Economics 142 Problem Set 1: Behavioral Decision Theory   Spring 2007
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This problem set, which is optional, covers the material in the first half of the course. The problems are meant to help you think about some of the issues we discuss in lectures, and also as practice for the midterm exam, which will be drawn mostly from these questions. Before the midterm you are encouraged to work together and/or consult TA Rosalin Wu or me on these problems.

Choice under uncertainty (or certainty)

Expected Utility Theory

1. You are risk neutral, and care only about your income. With probability p, you will catch a disease that reduces your income from y, its level when you are healthy, to $y - k$, where $k > 0$. A vaccine is available, at cost c, that reduces the probability of your catching the disease from p to $q < p$.

(a) Suppose that you know the values of p, q, y, k, and c, so that the only thing about which you are uncertain is whether you will catch the disease. Write the condition that determines whether or not you should buy the vaccine.

(b) Now suppose that you know y, k, and c, but neither p nor q. Which is more relevant to your decision, the percentage amount by which the vaccine reduces the probability of catching the disease (what is usually reported in the press), or the absolute amount? Explain.

(c) How do your answers to (a) and (b) change if you are a risk-averse expected-utility maximizer?

2. In the game Former Soviet Union Roulette, a number of bullets are loaded into a revolver with six chambers; an individual then points the revolver at his head, pulls the trigger, and is killed if and only if the revolver goes off. Assume the individual must play this game; that he is an expected-utility maximizer; and that each chamber is equally likely to be in firing position, so if the number of bullets is b his probability of being killed is $b/6$. Suppose further that the maximum amount he is willing to pay to have one bullet removed from a gun initially containing only one bullet is $\$x$, and the maximum amount he is willing to pay to have one bullet removed from a gun initially containing 4 bullets is $\$y$, where x and y are both finite. Finally, suppose that he prefers more money to less and that he prefers life (even after paying $\$x$ or $\$y$) to death. Let U_D denote his von Neumann-Morgenstern utility when dead, which is assumed to be independent of how much he paid (as suggested by empirical studies of the demand for money); and let U_{A0}, U_{Ax}, and U_{Ay} denote his von Neumann-Morgenstern utilities when alive after paying \$0, $\$x$, or $\$y$ respectively.

(a) What restrictions are placed on U_D, U_{A0}, U_{Ax}, and U_{Ay} by the assumption that he prefers more money to less when alive?

(b) What restrictions are placed on U_D, U_{A0}, U_{Ax}, and U_{Ay} by the assumption that he prefers life (even after paying $\$x$ or $\$y$) to death?

(c) Is it possible to tell from the information given above whether $x > y$ for an expected utility maximizer? Does it matter whether he is risk-averse? Explain.

3. Suppose that there are two states of the world, s_1 and s_2, and that an individual who knows the probabilities, p_1 and p_2, of the two states chooses among state-contingent consumption bundles to maximize the expectation of a state-independent, strictly increasing von Neumann-Morgenstern utility function.

(a) Suppose that the individual is risk-neutral, and that he is indifferent between (8, 2) and (4, 4). What must the value of p_1 be?

(b) Now suppose that the individual may be either risk-averse or risk-loving. What is the lowest possible value of p_1 for which the individual could weakly (or strictly) prefer the state-contingent consumption bundle (6, 2) to the bundle (2, 6)?

(c) Now suppose that the individual is risk-averse, and that he is indifferent between (6, 2) and (2, 6). Show (graphically or algebraically) that he must weakly prefer (4, 4) to either of these bundles.

4. Consider an expected utility-maximizing student, who cares only about his income. Cheating on his 142 exam adds a given amount to his income, whether or not he is caught at it. Suppose, however, that a student who is caught cheating is fined a given amount. It is observed that a 1% increase in the probability of being caught lowers the student's expected utility of cheating by more than a 1% increase in the amount of the fine.

(a) Is the student a risk-averter or a risk-lover? Explain.

5. An expected utility-maximizing person has von Neumann—Morgenstern utility function $u(\cdot)$, with $u'(\cdot) > 0$, and deterministic initial wealth w. He is just indifferent between losing $x > 0$ for certain, and losing $y > x$ with probability $p > 0$ and losing nothing with probability $1 - p$. (In other words, x is the most he will pay to be insured against a random loss of y with probability p.)

(a) Prove that for any given values of w and y, x is an increasing function of p.

(b) Prove that for any given values of w and p, x is an increasing function of y.

(c) Prove that if the person is risk-averse, then $x > py$.

(d) How does x vary with w when $u(w) \equiv a - be^{-cw}$ with $b, c > 0$, so that the person has constant absolute risk aversion? (Here, e is the base of natural logarithms.)

6. An individual has initial wealth w and holds a lottery ticket that will be worth zA with probability p and $-zB$ with probability $1 - p$. Here, A, B, and z are positive, with $pA \leq (1-p)B$. Let X be the maximum amount the individual would pay someone to take this ticket off his hands. Prove that if the individual is risk-averse, then X is an increasing function of z.

7. Consider a risk-averse, expected-utility maximizing agent with von Neumann—Morgenstern utility function u(·) and initial wealth y.

(a) Show how to determine (by giving an expression that implicitly defines it) the minimum probability of winning, p, needed to get the agent to accept a binary bet in which the outcomes are winning or losing $z > 0$. (Assume he will accept if indifferent.).

(b) Use Taylor's Theorem to derive an approximate expression for p when z is small, and use your expression to show that p is then an increasing function of z.

(c) What aspect of the agent's risk preferences determines whether p is an increasing function of y for small z? Explain.

8. A rich uncle gives you a gift certificate that entitles you to an insurance contract, of your own choosing, with an actuarial value (an expected return) of $1 million. The only uncertainty you face us about whether you will be involved in an accident that leaves you paralyzed; this will happen with probability ½. You may allocate the $1 million in actuarial value however you wish between the two states, paralyzed and not paralyzed. It has been observed that, when faced with this choice, some people choose to receive more money in the paralyzed state, some choose to receive less money in the paralyzed state, and some choose to receive equal amounts in both states.

(a) Which of these choices is/are consistent with expected-utility maximization with a strictly concave, differentiable, state-independent von Neumann—Morgenstern utility function? Explain.

(b) Which of these choices is/are consistent with expected-utility maximization with a strictly concave, differentiable state-dependent von Neumann—Morgenstern utility function? Explain.

9. Suppose that there are three money outcomes, x_1, x_2, and x_3, with $x_1 < x_2 < x_3$, and that you can observe which values of p make a person prefer getting x_2 for certain to getting a random outcome $\{x_1$ with probability p, x_3 with probability $(1-p)\}$. Is this enough to determine a person's preferences over arbitrary probability distributions over x_1, x_2, and x_3:

(a) if he is an expected-utility maximizer? Explain.

(b) if he chooses among distributions to maximize some differentiable preference function, not necessarily consistent with expected-utility maximization? Explain.

10. An individual chooses among lotteries according to preferences that are complete, transitive, and continuous. He cares only about money, and all the lotteries he faces have the same three possible outcomes, $1, $2, and $3; the probabilities of these outcomes are denoted p_1, p_2, and p_3, respectively. Labeling your diagrams carefully so that I can tell how they were constructed, draw an indifference map in (p_1, p_3)-space for an individual who:

(a) is risk-loving, likes money (always prefers first-order stochastically dominating shifts in the distribution of money outcomes), and satisfies the independence axiom

(b) is risk-averse, likes money, but does not satisfy the independence axiom

(c) is risk neutral, but does not like money. Is it possible for a risk-neutral individual whose preferences depend only on the probability distribution of money outcomes to violate the independence axiom? Explain why or why not.

11. Formulate and prove the statement that an expected utility-maximizing risk averter who prefers more money to less and has a deterministic initial wealth will never take a bet that does not have a strictly positive expected return.

12. Consider an expected-utility maximizer who prefers more money to less and has a deterministic initial wealth, who is offered a bet with a strictly positive expected return, with the option to scale it up or down proportionally as much as he wishes. Making whatever assumptions you find necessary, formulate the problem that determines his optimal choice of scale, and prove that he will always take the bet at some strictly positive scale. Use your argument to decide whether this conclusion depends on whether he is risk-averse or on whether his von Neumann-Morgenstern utility function is differentiable. Explain.

13. An individual with a state-independent, differentiable von Neumann—Morgenstern utility function, $u(\cdot)$, with $u'(\cdot) > 0$, and deterministic initial wealth, W, is offered a bet that will add a random amount, X, to his initial wealth, where $\mathrm{E}X > 0$.

(a) Suppose it is known that he is either (globally) risk-averse or (globally) risk-loving. What can you infer about his risk preferences if he declines the bet?

(b) Use your answer to (a) to show that if he declines the bet, he must also decline all scaled bets of the form aX with $a > 1$, but might accept some bets of this form with $a < 1$.

14. According to Paul Samuelson, the mathematician Stanislaw Ulam defined a coward as someone who will not bet even when you offer him two-to-one odds and let him choose his side. (A gamble with two-to-one odds is one in which the individual wins \$$2x$ if an event A occurs and loses \$$x$ if A does not occur. Letting the individual choose his side means letting him choose between winning \$$2x$ if A occurs and losing \$$x$ if A does not occur, or winning \$$2x$ if A does not occur and \$$x$ if A occurs.)

(a) Show by example (graphical, if you prefer) that it is possible for an expected-utility maximizer who likes money to be a coward according to Ulam's definition.

(b) Show (graphically, if you prefer) that an expected-utility maximizer who likes money, and whose von Neumann—Morgenstern utility function is differentiable, cannot be a Ulam-coward for all values of $x > 0$.

15. There are two states of the world, 1 and 2, and a single consumption good; the state-contingent consumption vector $e \equiv (e_1, e_2)$ represents consumption of e_i units of the consumption good if state i occurs. The probability of state i is p_i. Suppose that an individual chooses among state-contingent consumption vectors to maximize the expectation of the state-independent von Neumann—Morgenstern utility function $u(\cdot)$.

(a) Write the equation of a typical indifference curve for the individual.

(b) Derive an expression for $\mathrm{MRS}_{12}(e_1, e_2)$, the individual's marginal rate of substitution between consumption in states 1 and 2 at consumption vector (e_1, e_2).

(c) Suppose that $e^a \equiv (\underline{e}_1, \underline{e}_2)$, $e^b \equiv (\underline{e}_1, \bar{e}_2)$, $e^c \equiv (\bar{e}_1, \underline{e}_2)$, and $e^d \equiv (\bar{e}_1, \bar{e}_2)$, so that these four consumption vectors form a rectangle in (e_1, e_2)-space. Show that $\mathrm{MRS}_{12}(e^a)/\mathrm{MRS}_{12}(e^b) = \mathrm{MRS}_{12}(e^c)/\mathrm{MRS}_{12}(e^d)$.

(d) Does the result of part (c) remain valid when the utility function is state-dependent? Explain.

(e) Does the result of part (c) remain valid when the individual is not an expected-utility maximizer? Explain.

16. Suppose that there are two states of the world, s_1 and s_2, and that an individual who knows their probabilities, p_1 and p_2, chooses among state-contingent consumption bundles (x_1, x_2), facing budget sets with varying prices as in Figure 1 (next page; the dotted line is the 45-degree certainty line). Figure 2 graphs $x_1/(x_1+x_2)$ against $\ln(p_1/p_2)$ for experimental subjects who faced many such choices, with varying prices. (The paper is at http://socrates.berkeley.edu/~kariv/CFGK_III.pdf, but it won't help you answer this question.) Subject A: ID 304 always chooses $x_1 = x_2$ without regard to the price ratio, while Subject C: ID 307 chooses $x_1 = x_2$ for price ratios very near one, but otherwise puts "all his eggs" in the basket for which eggs are cheaper. (The other three subjects have demands that vary smoothly with prices.) This question will focus entirely on Subjects A and C.

(a) Can A's and C's choice behavior be described as (approximately) maximizing a preference function over (x_1, x_2) bundles? If so, graph an indifference map for A, and one for C, such that (approximately) maximizing the associated preference function generates his observed choicees.

(b) Can your indifference maps for A and C from (a) be generated by maximizing differentiable, state-independent von Neumann-Morgentern utility functions? Explain. (Allowing non-differentiable utility functions doesn't change the answer to this, but you are not asked to show that.)

Figure 1: An example of a budget constraint with two states and two assets.

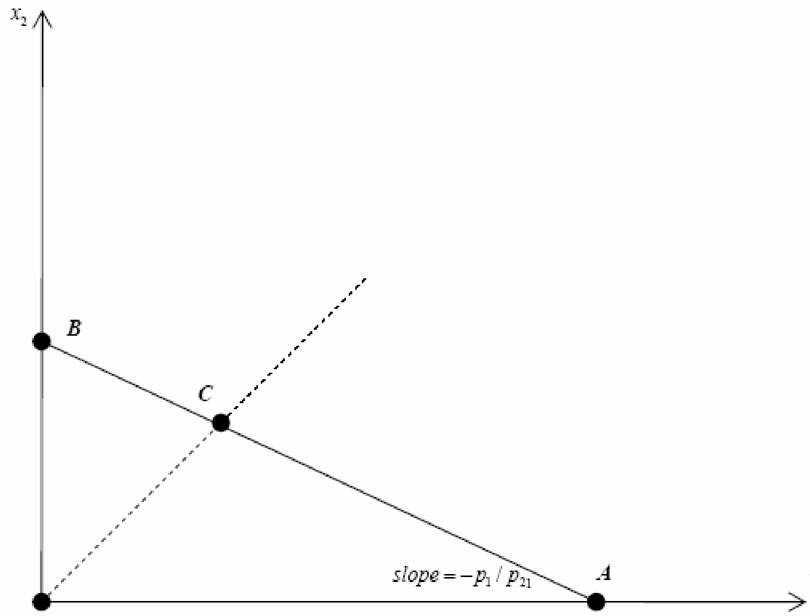
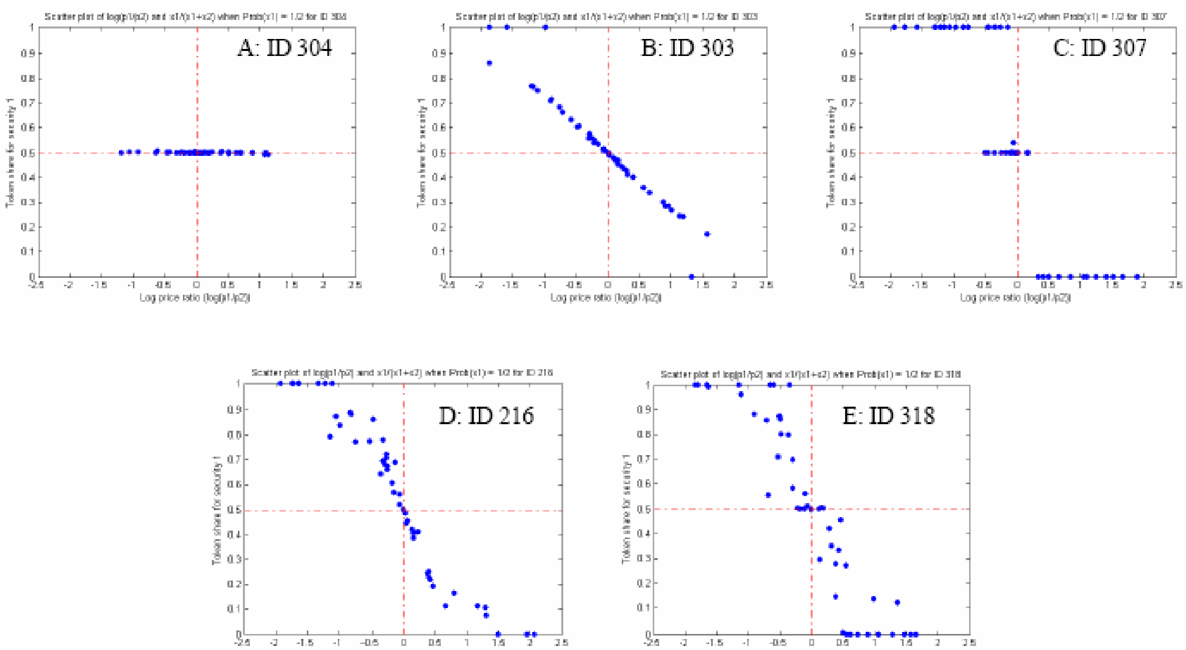


Figure 2: The relationship between the log-price ratio $\ln(p_1 / p_2)$ and the token share $x_1 / (x_1 + x_2)$ for selected subjects.



17. In 1953, Maurice Allais proposed the following thought-experiment. You must make a choice between Gamble A and Gamble B (you can interpret these dollar amounts as final wealth levels):

Gamble A: $1 million for sure

Gamble B: $1 million with probability 0.89
$5 million with probability 0.10
$0 with probability 0.01

Which would you choose?

Next, you must make a choice between Gamble C and Gamble D:

Gamble C: $1 million with probability 0.11
$0 with probability 0.89

Gamble D: $5 million with probability 0.10
$0 with probability 0.90

Which would you choose?

Most people choose Gambles A and D. Explain why (no matter what utility function a person has!) this pattern of choices violates expected utility theory.

18. Daniel Ellsberg (long before the Pentagon papers) proposed the following thought-experiment. An urn contains 90 balls, 30 of which are red. The other 60 are black or yellow, in unknown proportions. One ball will be drawn randomly from the urn. You must make a choice between Gamble A and Gamble B:

Gamble A: You win $100 if the ball is red.
Gamble B: You win $100 if the ball is black.

Which would you choose?

Next, you must make a choice between Gamble C and Gamble D:

Gamble C: You win $100 if the ball is either red or yellow.
Gamble D: You win $100 if the ball is either black or yellow.

Which would you choose?

Most people strongly prefer Gambles A and D.

(a) Explain why this pattern of choices violates expected utility theory.

(b) Explain why this pattern of choices is also inconsistent with prospect theory.

(c) Ellsberg argued that people treat uncertainty (situations without known probabilities) differently than they treat risk (situations with known probabilities). Can you think of real-world examples where people go out of their way to avoid uncertainty?

19. Consider a gamble to win \$200 with probability 0.50 and lose \$100 with probability 0.50. Consider an expected utility-maximizing consumer with initial wealth of \$10,000 and a constant relative risk aversion (CRRA) utility of wealth function $U(w) = w^{(1-\rho)}/(1-\rho)$, where $\rho>0$.

(a) Show that for $\rho = 45$, the consumer would accept this gamble but that for $\rho = 55$, the consumer would reject it. Argue that ρ would have to be much, much larger for the consumer to reject the gamble if initial wealth were \$1 million.

(b) Now suppose the consumer has utility of wealth function $U(w) = w^{(1-\rho*)}/(1-\rho^*)$, where $\rho^* = 55$. You offer the consumer a gamble to win \$$z$ with probability 0.50 and lose \$1,000 with probability 0.50. What is the smallest value of z such that the consumer will accept your gamble? (Hint: This is a trick question, but try to answer it!) Does this seem like reasonable behavior?

(c) Explain intuitively why rejecting a small-stakes gamble that has positive expected value is qualitatively consistent with Expected Utility Theory but *quantitatively* inconsistent with it.

Prospect Theory

20. For each of the following anecdotes, briefly explain (i) why the person's behavior is *prima facie* inconsistent with expected utility theory, (ii) why it is consistent with prospect theory, and (iii) how the behavior might be reconciled with expected utility theory.

(a) Some students who were about to buy season tickets to a campus theater group were randomly selected and given a discount. During the first part of the season, those who paid full price attended significantly more plays than those who received discounts.

(b) Cab drivers in New York City work longer hours on warm, sunny days when their per-hour wage is low.

(c) People purchase insurance against damage to their telephone wires at 45 cents a month even though the probability that they'd incur the \$60 repair cost in any month is 0.4%.

(d) Bettors tend to shift their bets toward longshots, and away from racetrack favorites, later in the racing day.

(e) Unionized workers have their wages set 1 year in advance and they receive some bad news that their wages will be cut next year, but they do not cut their spending. However, the previous year when they learned that their wages would increase, they increased their spending.

21. Throughout life, we face many positive-expected-value small-scale risks. What are some examples? Normally, we consider each risk in isolation – this is called *narrow bracketing*. When we narrowly bracket the risks we face, loss aversion may lead us to turn down positive-expected-value gambles. Explain why this is a mistake. Argue that it is good advice even to a loss-averse person to accept positive-expected-value gambles. Do you think it is generally a good idea to pay extra for a one-year warranty on a CD player?

22. Tim owns a house. His company has offered him a job elsewhere, which he has accepted, and he has therefore decided to sell the house. He does not have much time, thus he just plan to post a take-it-or-leave-it offer with price x. For any price x from \$1 million to \$2 million, Tim assesses the probability q of selling as $q = 2 - x$. If he doesn't find a buyer, he can always sell the house to a friend for \$1 million.

Tim is a prospect theory maximizer, and he integrates over different accounts (house and money). In particular, he values any two-outcome distribution of changes to his reference point, say s with probability p and t with probability $1 - p$, at $V = v(t) + (v(s) - v(t))p$ whenever $s > t \geq 0$ or $s < t \leq 0$. Here $v(z) = |z|^{1/2}$ if $z \geq 0$ and $v(z) = -2|z|^{1/2}$ if $z < 0$. Tim's reference point already includes all the changes required by the move to Europe other than the sale of the house.

(a) Assume that Tim is a pessimist and his reference point is based on presumption that he will sell the house for \$1 million. Thus, he will see it as a gain of $x - 1$ if he obtains a price x higher than \$1 million. What price x would Tim ask for?

(b) Now assume that Tim is an optimist and his reference point is based on presumption that he will sell the house for \$2 million. Thus, he will see any price x below \$2 million as a loss of $2 - x$. What price x would Tim ask for?

(c) Is there a difference between the optimal prices in questions 1 and 2? If not, try to explain why not. If yes, tell which one is higher and explain intuitively why the prices are different.

23. G is a 50-50 win \$1000 lose \$550 gamble. Consider an agent with a non-decreasing probability weighting function $\pi(p)$ and with the following prospect theory value function:

$$V(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2.5x & \text{for } x < 0. \end{cases}$$

(a) What will this agent choose among:

(i) do not participate,
(ii) play G one time,
(iii) play G two times with a single payment done at the end by adding up the two results.

(b) What will he do if he has also the extra option:

(iv) play G one time, see the result and have the option of playing it a second time. A single payment is done at the end.

(c). Give an example of a situation:

(i) where people will aggregate the risks and take their decision based on the final outcome,
(ii) where they will do the opposite

24. Comment, using ideas from this course (but not necessarily restricted to prospect theory ideas). What kinds of models of consumer behavior might be able to explain this phenomenon?

Entrees Reach $40, and, Sorry, the Sides Are Extra
By JODI KANTOR

A new dish is appearing on menus across the nation. Restaurateurs say they have little choice other than offer it, though it horrifies many customers.

That item is the $40 entree.

Until recently, such prices were the stuff of four-star, white-tablecloth meals, the kind that ended with a diamond ring on the petit four tray. But now entrees over $40 can be found in restaurants that are merely upscale, where diners wear jeans and tote children. In geographic terms, New York and Las Vegas have led the charge, and in culinary ones, luxury items like steak and lobster were first and are still most prevalent.

But the $40 entree is migrating: to restaurants in Philadelphia, Fort Lauderdale and Denver, and to ingredients like fish and even pasta. Several national chains serve entrees priced above $40.

"Forty is the new 30," said Richard Coraine, the chief operating officer of Union Square Hospitality Group, which recently began charging $42 for a 1¾-ounce appetizer portion of lobster at lunchtime at the Modern in New York. Ten percent of its lunch patrons order the dish, it says.

Hovering just below the $40 mark is an even vaster group of $38 and $39 entrees, waiting to cross the line like thirtysomethings approaching a zero-ended birthday. The arctic char at the Indianapolis branch of the Oceanaire Seafood Room chain is $38.50. Metropolitan Grill in Seattle serves shrimp scampi for $39.95. At Mike's, a new steakhouse in Brooklyn Heights, $9.95 chicken nuggets share the menu with $38.95 veal chops.

Like the $100 Broadway ticket, $200 jeans and the $20 museum admission, the $40 entree is provoking a righteous burst of populist outrage, especially among those who pay their own way. When Angela Dansby, a Chicago diner, sees a 4 in front of a price, she thinks: "Either this must be out of this world, or it's totally overpriced and I'm not going to order it. It's usually the latter." When she does pay, she compensates by skimping on appetizers and wine.

Restaurateurs say rising rents, ever more elaborate interior-decoration schemes and the increasing cost of premium ingredients — especially beef and fish — leave them little choice. Chefs, so fond of listing purveyors on menus, do not want those names to be Tyson and Del Monte. They "take pride in getting carrots or beets that no one has," Mr. Coraine said.

Bobby Flay acknowledges that "the needle has moved very fast." Mr. Flay recently crossed the $40 mark in his Las Vegas and Atlantic City outposts, though he says he intentionally loses money on many other entrees in order to keep prices reasonable. His entrees at Mesa Grill in New York top out at $34. (When it opened in 1991, the steepest entree was $19, or $28.30 when adjusted for inflation.)

But what makes the rise of the $40 entree so significant is not just the price creep, it's the sophisticated calculation behind it. A new breed of menu "engineers" have proved that highly priced entrees increase revenue even if no one orders them. A $43 entree makes a $36 one look like a deal.

"Just putting one high price on the menu will take your average check up," said Gregg Rapp, one such consultant. "My mom taught me to never order the most expensive thing on the menu, but you'll order the second."

With just a few keystrokes, restaurateurs can now digitally view the entire history of a dish: how the lamb sold around this time last year, whether it did better when paired with squash or risotto, and how orders rose or fell when the price went from $39 to $41.

With a few more clicks and a new stack of paper in the office printer, the menu can be revised to test new prices.

"In the old days, restaurateurs printed up menus and they were stuck with them for six months or a year; now they can do it daily, experimenting with price or placement," said Tim Ryan, president of the Culinary Institute of America, which teaches menu engineering to all its chefs in training.

The towering prices at wildly luxurious restaurants like Per Se and Masa in New York and Alinea in Chicago have set a new price in the collective dining consciousness for a truly top meal, nudging up what diners will pay for far more modest dinners. In Las Vegas, the current talk is about Guy Savoy at Caesars Palace, where desserts alone are $22 each and a meal for two can easily run $500.

"I love when I hear about that stuff, because then Craft becomes inexpensive," said Tom Colicchio, chef of the quickly multiplying restaurants, including a steakhouse in Las Vegas.

Oddly, as entrees rise in price, they seem to be shedding their traditional accompaniments. Today a $40 main dish is often now just that. Order a side dish, and the entree price climbs dizzyingly close to the 50's. At the highly influential Craft, Mr. Colicchio serves pricey, naked hunks of protein and charges extra for vegetables. (He says the portions are enough for two.) Porter House, a new steakhouse at the Time Warner Center in New York, even charges diners separately for sauce.

"I blame Tom Colicchio for this," said Barry Okun, a New York lawyer who has established a personal price limit of "between $50 and $60" per entree. "It's not that I'm happy about it," he added.

Mr. Colicchio acknowledged the influence of his pricing, adding that restaurants like those of the Bistro Laurent Tourondel group in New York "completely ripped off the concept" of focusing on individual elements.

To which Mr. Tourondel replied, "He should look back at the old-time steakhouse menus that were around way before Craft ever existed."

Liz Johannesen, senior manager of restaurant marketing at OpenTable, which takes online reservations for 6,000 restaurants nationwide, said that in the last year diners had started occupying tables for longer periods, mimicking the leisurely pace at the very top establishments and forcing restaurants to raise entree prices because they were turning fewer tables.

“Just like in other cultural pursuits, trends filter downwards,” Ms. Johannesen said.

That applies to a taste for splurging as well. Kobe and Wagyu beef, from pampered Japanese cattle renowned for their tender meat, is cropping up at restaurants around the nation, according to Technomic, an industry research concern. Steakhouses and sushi restaurants, now so ubiquitous, have trained diners to pay large sums for specific ingredients, leading some to fall for the old “if it’s expensive it must be great” trick.

Indeed, no chef needs a menu engineer to explain a time-honored truth of the restaurant industry: many business diners look to spend money, not save it.

“If I’m entertaining clients, it’s all about making sure my clients are having the best time,” said Andrew Passeri, a private banker in New York who last week dined at davidburke & donatella, where he chose the $44 lobster over the $46 Dover sole and the $44 ostrich scramble.

At those prices, dinner is garnished with a large dusting of skepticism. Underattentive service or an overcooked piece of fish is not merely a minor annoyance but an unjustifiable offense.

“I’m happy to pay good money for something I can’t replicate at home,” said Ms. Dansby, of Chicago, “but when you get charged these prices for bad service, or quality, or visual presentation that isn’t so great, it’s really irritating.”

Two years ago, when Gray Kunz opened Cafe Gray in the Time Warner Center, he said he hoped it would become a destination for secretaries who work in the surrounding office buildings. The priciest entree then was the short ribs at $34. Now they are $38, the chicken is $37, and Mr. Kunz just introduced a lobster ravioli at $41.

“The biggest gasp I ever had at menu prices was the first time I went to Cafe Gray,” Mr. Okun said. “It looks like it should be a casual ‘stop in here for a bite’ place. For the amount of money you’re spending, you really want a special experience.”

Mr. Kunz now calls his restaurant “right in between the high end and low end” and said he provided “good product for very good value.”

According to Zagat, which measures what diners estimate paying, not actual prices, the average check at the most expensive 200 restaurants in San Francisco has risen 14 percent in the last two years, after remaining fairly stable earlier in the decade. At the 200 priciest restaurants in New York, Zagat users say, checks have followed the same pattern.

For his part, Tim Zagat, publisher of the guides that bear his name, said he was almost over the shock of entree prices. But now, he said, he finds himself startled by another development.

“Your $40 plate?” Mr. Zagat said. “It comes with a $20 first course.”

Probabilistic judgment

Representativeness

25. Suppose that, in the course of a regular check-up, a doctor discovers that the patient has a potentially cancerous lesion. Most lesions are benign (non-cancerous), say 99%. The doctor orders an x-ray just in case. In laboratory tests on malignant (cancerous) lesions, the x-ray returns positive (cancer-affirming) results 79.2% of the time and negative results 20.8% of the time. In laboratory tests on benign lesions, the x-ray returns positive results only 9.6% of the time and negative results 90.4% of the time.

(a) The patient's x-ray comes back positive. What is the probability that the patient has cancer?

(b) Suppose that the doctor calculates the probability that the patient has cancer without regard to the base rate of cancer in the population – that is, the doctor uses Bayes' Rule but assumes that cancerous and non-cancerous lesions are equally likely. What mistaken conclusion will the doctor draw from the test? How is this mistake an example of representativeness? Explain why it is important in these situations to have hospital procedures that require additional tests to be performed before a patient undergoes treatment.

26. Consider the Kahneman and Tversky base-rate neglect experiment. In Problem A, subjects are told that Jack has been drawn from a population of 30% engineers and 70% lawyers and that Jack wears a pocket protector.

(a) Let p_1 denote the probability that Jack is an engineer, given that he wears a pocket protector. Using Bayes' Rule, show that the odds that Jack is an engineer as opposed to a lawyer is given by:

$$p_1/(1-p_1) = [0.30 \text{ Pr(pocket protector | Jack is engineer)}] \\ / [0.70 \text{ Pr(pocket protector | Jack is lawyer)}].$$

In Problem B, subjects are told that Jack has been drawn from a population of 70% engineers and 30% lawyers and that Jack wears a pocket protector.

(b) Let p_2 denote the probability that Jack is an engineer, given that he wears a pocket protector. Show that:

$$p_2/(1-p_2) = [0.70 \text{ Pr(pocket protector | Jack is engineer)}] \\ / [0.30 \text{ Pr(pocket protector | Jack is lawyer)}].$$

Conclude that, if subjects form beliefs according to the laws of probability, it must be the case that:

$$[p_1/(1-p_1)] / [p_2/(1-p_2)] = (3/7)^2.$$

(c) Explain intuitively why this ratio of odds does *not* depend on Pr(pocket protector | Jack is engineer).

(d) Explain why Kahneman and Tversky set up the experiment in this way.

(e) What values for $[p_1/(1-p_1)] / [p_2/(1-p_2)]$ imply that subjects exhibit base-rate neglect?

(f) Kahneman and Tversky ran this experiment as a *between-subjects* design – different groups of subjects responded to Problems A and B. How might their results have changed if they had run a *within-subjects* design – where each subject responded both problems? Why do you think Kahneman & Tversky chose a between-subjects design?

27. Consider the following hypothetical facts: "One percent of people in the world are rational. We have a test for rationality. If someone is rational, they have a 60% chance of passing. If someone is irrational, they have a 40% chance of passing. Adam was just given the test, and he passed."

(a) Assume that Adam was drawn randomly from the world population. What is the probability that he is truly rational? Don't bother with the long division. Expressing your answer as a ratio is fine.

(b) Predict the responses of a population of naive subjects who are asked to estimate the probability of Adam's rationality, given the information above. Justify your answer. Describe the kinds of errors that they are likely to make.

28. Another manifestation of the representativeness heuristic is that people believe in the "Law of Small Numbers."

(a) Define the Law of Small Numbers, and explain why it is an error. Describe some evidence for the "Law of Small Numbers."

(b) Suppose that basketball players are, during any given game, in one of three states: Hot (they make 75% of their shots), Normal (they make 50% of their shots), or Cold (they make only 25% of their shots). Suppose Paul Pierce is Hot. What is the probability that he will make 3 baskets in a row? What if he is Normal? Cold? If you have no idea what state he'll be in before the game (that is, each state is equally likely), what would you believe about the likelihood that he is Hot after he makes his first 3 baskets in a row?

(c) One of the fans at this game believes in the Law of Small Numbers. She has the wrong model of how likely Paul Pierce is to make a basket. Here's how her model works. The fan imagines that there is a deck of 4 cards. When Paul is Hot, 3 of these cards say "hit" on them, and only 1 says "miss." Every time Paul takes a shot, one of these cards is drawn randomly *without replacement* from the deck, and the outcome is whatever the card says. Therefore, when Paul is Hot, he *always* makes 3 out of every 4 shots he takes. (When the deck is used up, the 4 cards are replaced, the deck is shuffled, and the process begins again – but that isn't important for this problem.) Similarly, when Paul is Normal or Cold, the outcome of every shot is determined by the draw of a card without replacement from a deck of 4 cards. When Paul is Normal, the deck has 2 "hit" cards and 2 "miss" cards. When Paul is Cold, the deck has 1 "hit" card and 3 "miss" cards. Explain how her model corresponds to the Law of Small Numbers.

Suppose Paul Pierce is Hot. According to the fan, what is the probability that Paul will make his first basket? After Paul makes his first basket, what does the fan believe about the probability that Paul will make his next basket? Explain why it is lower than the fan's belief about the probability that Paul will make his first basket.

According to the fan, what is the probability that Paul will make 3 baskets in a row? What if he is Normal? Cold? If the fan has no idea what state Paul will be in before the game (that is, each state is equally likely), what would she believe about the likelihood that he is Hot after he makes his first 3 baskets in a row? Explain intuitively why the fan's beliefs differ from the normatively correct probability that you calculated in part (ii).

(d) Suppose that, in reality, there is no such thing as being "Hot" or "Cold." Paul Pierce is, in fact, always Normal. Over many games, with what frequencies will Paul score 0, 1, 2, and 3 baskets in his first 3 attempts? Suppose the fan attends many games and observes these frequencies. Explain why the fan would not believe you if you tried to convince her that there is no such thing as being "Hot" or "Cold."

In this example, the fan's misunderstanding of probability leads her to believe (falsely) in "hot hands" and "cold hands." Something similar may be going on in the mutual fund industry. Even if mutual fund returns are almost entirely due to luck, there will be some mutual funds that have done exceptionally well and others that have done exceptionally poorly in recent years due entirely to chance. Explain how the Law of Small Numbers would lead some investors to conclude (falsely) that mutual fund managers differ widely in skill.

Probabilistic inferences

29. In Monte Hall's game show "Let's Make a Deal," contestants chose one of three doors. Behind one of the doors was the grand prize (a car), and behind the other two doors were booby prizes (goats). Each door was equally likely to contain the grand prize. After the contestant chose a door and *before* Monte Hall opened the chosen door, Monte Hall opened a *different* door – and he *always* intentionally opened a door that contained a booby prize. Then Monte Hall gave the contestant a choice between sticking with the original choice or switching to the third, unopened door. Before reading on – would you switch or stick with your original choice?

(a) In fact, the best strategy is to switch. Explain why. (Many actual contestants did not switch, and many eminent scholars have been confused about this problem.)

(b) Explain why people get so confused about problems like this one. (Think about how conditional probabilities differ from unconditional probabilities. In particular, did the fact that Monte Hall opened one of the doors give you information about whether the door you originally chose has the grand prize? Did the fact that Monte Hall *didn't* open the third door give you information about whether *that* door has the grand prize?)

30. Suppose that you are making a take-it-or-leave-it offer to Joel Watson, brilliant but compassionate CEO of the UCSD Economics Department. The value of the Department to Watson is distributed uniformly between \$0 and \$10 million. Watson is shrewd, and knows the Department's true value, but you don't know its value (and Watson knows that you don't, etc.). However, you do know that whatever the Department is worth, it is worth 2.5 times more to you than it is to Watson. For example, if the Department is worth \$10 million to Watson, then it is worth \$25 million to you. If your take-it-or-leave-it offer is above Watson's value, he will sell you the Department. If not, he won't. (Ties have zero probability here, so don't worry about them.)

(a) What is your optimal offer? Explain.

(b) Predict what a population of naive people would offer in this situation. Explain by describing the kinds of errors they are likely to make.

31. First suppose that you are bidding for a single, indivisible object in an independent-private-value sealed-bid auction in which each bidder's value is an i.i.d. draw from a common, continuous distribution. It is a second-price ("Vickrey") auction, in which the highest bidder wins the object, but pays only the second-highest bid. (The other bidders win nothing and pay nothing; ignore ties.)

(a) What is your optimal bidding strategy (relating your optimal bid to the realization of your own value)? Explain why it is optimal, no matter what bidding strategies other bidders adopt. (That is, explain why it is a dominant strategy.)

(b) Would you expect naïve experimental subjects to bid optimally in this situation? If not, what kind of errors do you think they are most likely to make?

(This part is way too hard for an exam question) Now suppose that all is as in the first part, but the auction is a common-value one. That is, the value is commonly known to be the same to all bidders ex post, but ex ante they do not know what the value will be. Instead they observe only noisy, i.i.d. signals of its value. (In a common-value auction, there is a well-known phenomenon known as the winner's curse, in that in equilibrium, bidders' bidding strategies are increasing in their own values, so that the winner is normally the bidders whose value signal was highest among all bidders' signals. This means that his signal, taken by itself, overestimates the value, so that to bid optimally he must correct for this.)

(c) What is your optimal bidding strategy? Explain why it is optimal. Is it still a dominant strategy? Explain why or why not.

(d) Would you expect naïve experimental subjects to bid optimally in this situation? If not, what kind of errors do you think they are most likely to make (over and above those in (b))?

Intertemporal Choice

Hyperbolic Discounting

32. Consider a consumer faced with a "vice" good like potato chips, which he is tempted to consume rapidly. The consumer can buy a large (2-serving) or small (1-serving) pack at period 0. In period 1, she must decide how much to consume. If she bought only the small pack, she consumes one serving. If she bought the large pack, she can consume two servings right away, or one serving and save another serving for the future (which is automatically consumed in period 2).

Assume there is positive utility in period 1 from consumption, and negative utility in period 2 (a reduced-form expression for poor health, say). Because the large size has some production economies, it is cheaper, which is reflected in higher immediate consumption utility. The Table below shows numerical utilities. (If she chooses to eat 1 serving from the large pack in period 1, then she gets utility of +3 in period 2, and –2 in period 3, from the second pack.)

Consider a β-δ quasi-hyperbolic framework. For simplicity assume $\delta = 1$ to focus attention on the β term. Analyze the optimal consumption decisions of three types of agents: Exponential ($\beta = 1$, $\beta' = 1$); naïve hyperbolic ($\beta < 1$, $\beta' = 1$); and sophisticated hyperbolic ($\beta < 1$, $\beta' = \beta$).

Purchase Decision **Consumption Decision**	**Instantaneous Utility in Period 1**	**Instantaneous Utility in Period 2**
Small		
1 serving	2.5	-2
Large		
1 serving	3	-2
2 servings	6	-7

For each agent, figure out:

(i) What will they expect to do, at time 0, if they buy either the large and small packages?

(ii) Given your answer in (i), which package will the period-0 "self" purchase, for each of the three types?

(iii) After they buy their optimal package, how much will they consume in period 1?

(iv) Which of the type's (if any) plans embedded in (i) are actually violated in (iii)

(v) Suppose agents could purchase external commitment, in which they could only consume 1 of the 2 servings in the large pack in period 1, at a price of $P > 0$ (think of this as buying pre-packaged dietary portions of food). Which agents would commit at time zero to pay P, and how much would they pay?

33. As question 32 indicates, an important empirical demarcation between naïve and sophisticated hyperbolic agents is whether they will pay in advance for planned self-control (a la Ulysses and the Sirens). Give an example of external self-control that is *voluntarily* chosen by agents (other than those discussed in class). Try to think of the biggest examples in the economy that you can think of.

34. (ignore point values)

Consider a (β, δ) agent. His utility at time t is

$$U_t(u_t, u_{t+1}, ..., u_T) = u_t + \beta \sum_{s=1}^{T-t} \delta^s u_{t+s}$$

Assume for simplicity that $\beta = .5$ and $\delta = 1$.

This agent can watch 3 movies over the next 4 weeks. He has to spend one week without watching a movie, not seeing a movie gives him 0 utils.

- week 1: mediocre movie = 3 utils
- week 2: good movie = 5 utils
- week 3: great movie = 8 utils
- week 4: excellent movie = 13 utils

1. [10 points] What movie will a sophisticate miss? Solve the game played by the different selves by backward induction.
2. [10 points] What movie will a naive skip? He is unaware of the changes in his preferences, he thinks the future selves will do what the present self thinks they should do.
3. [5 points] Suppose an economist who thinks $\beta = 1$ wants to estimate δ from the naif's behavior. Find an upper bound for δ, i.e. find a condition on δ such that an exponential discounting agent chooses to miss the same movie as the naif agent.

   Assume now that the agent can go to only one movie during those 4 weeks.
4. [5 points] What movie will a sophisticate see?
5. [5 points] What movie will a naive see?
6. [5 points] Interpret in terms of who is optimist, who is pessimist, when does it help to be either of those?

35. (ignore point values)

2 Intertemporal choice

Consider a consumer with temporaneous utility of consumption

$$u(c) = \frac{c^{1-\rho} - 1}{1 - \rho}$$

for some parameter $\rho \in (0, 1)$ who discounts future temporanous utilities that are one period ahead by δ_1 and utilities that are two period ahead by δ_2. The consumer has positive wealth W_0 and is going to consume it over three periods $0, 1, 2$. He has access to bank deposits which pays the interest rate $r > 0$. Thus we can formulate his problem as:

$$\max_{\{c_0, c_1, c_2\}} u(c_0) + \delta_1 u(c_1) + \delta_2 u(c_2)$$

subject to the constraint:

$$W_0 = c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2}.$$

1. [15 points] Compute consumptions c_0, c_1, c_2.
2. [10 points] What condition on δ_1 and δ_2 ensures time consistency of the consumer?