1	2	3	4	Total
/20	/30	/30	/20	/100

Economics 142 Midterm Exam **NAME________________________________**

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Your grade from this exam is one third of your course grade. The exam ends promptly at 1:50, so you have 80 minutes. You may not use books, notes, calculators or other electronic devices. (Calculators shouldn't be needed.) There are four questions, weighted as indicated. Answer them all. If you cannot give a complete answer, try to explain what you understand about the answer. Write your name in the space above, now. Write your answers below the questions, on the back of the page, or on separate sheets. Explain your arguments and show your work. Good luck!

1. (20 points; slightly modified from number 9 on Problem Set 1) Suppose that there are three money outcomes, x_1, x_2, and x_3, with $x_1 < x_2 < x_3$, and that you can observe which values of p make a person prefer getting x_2 for certain to getting a random outcome {x_1 with probability p, x_3 with probability (1-p)}. By graphing indifference maps in (p_1,p_3)-space, show whether such observations are enough to determine the person's preferences over probability distributions over x_1, x_2, and x_3:

(a) If the person is an expected-utility maximizer? Explain.
[Yes. Observations determine an indifference curve that goes through (0,1,0) in (p_1,p_3)-space and an "up" direction, which are enough to determine the whole indifference map for an expected-utility maximizer.]

(b) If the person likes money and maximizes some differentiable preference function, but the preference function is not necessarily consistent with expected-utility maximization? Explain.
[No. Observations determine an indifference curve that goes through (0,1,0) in (p_1,p_3)-space and an "up" direction, which are not enough to determine the whole indifference map if indifference curves are not linear and parallel.]

2. (30 points; substantially modified from number 14 on Problem Set 1) According to Paul Samuelson, the mathematician Stanislaw Ulam defined a coward as someone who will not bet even when you offer him two-to-one odds and let him choose his side. (A gamble with two-to-one odds is one in which the individual wins \$2*x* if an event A occurs and loses \$*x* if A does not occur. Letting the individual choose his side means letting him choose between winning \$2*x* if A occurs and losing \$*x* if A does not occur, or winning \$2*x* if A does not occur and \$*x* if A occurs.)

(a) Assuming for simplicity that the events that A occurs and A does not occur are equally likely, and letting initial wealth be y, draw a graph for an expected-utility maximizer who likes money, with a differentiable von Neumann-Morgenstern utility function, and who is a coward according to Ulam's definition. (Put von Neumann-Morgenstern utility u(y) on the vertical axis and final wealth y on the horizontal axis.)
[Utility should be concave and increasing, with u(y) > ½u(y-x) + ½u(y+2x). That is, on the vertical axis the point u(y) should be higher than the point halfway on the line segment between u(y-x) and u(y+2x).]

(b) Use your graph to show that he cannot be a Ulam-coward for all values of *x* > 0.
[In the same graph, shrink x toward 0, say to ax, enough that the point u(y) is lower than the point halfway on the line segment between u(y-ax) and u(y+2ax). Since u( ) is differentiable, it will always be possible to do this.]

(c) Draw a graph for a Prospect Theory expected-value maximizer who likes money, with a piecewise linear value function with a kink at winning or losing 0, and who is a coward according to Ulam's definition. (Put value v(y) on the vertical axis and gains or losses on the horizontal axis.)
[Value should be piecewise linear and increasing, with loss aversion so the slope is less for gains than for losses, with v(0) > ½v(-x) + ½v(2x). That is, on the vertical axis the point v(0) should be higher than the point halfway on the line segment between v(-x) and v(2x).]

(d) Use your graph to show that such a Prospect Theory expected-value maximizer is a Ulam-coward for large x if and only if he is a Ulam-coward for small x.
[On the piecewise linear graph from c, shrinking x toward 0 just shrinks the picture, so the person is a Ulam-coward either for all x or for no x.]

3. (30 points) Consider a Prospect Theory expected-value maximizer with value function defined over gains and losses relative to a reference point 0, defined as no gains or losses:

$$v(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2x & \text{for } x < 0. \end{cases}$$

He has some money invested, and each day the value of his investments goes up by \$3000 with probability ¼ or down by \$1000 with probability ¾, and the probability of "up" or "down" on the second day is independent of what happened on the first day.

Suppose first that he has the choice of checking his portfolio's performance either at the end of each day, or only at the end of the second day. However, even if he chooses to check at the end of each day, he still cannot change his portfolio after the first day. His expected value is additive across days, so that if he checks at the end of each day, his total expected value equals his expected value from the first day plus his expected value from the second day. But if he checks only at the end of the second day, his total expected value is just his expected value from the sum of both days' outcomes. (That is, he experiences his gains or losses whenever he checks, whether it is at the end of each day or only at the end of both days.)

(a) Which will he prefer, to check his portfolio's performance at the end of each day or to check only at the end of the second day? Explain, both algebraically and intuitively.
[If he checks at the end of each day, his expected value for each day is ¼3000 – ¾×2×1000 = 750 - 1500 = -750, for a total expected value of -1500 over the two days. If he checks only at the end of the second day, his total expected value is (1/16)×(3000+3000) + 2×(3/16)×(3000-1000) – (9/16)×2×(1000+1000) = -18000/16 = -1125. So it's better to check only at the end of the second day. Intuitively, checking only at the end of the second day gives him a chance of offsetting some losses against possible gains. With loss aversion, this increases his total expected value.]

Now suppose that he faces the same choice, but that if he decides to check at the end of each day, he can pull all of his money out of the stock market (the only option) at the end of the first day is he wishes. *Further suppose that even if he decides to check at the end of each day, his value is still determined by his total gains or losses over both days, with reference point 0.*

(b) What would his investment decision be at the end of the first day, if he finds that his stocks have gone up by \$3000? Explain, both algebraically and intuitively.
[Given that his stocks have gone up by \$3000, if he leaves his money in for the second day he has a ¼ chance of ending up with a total gain of \$6000 and a ¾ chance of ending up with a total gain of \$2000. This yields him total expected value ¼6000 + ¾2000 = 3000, so he is indifferent between leaving his money in for the second day and pulling it out. Intuitively, if his stocks go up in the first day, he is assured of a gain. He is then effectively risk-neutral, and the investment has zero expected return, so he is indifferent between leaving his money in and taking it out.]

(c) What would his investment decision be at the end of the first day, if he finds that his stocks have gone down by $1000?
[**Given that his stocks have gone down by $1000, if he leaves his money in for the second day he has a ¼ chance of ending up with a total gain of $2000 and a ¾ chance of ending up with a total loss of $2000. Correction (thanks to Stephanie Hitchcock for pointing out my mistake): This yields him total expected value ¼2000 - ¾×2×2000 = -2500. If he pulls his money out after the first day, his total expected value is -2000, so it's better to pull it out after the first day.**
(Originally posted answer to part (c): This yields him total expected value ¼2000 - ¾×2×2000 = -1500. If he pulls his money out after the first day, his total expected value is -2000, so it's better to leave it in for the second day. Intuitively, leaving his money in gives him a chance of offsetting some losses against possible gains. With loss aversion, this increases his total expected value. This is like the "longshot bias" whereby bettors who have experienced losses over the day become less risk averse.)]

(d) Given the investment decisions in (b) and (c), and assuming that he breaks any ties in his optimal decisions by leaving his money in the investment, which will he prefer, to check at the end of each day or to check only at the end of the second day?
[Correction resulting from the correction in part (c): If he checks at the end of each day, under the tie-breaking assumption he will leave his money in if he has a gain, realizing total expected value 3000; and pull it out if he has a loss on the first day, realizing total expected value -2000. Thus if he checks at the end of each day, his total expected value is ¼3000 - ¾2000 = -750. If he checks only at the end of the second day, his total expected value is -1125 from part (a). So he will prefer to check at the end of each day.
(Originally posted answer to part (c): If he checks at the end of each day, under the tie-breaking assumption he will leave his money in no matter what. Thus the comparison reduces to the comparison for part (a), and so he will again prefer to check only at the end of the second day.)]

4. (20 points; slightly modified from number 27 on Problem Set 1) Consider the following hypothetical facts: “1% of people in the world population are rational and 99% are irrational. We have a test for rationality. If someone is rational, they have a 60% chance of passing. If someone is irrational, they have a 40% chance of passing. JJ was just given the test, and she passed.”

(a) Assume that JJ was drawn randomly from the world population. What is the probability that she is rational? Don’t bother with the long division. Expressing your answer as a ratio without simplifying it is fine.
[By Bayes’ Rule, the probability is the probability that {JJ is rational and passes the test} divided by the unconditional probability that {JJ passes the test}. The probability that {JJ is rational and passes the test} = $0.01 \times 0.6 = 0.006$. The probability that {JJ passes the test} = $0.01 \times 0.6 + 0.99 \times 0.4 = 0.006 + 0.396 = 0.402$. Thus the probability that JJ is rational is $0.006/0.402 \approx 0.015$. So much for tests!]

(b) Predict the responses of a population of normal people who have not studied either probability theory/statistics or behavioral economics, who are asked to estimate the probability that JJ is rational, given the information in the question. Justify your answer. Describe the kinds of errors that they are likely to make.
[Representativeness and base rate neglect suggest that most people will greatly over estimate the probability that JJ is rational, putting it much closer to 60% (or even higher) than it should be.]