

1	2	3	4	Total
/20	/30	/30	/20	/100

Economics 142 Midterm Exam
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NAME _____
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Your grade from this exam is one third of your course grade. The exam ends promptly at 1:50, so you have 80 minutes. You may not use books, notes, calculators or other electronic devices. (Calculators shouldn't be needed.) There are four questions, weighted as indicated. Answer them all. If you cannot give a complete answer, try to explain what you understand about the answer. Write your name in the space above, now. Write your answers below the questions, on the back of the page, or on separate sheets. Explain your arguments and show your work. Good luck!

1. (20 points; slightly modified from number 9 on Problem Set 1) Suppose that there are three money outcomes, x_1 , x_2 , and x_3 , with $x_1 < x_2 < x_3$, and that you can observe which values of p make a person prefer getting x_2 for certain to getting a random outcome $\{x_1$ with probability p , x_3 with probability $(1-p)\}$. By graphing indifference maps in (p_1, p_3) -space, show whether such observations are enough to determine the person's preferences over probability distributions over x_1 , x_2 , and x_3 :

- (a) If the person is an expected-utility maximizer? Explain.
- (b) If the person likes money and maximizes some differentiable preference function, but the preference function is not necessarily consistent with expected-utility maximization? Explain.

2. (30 points; substantially modified from number 14 on Problem Set 1) According to Paul Samuelson, the mathematician Stanislaw Ulam defined a coward as someone who will not bet even when you offer him two-to-one odds and let him choose his side. (A gamble with two-to-one odds is one in which the individual wins $\$2x$ if an event A occurs and loses $\$x$ if A does not occur. Letting the individual choose his side means letting him choose between winning $\$2x$ if A occurs and losing $\$x$ if A does not occur, or winning $\$2x$ if A does not occur and $\$x$ if A occurs.)

(a) Assuming for simplicity that the events that A occurs and A does not occur are equally likely, and letting initial wealth be y , draw a graph for an expected-utility maximizer who likes money, with a differentiable von Neumann-Morgenstern utility function, and who is a coward according to Ulam's definition. (Put von Neumann-Morgenstern utility $u(y)$ on the vertical axis and final wealth y on the horizontal axis.)

(b) Use your graph to show that he cannot be a Ulam-coward for all values of $x > 0$.

(c) Draw a graph for a Prospect Theory expected-value maximizer who likes money, with a piecewise linear value function with a kink at winning or losing 0, and who is a coward according to Ulam's definition. (Put value $v(y)$ on the vertical axis and gains or losses on the horizontal axis.)

(d) Use your graph to show that such a Prospect Theory expected-value maximizer is a Ulam-coward for large x if and only if he is a Ulam-coward for small x .

3. (30 points) Consider a Prospect Theory expected-value maximizer with value function defined over gains and losses relative to a reference point 0, defined as no gains or losses:

$$v(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2x & \text{for } x < 0. \end{cases}$$

He has some money invested, and each day the value of his investments goes up by \$3000 with probability $\frac{1}{4}$ or down by \$1000 with probability $\frac{3}{4}$, and the probability of “up” or “down” on the second day is independent of what happened on the first day.

Suppose first that he has the choice of checking his portfolio’s performance either at the end of each day, or only at the end of the second day. However, even if he chooses to check at the end of each day, he still cannot change his portfolio after the first day. His expected value is additive across days, so that if he checks at the end of each day, his total expected value equals his expected value from the first day plus his expected value from the second day. But if he checks only at the end of the second day, his total expected value is just his expected value from the sum of both days’ outcomes. (That is, he experiences his gains or losses whenever he checks, whether it is at the end of each day or only at the end of both days.)

(a) Which will he prefer, to check his portfolio’s performance at the end of each day or to check only at the end of the second day? Explain, both algebraically and intuitively.

Now suppose that he faces the same choice, but that if he decides to check at the end of each day, he can pull all of his money out of the stock market (the only option) at the end of the first day if he wishes. *Further suppose that even if he decides to check at the end of each day, his value is still determined by his total gains or losses over both days, with reference point 0.*

(b) What would his investment decision be at the end of the first day, if he finds that his stocks have gone up by \$3000? Explain, both algebraically and intuitively.

(c) What would his investment decision be at the end of the first day, if he finds that his stocks have gone down by \$1000?

(d) Given the investment decisions in (b) and (c), and assuming that he breaks any ties in his optimal decisions by leaving his money in the investment, which will he prefer, to check at the end of each day or to check only at the end of the second day?

4. (20 points; slightly modified from number 27 on Problem Set 1) Consider the following hypothetical facts: “1% of people in the world population are rational and 99% are irrational. We have a test for rationality. If someone is rational, they have a 60% chance of passing. If someone is irrational, they have a 40% chance of passing. JJ was just given the test, and she passed.”

(a) Assume that JJ was drawn randomly from the world population. What is the probability that she is rational? Don't bother with the long division. Expressing your answer as a ratio without simplifying it is fine.

(b) Predict the responses of a population of normal people who have not studied either probability theory/statistics or behavioral economics, who are asked to estimate the probability that JJ is rational, given the information in the question. Justify your answer. Describe the kinds of errors that they are likely to make.