Economics 172A: Introduction to Operations Research **Winter 2008**
Problem set 1 **Due Thursday, January 31 at start of class (no late papers)**

Instructions

Unless otherwise noted on homework assignments and on examinations, you are required to supply complete answers and explain how you got them. Simply stating a numerical answer is insufficient.

For this assignment, attach printouts of Excel spreadsheets when requested and indicate where to find the answers for each question the spreadsheet covers. This assignment asks you to solve many linear programming problems, but most are variations on the same basic problem. Set up one template for the Excel computations and then make changes to get the answers for variations of the problem. You need not include a separate printout for every simplex computation as long as you provide a clear description of how you got the answers. You are responsible for using the notes on Excel on the website to figure out how to get Excel answers yourself (I won't lecture on it).

For this assignment there is no need to provide answer reports and sensitivity reports, but please do indicate which cells on your spreadsheet have the solution. For graphs, clearly label the graph and show where the objective function is and how you identified a solution. If I ask you to solve a problem, please give both the solution (the optimal x) and the value (the objective function value for the optimal x).

1. Consider the linear programming problem:

Choose $x_1, x_2 \geq 0$ to solve max y subject to $-x_1 + 2x_2 \leq 5$
$3x_1 + x_2 \leq 3$,

where the objective function y is a function of x_1 and x_2 to be specified.

(a) Graph the feasible region. Put x_1 on the vertical axis and x_2 on the horizontal axis.

(b) Solve the problem graphically when:

(i) $y = x_2$.
(ii) $y = x_1 + x_2$.
(iii) $y = x_1 - x_2$.

(c) Identify the corners of the feasible region. For each corner, give an example of a linear objective function y (a linear function of x_1 and x_2) such that the solution of the problem occurs at (and only at) that corner.

(d) Now solve each of the problems in part (b) using Excel. Compare your answers to the graphical solutions. Are there any differences? Explain.

(e) Now multiply each of the objective functions in part (b) by 3. Solve the new problem (graphically or using Excel, whichever you prefer). How do the optimal x's and values change?

(Note that parts (e), (f), and (g) are independent. For example, when you do part (f) do not multiply the objective functions by 3: leave them as they were originally. Thinking should allow you to do (e)-(h) with little computation. But even if you don't see why, you should be able to do these parts and use them to understand the ideas they get at. Please be sure to compare the answers and comment on the changes as requested.)

(f) Now multiply the second constraint of the problem by 12 (so that it becomes $36x_1 + 12x_2 \leq 36$). With the new constraint, solve each of the problems in part (b) again (graphically or using Excel). How do the optimal x's and values change?

(g) Now multiply the coefficient of x_1 in each constraint of the problem (except the nonnegativity constraints) and in each of the objective functions in part (b) by 12. With these changes, solve each of the problems in part (b) again (graphically or using Excel). How do the optimal x's and values change?

(h) Now repeat part (f), except this time multiply as in (f) but by -12 instead of 12.

2. Reconsider the formulation example discussed in class: A UCSD degree … and your own MS-burger (pronounced “Messburger”) franchise! You have three “profit centers”:

● the MS-Burger (“a quarter of a quarter of a pound of USDA choice beef on a fresh-baked bun”)
● the Beefburger (“for the total carnivore, a USDA choice beef pattie on a ‘bun’ also made entirely of beef”), and
● the Breadburger (“a fresh-baked bun in … what else? ... another fresh-baked bun”)

Producing x_1 MS-burgers, x_2 Beefburgers, and x_3 Breadburgers yields total profit (taking costs into account) of $60x_1 + 50x_2 + 10x_3$.

MS-burger CEO Joel Watson sends you $b_1 > 0$ quarter-of-a-quarter-of-a-pound units of beef and $b_2 > 0$ buns every month; it takes one unit of beef and one bun to make an MS-burger, two units of beef (and no buns) to make a Beefburger, and two buns (and no beef) to make a Breadburger.

(a) Formulate the linear programming problem that determines the profit-maximizing use of your monthly supply of beef and buns, assuming that the x_i must be nonnegative, but ignoring integer restrictions. Do not assume that all the beef or all the buns must be used.

(b) Write the dual of the problem.

(c) Interpret the dual.

(d) Setting $b_1 = 20$ and $b_2 = 30$, solve the primal and the dual using Excel.

(e) Compare your answers for the primal and the dual and confirm the conclusion of the duality theorem of linear programming and all complementary slackness conditions.

3. You must assign three people, A, B, and C, to fill five jobs, 1, 2, 3, 4, and 5. Each person *must* be given either one or two jobs, but you are otherwise free to make the assignment in any way you like. The costs are given in the following table; if a person is assigned to two jobs, the total cost of that part of the assignment is computed by adding the costs for the two jobs. c_{ij} is the cost of having worker i assigned to job j.

	1	2	3	4	5
A	4	9	3	5	3
B	3	6	2	6	1
C	1	7	7	3	4

(a) Show how to formulate this problem as a linear programming problem. (Hint: I found it helpful to create two mathematical duplicates of each worker (A1 and A2, and so on) and to use the variable x_{ij} to represent whether worker i is assigned to job j), with $x_{ij} = 1$ meaning that worker i is assigned to job j and $x_{ij} = 0$ meaning worker i is not assigned to job j.) Your formulation *must* include a definition of the variables and a clear statement (in both algebra and words) of the objective function and all relevant constraints.

(b) Is a linear programming formulation fully appropriate? Comment on whether there are any important assumptions made in the formulating the problem as a linear program.

(c) Use Excel to solve the problem.

(d) Now suppose that job 5 has been eliminated, but the rest of the problem is unchanged. Can this problem still be formulated as a linear programming problem? Explain why, or why not.