationalization (B.8) must hold for any \( q \) with \( q_2 \leq r_{22} \) in observation 2’s budget set, or without loss of generality on its budget line \( p_{12}q_1 + p_{22}q_2 = \) constant. Along that budget line (B.8) is hardest to satisfy when \( p_{11}q_1 + p_{21}q_2 = \) constant. Along that budget line (B.8) is maximized. Given the example’s budget lines, that maximum occurs when \( q_1 = (p_{12}q_{12} + p_{22}q_{22})/p_{12} \) and \( q_2 = 0 \), which satisfies \( q_2 \leq r_{22} \). Plugging in \( \lambda_1^g = \lambda_2^g = 1 \) and \( q_1 = (p_{12}q_{12} + p_{22}q_{22})/p_{12} \) and \( q_2 = 0 \), reduces the infinity of inequalities in (B.8) to the inequality

(B.9) \( p_{11}(p_{12}q_{12} + p_{22}q_{22})/p_{12} \leq p_{21}r_{22} \) or \( p_{11}p_{12}q_{12} + p_{11}p_{22}q_{22} - p_{12}p_{21}r_{22} \leq p_{12}p_{12}q_{12} + p_{12}p_{22}q_{22} - p_{12}p_{22}r_{22} \).

Given the example’s budget lines, if you spend all your money on good 1, observation 1’s budget set yields less of good 1 than observation 2’s. That is, for good 1,

(B.10) \( (p_{11}q_1 + p_{21}q_{21})/p_{11} < (p_{12}q_{12} + p_{22}q_{22})/p_{12} \).

Combining that inequality with (B.9)

(B.11) \( p_{11}q_{11} + p_{21}q_{21} - p_{21}r_{22} < p_{11}(p_{12}q_{12} + p_{22}q_{22})/p_{12} \) or \( p_{11}q_{11} + p_{21}q_{21} - p_{21}r_{22} < p_{12}q_{12} + p_{22}q_{22} - p_{22}r_{22} \).

That is, the utility of defecting from observation 2 to the best defection in regime \( g \) (whose utility is constant along observation 1’s budget line), taking loss costs into account (with \( r_{12} = 0 \), is \( < 0 \).

Combining the conditions for observations 1 and 2 yields necessary and sufficient conditions for a rationalization using the Afriat regime preferences

(B.12) \( p_{12}q_{12} + p_{22}q_{22} - p_{22}r_{21} < p_{11}q_{11} + p_{21}q_{21} - p_{21}r_{21} \)

or \( p_{12}q_{12} + p_{22}q_{22} - p_{11}q_{11} - p_{21}q_{21} < (p_{22} - p_{21})r_{21} \)

and

(B.13) \( p_{11}q_{11} + p_{21}q_{21} - p_{21}r_{22} < p_{12}q_{12} + p_{22}q_{22} - p_{22}r_{22} \)

or \( (p_{22} - p_{21})r_{22} < p_{12}q_{12} + p_{22}q_{22} - p_{11}q_{11} - p_{21}q_{21} \)

Chaining the second lines yields a contradiction. ■
ONLINE APPENDIX C. Farber’s (2005, 2008) dataset

Figure C.1: Hours and earnings choices, driver by driver
ONLINE APPENDIX D. Models that relax additive separability across goods

Figure D.1: Pass rates by reference-point model with reference-dependence in hours only, relaxing additive separability across goods

Notes: **Horizontal axis**: pass rate [0,1]. **Vertical axis**: driver identifier {1,..21}.
**Horizontal lines**: extent of the bounds on pass rate for each reference-point model; where the line is a point the upper and lower bounds coincide and the pass rate is point-identified. **Vertical line**: the pass rate for the neoclassical model.
Figure D.2: Pass rates by reference-point model with reference-dependence in earnings only, relaxing additive separability across goods.

**Notes:**
- **Horizontal axis:** pass rate [0,1].
- **Vertical axis:** driver identifier \{1,...,21\}.
- **Horizontal lines:** extent of the bounds on pass rate for each reference point model; where the line is a point the upper and lower bounds coincide and the pass rate is point-identified.
- **Vertical line:** the pass rate for the neoclassical model.
Figure D.3: Pass rates by reference-point model with reference-dependence in both hours and earnings, relaxing additive separability across goods

**Reference Dependence Active With Respect To Hours & Earnings**

Notes: *Horizontal axis*: pass rate [0,1]. *Vertical axis*: driver identifier{1,..21}.

*Horizontal lines*: extent of the bounds on pass rate for each reference point model; where the line is a point the upper and lower bounds coincide and the pass rate is point-identified. *Vertical line*: the pass rate for the neoclassical model.
Figure D.4: Selten measures by reference-point model with reference-dependence in hours only, relaxing additive separability across goods.

Reference Dependence Active With Respect To Hours

Notes: Horizontal axis: Selten Index [-1,1]. Vertical axis: driver identifier [1..21]. Horizontal lines: extent of the bounds on the Selten index for each reference point model; where the line is a point the upper and lower bounds coincide and the Selten Index is point-identified. Vertical line: the Selten Index for the neoclassical model.
Figure D.5: Selten measures by reference-point model with reference-dependence in earnings only, relaxing additive separability across goods

Notes: Horizontal axis: Selten Index [-1,1]. Vertical axis: driver identifier [1..21]. Horizontal lines: extent of the bounds on the Selten index for each reference point model; where the line is a point the upper and lower bounds coincide and the Selten Index is point-identified. Vertical line: the Selten Index for the neoclassical model.
Figure D.6: Selten measures by reference-point model with reference-dependence in both hours and earnings, relaxing additive separability across goods

Notes: *Horizontal axis*: Selten Index [-1,1]. *Vertical axis*: driver identifier {1,..21}. *Horizontal lines*: extent of the bounds on the Selten index for each reference point model; where the line is a point the upper and lower bounds coincide and the Selten Index is point-identified. *Vertical line*: the Selten Index for the neoclassical model.
ONLINE APPENDIX E. Models that impose additive separability across goods

Figure E.1: Pass rates by reference-point model with reference-dependence in hours only, imposing additive separability across goods

Notes: **Horizontal axis**: pass rate [0,1]. **Vertical axis**: driver identifier [1..21]. **Horizontal lines**: extent of the bounds on pass rate for each reference point model; where the line is a point the upper and lower bounds coincide and the pass rate is point-identified. **Vertical line**: the pass rate for the neoclassical model.
Figure E.2: Pass rates by reference-point model with reference-dependence in earnings only, imposing additive separability across goods.

Notes: Horizontal axis: pass rate [0,1]. Vertical axis: driver identifier {1,..,21}. Horizontal lines: extent of the bounds on pass rate for each reference point model; where the line is a point the upper and lower bounds coincide and the pass rate is point-identified. Vertical line: the pass rate for the neoclassical model.
Figure E.3: Pass rates by reference-point model with reference-dependence in both hours and earnings, imposing additive separability across goods

Notes: *Horizontal axis*: pass rate [0,1]. *Vertical axis*: driver identifier \{1,..,21\}. *Horizontal lines*: extent of the bounds on pass rate for each reference point model; where the line is a point the upper and lower bounds coincide and the pass rate is point-identified. *Vertical line*: the pass rate for the neoclassical model.
Figure E.4: Selten measures by reference-point model with reference-dependence in hours only, imposing additive separability across goods

Notes: *Horizontal axis:* Selten Index [-1,1]. *Vertical axis:* driver identifier {1,..21}. *Horizontal lines:* extent of the bounds on the Selten index for each reference point model; where the line is a point the upper and lower bounds coincide and the Selten Index is point-identified. *Vertical line:* the Selten Index for the neoclassical model.
Figure E.5: Selten measures by reference-point model with reference-dependence in earnings only, imposing additive separability across goods.

**Reference Dependence Active With Respect To Earnings**

Notes: *Horizontal axis*: Selten Index [-1,1]. *Vertical axis*: driver identifier [1..21].

*Horizontal lines*: extent of the bounds on the Selten index for each reference point model; where the line is a point the upper and lower bounds coincide and the Selten Index is point-identified. *Vertical line*: the Selten Index for the neoclassical model.
Figure E.6: Selten measures by reference-point model with reference-dependence in both hours and earnings, imposing additive separability across goods.

Reference Dependence Active With Respect To Hours & Earnings

Notes: *Horizontal axis*: Selten Index [-1,1]. *Vertical axis*: driver identifier {1,..21}. *Horizontal lines*: extent of the bounds on the Selten index for each reference point model; where the line is a point the upper and lower bounds coincide and the Selten Index is point-identified. *Vertical line*: the Selten Index for the neoclassical model.