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**Economics 142 Final Exam**  
**Vincent Crawford**

NAME \_\_\_\_\_

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Your grade from this exam is two thirds of your course grade. The exam ends promptly at 2:30, so you have three hours. You may not use books, notes, calculators or other electronic devices. There are six questions, weighted as indicated; within a question, each part identified by a letter (a), (b), (c), etc., is equally weighted. Answer as many parts of as many of the questions as you can, but you are not expected to answer them completely. If you cannot give a complete answer, explain what you understand about the answer. Write your name in the space above, now. Write your answers below the questions, on the back of the page, or on separate sheets. Explain your arguments. Good luck!

1. For each of the following phenomena, briefly discuss the difficulties that might be encountered trying to explaining the phenomenon using standard economic ideas, and then suggest a possible explanation using ideas from this course. Please explain clearly (but briefly) how the ideas you use apply to the phenomenon. (You are *not* asked for any mathematical modeling, only to identify relevant concepts and explain how they apply. The quotations are from *New York Times* articles, supplied on request after the exam.)

(a) “From 1989 to 1992, prices in Boston fell sharply, with condominium prices dropping as much as 40 percent. For a great many of those who bought condominiums during that period, selling could be done only at a significant loss. And, basically, many people refused to sell....For essentially identical condominiums, people who had bought at the peak and were facing a loss generally listed their properties for significantly more than those who had bought at a time when prices were lower....Properties listed above the market price just sat there. In the Boston market over all, sellers listed their properties for an average of 35 percent above the expected sale price, and less than 30 percent of the properties sold in fewer than 180 days.”

**(a) In standard theory, the purchase price is almost completely irrelevant to the decision of what selling price to list at. Controlling for house quality, there should be no significant difference on average between the selling prices of those who had bought at the peak and those who had bought at a time when prices were lower. (One could make up reasons for differences involving strong income effects, but they would be “strained”.) However, if people have reference-dependent preferences with loss aversion, and if they (plausibly) bracket the transaction of buying a given house with selling it, then selling at a loss lowers their value beyond the associated reduction in money gains. (That is, selling at a loss yields negative gain-loss utility, in addition to its effect on consumption utility.) Thus (as in the mugs example, but for somewhat different reasons because of the different bracketing) people’s selling prices are higher, the more they paid for their houses.**

(b) “Genes play a role in Alzheimer’s disease, but in most cases the role is not fully understood. In...late-onset Alzheimer’s, there is no single yes/no gene. Instead, researchers think a combination of genes work together, ...[and each] gene merely adds to the risk....So far, the strongest influence comes from a gene called APOE....But APOE is by no means definitive. Many people with [the APOE gene type that adds to the risk] never become demented, and more than a third of Alzheimer’s patients have [none of that gene type]....Because of the uncertainty, the medical profession, the Alzheimer’s Association and genetic counselors have for years steadfastly advised against APOE testing, saying that the results are not definitive and if misunderstood could be needlessly upsetting, especially since there is no way to prevent or cure the disease.” However, there is now a movement in the medical profession in favor of voluntary APOE testing and revealing the results to patients: “Not everyone wants to know, but the people who want to know really want to know, and they have their own reasons,” Dr. Green said. “I think it’s a little patronizing for the medical establishment to say, ‘We could give you that test, but we don’t think you can handle it.’”....“People are eventually going to understand that genetic risk factors are just risk factors, not determinants,” Dr. Green said. “I think this blanket resistance to APOE exposure is not going to last too much longer.”

**(b) In standard theory, more information (on the individual level) is always better, and it would be irrational for patients not to want it, or for doctors working in patients’ interest to prevent them from getting it. However, if there is a perception that patients will ignore the base rate of Alzheimer’s in the population and focus exclusively on the results of a test that is less than perfectly accurate, the tests might cause costs to patients who are wrongly convinced they will (or will not) get Alzheimer’s greater than the benefits to those for whom the test results are correct. If people did the Bayesian updating correctly, there might still be some emotional harm that outweighs the benefits, just as people might not want to know when they will die; but the standard non-emotional framework will have a hard time reaching the conclusion that the expected costs of having more information are greater than the expected benefits. But if the test brings a strong bias like ignoring the base-rate into play, it is much more likely that the test will reduce expected welfare.**

(c) “With the popularity of traditional lotteries waning across the country, many states are turning to instant games priced at \$20, \$30 and as high as \$50 to lure new players and raise revenue. Scratch-off tickets, for example, now account for more than 75 percent of lottery sales in Texas, which this year became the first state to introduce a \$50 scratch-off game. But critics in Texas and elsewhere say games promising this kind of instant gratification are more likely to contribute to the kind of problem gambling that is usually associated with fast-paced casino betting, and they are now trying to limit them... ‘Scratch-off tickets are to the lottery what crack is to cocaine,’ said State Senator Eliot Shapleigh, a Democrat who represents El Paso.”

**(c) In standard theory, a delay in payment to winners of at most a few days should have little or no effect on gambling behavior, other things equal. But with present-biased or “hyperbolic” preferences, modeled say by  $\beta$ - $\delta$  discounting as discussed in class, making the payment immediate greatly increases the weight of gains (as well as losses). This alone might not be enough to make gambling more attractive in the short run, because the losses might be experienced instantly along with the gains. But if the losses are experienced only when the person runs out of money that month, while the gains are spent “instantly,” then instant games may be more attractive.**

2. A student must do a problem set, but can do it in any one of the three periods  $t = 0, 1, 2$ . The immediate utility cost of doing it in period  $t = 0$  is 4; in period  $t = 1$  it is 6; and in period  $t = 2$  it is 9. The student is a hyperbolic discounter with  $\beta = \frac{1}{2}$  and  $\delta = 1$ . (That is, “self 0”—the student from the point of view of period 0—makes decisions to maximize 0<sup>th</sup>-period utility plus  $\frac{1}{2}$  times the (undiscounted) sum of 1<sup>st</sup>-period and 2<sup>nd</sup>-period utility. Self 1 makes decisions to maximize 1<sup>st</sup>-period utility plus  $\frac{1}{2}$  times 2<sup>nd</sup>-period utility. And self 2 makes decisions to maximize 2<sup>nd</sup>-period utility.)

For the first two parts of this question, assume that the student *cannot* make commitments or limit the freedom of choice of future selves in any way. Note however that if the problem set is not done by the start of period 2, self 2 has no choice but to do it.

(a) First assume that the student is *naïve*, in the sense that self 0 expects selves 1 and 2 to carry out the period-1 and period-2 parts of self 0’s optimal plan, even though selves 1 and 2 have different tradeoffs between periods. Explaining your argument carefully, show that a naïve student actually ends up doing the problem set in period 2. (Hint: Start by figuring out what self 1 will do if the problem set is not done by period 1, and then work backwards to figure out what naïve self 0 will do.)

**(a) If self 1 does it, self 1 experiences period-1 cost 6 and period-2 cost 0, for a total of 6. If self 1 does not do it, self 2 will do it, so self 1 experiences period-1 cost 0 and period-2 cost 9, but the latter is weighted by  $\beta = \frac{1}{2}$ , so the total cost is  $4.5 < 6$ . So self 1 will put it off to period 2. If self 0 does it, self 0 experiences period-0 cost 4 and period-1 and period-2 costs 0, for a total of 4. If self 0 plans to do it in period 1, self 0 anticipates period-1 cost 6 and period-2 cost 0, each weighted (from the point of view of self 0) by  $\beta = \frac{1}{2}$ , so the total anticipated cost is 3. If self 0 plans to do it in period 2, self 0 anticipates period-1 cost 0 and period-2 cost 9, each weighted (from the point of view of self 0) by  $\beta = \frac{1}{2}$ , so the total anticipated cost is 4.5. Thus self 0 will plan to do it in period 1. But self 1 will then put it off to period 2 (deviating from self 0’s plan, although a naïve self 0 does not anticipate this).**

(b) Now assume that the student is *sophisticated*, in the sense that self 0 can correctly predict what selves 1 and 2 will do in whatever situations they find themselves in. Explaining your argument carefully, show that a sophisticated student ends up doing the problem set in period 0.

**(b) Recycling the calculations from part (a), the sophisticated student will correctly predict that if s/he does not do the problem set in period 0, it won’t get done till period 2. The total cost (from the point of view of self 0) will then be 4.5. Because the total cost to self 0 of doing it in period 0 is  $4 < 4.5$ , self 0 will do it in period 0.**

For part (c), assume that self 0 can commit in period 0, completely determining the future decisions of selves 1 and 2 in any way self 0 wishes.

(c) When will a sophisticated student who can commit in period 0 to determine the future decisions of selves 1 and 2 end up doing the problem set? When will a naïve student who can commit end up doing the problem set?

**(c) Recycling the calculations from part (a), the best plan is to commit in period 0 to do the problem set in period 1, for a total cost (from the point of view of self 0) of 3. The answer is the same for a naïve student, because the commitment overrides the student's future decisions, and a naïve student's inability to predict those decisions is the only thing that distinguishes her/him from a sophisticated student.**

3. In the 3×3 game discussed in class:

	L	C	R
T	7, 0	0, 5	0, 3
M	5, 0	2, 2	5, 0
B	0, 7	0, 5	7, 3

(a) Find each player's strictly dominated strategy or strategies, if any, and each player's strategies that survive iterated deletion of strictly dominated strategies.

**(a) Row has no strictly dominated strategies. For Column R is strictly dominated by C. With R eliminated, for Row B is strictly dominated by M. With R and B eliminated, for Column L is strictly dominated by C. With R and B and L eliminated, for Row T is strictly dominated by M. Only (M, C) (for (Row, Column)) survives iterated deletion of dominated strategies.**

(b) Find the Nash equilibrium or equilibria. Justify your Nash equilibrium or equilibria as the only possible outcome(s) of players' strategic thinking, making whatever assumptions about their rationality and/or knowledge of each other's rationality you need.

**(b) (M, C) is the only equilibrium. Assume that each player is rational, and each knows that the other is rational. This reduces the possible outcomes to the 3 × 2 game on the left (omitting R for Column). If in addition Column knows that Row knows that Column is rational, B for Row is also eliminated. If in addition Row knows that Column knows that Row knows that Column is rational, L for Column and T for Row are also eliminated.**

For parts (c) and (d), assume that players are repeatedly paired at random from a large population to play this game, and that they adjust their strategies over time in some way that always reduces the population frequency of each player's pure strategy that has the lowest expected payoff among all of her/his strategies, given the current mix of strategies in the population.

(c) Explain why the population frequency with which Column players play R will decline over time. Show that if it eventually declines close enough to 0, then the population frequency with which Row players play B will also start to decline.

**(c) For Column R is strictly dominated by C, so it always has lower payoff, and will therefore decline over time. Once the frequency of R gets close enough to 0, for Row B will have lower payoff than T or M, and the frequency of B will start to decline.**

(d) Can this process stop anywhere but with Row playing M and Column playing C with probability one? Do you think it is likely to get there? Explain.

**(d) No: Anywhere else, some strategy for one player or the other will have lower expected payoff than that player's other strategies, and its frequency will therefore decline. The process does seem likely to converge to Row playing M and Column playing C with probability one, but it depends on how fast the frequencies decline.**

4. Consider a Matching Pennies game but with the Row player's payoff for (Heads, Heads) 2 instead of 1 (and the Column player's payoff for (Heads, Heads) -2 instead of -1, so that the payoffs still add to 0 for each strategy combination). The Row and Column players' payoffs for (Tails, Tails) remain unchanged at 1 and -1.

(a) Write the payoff matrix of this version of Matching Pennies.

(a)

	Heads	Tails
Heads	2                      -2	-1                      1
Tails	-1                      1	1                      -1

(b) Find the Nash equilibrium and players' equilibrium expected payoffs in this game.

**(b) If  $\Pr\{\text{Heads}\} = p$  for Column, then  $2p - 1(1-p) = -1p + 1(1-p)$  so that  $p = 2/5$  and Row's equilibrium expected payoff is  $1/5$ . Similarly (even though the game is not symmetric)  $\Pr\{\text{Heads}\} = q$  for Row =  $2/5$  and Column's equilibrium expected payoff is  $-1/5$ .**

(c) Compared with the original Matching Pennies game, in which direction does Row's equilibrium probability of playing Heads change? In which direction does Column's equilibrium probability of playing Heads change? Do these directions correspond to your intuitions about the effect of increasing the payoff to matching on Heads on Row's and Column's probabilities of playing Heads? Explain.

**(c) Row's equilibrium probability of playing Heads goes down, counterintuitively because Heads has higher expected payoff for Row, other things equal. Column's equilibrium probability of playing Heads goes up, intuitively because Heads has lower expected payoff for Column, other things equal.**

(d) In which direction do Row's and Column's equilibrium expected payoffs change? Do these directions correspond to your intuitions about the effect of increasing the payoff to matching on Heads? Explain.

**(d) Row's equilibrium expected payoff goes up and Column's equilibrium expected payoff goes down. These changes are both in the intuitive direction.**

(e) How would you expect most real people, not trained in game theory, to respond to the increased payoff to matching on Heads (relative to standard Matching Pennies) in the Row player's role? In the Column player's role?

**(e) For real people, the responses are more likely than not to be in the intuitive directions in each role, as for example they would be for level-1 players.**

5. In the ancient Chinese historical novel *Three Kingdoms*, by Luo Guanzhong, defeated General Cao Cao must choose which of two roads on which to try to escape from victorious General Kongming, who must simultaneously choose which of the roads to wait in ambush on. If Cao Cao is captured, Cao Cao loses 2 and Kongming gains 2. Whether or not Cao Cao is captured, both Cao Cao and Kongming gain 1 additional unit of payoff by taking the comfortable Main Road instead of the awful Huarong Road. (However, because  $2 > 1$  they both think that whether Cao Cao is captured is more important than being comfortable.) The payoff matrix is:

		<b>Kongming</b>	
		<b>Main Road (<math>q</math>)</b>	<b>Huarong Road</b>
<b>Cao Cao</b>	<b>Main Road (<math>p</math>)</b>	-1                      3	1                      0
	<b>Huarong Road</b>	0                      1	-2                      2

(a) Compute the mixed strategy equilibrium in the game, letting  $p$  be the probability with which Cao Cao takes the Main Road and  $q$  be the probability with which Kongming takes the Main Road.

**(a) The game is just like Far Pavilions Escape in the notes. It has a unique equilibrium in mixed strategies, in which  $3p + 1(1 - p) = 0p + 2(1 - p)$  or  $p = 1/4$ , and  $-1q + 1(1 - q) = 0q - 2(1 - q)$  or  $q = 3/4$ .**

(b) Assuming that a level-0 Cao Cao or Kongming randomizes 50-50 between Main Road and Huarong Road, which strategy would a level-1 Cao Cao choose? Which would a level-1 Kongming choose? Which would a level-2 Cao Cao choose? Which would a level-2 Kongming choose? Which would a level-3 Cao Cao choose? Which would a level-3 Kongming choose?

**(b)**

Types	Cao Cao	Kongming
<i>L0</i>	uniform random	uniform random
<i>L1</i>	Main Road	Main Road
<i>L2</i>	Huarong Road	Main Road
<i>L3</i>	Huarong Road	Huarong Road

(c) Suppose both Cao Cao and Kongming are equally likely to be level-1, level-2, or level-3. Using your answer to (b), compute the probabilities with which Cao Cao and Kongming take the Main Road.

**(c) The probability with which Cao Cao takes the Main Road is  $1/3$ , and the probability with which Kongming takes the Main Road is  $2/3$ .**

[The actual story was more interesting than this question, because Kongming waited in ambush along the Huarong Road but also had campfires lit there, anticipating that Cao Cao would think the campfires were meant to fool him into taking the Main Road, and would therefore take the Huarong Road. (*Three Kingdoms* gives Kongming’s rationale for sending this deceptively truthful message: “Have you forgotten the tactic of ‘letting weak points look weak and strong points look strong’?” In other words, put the campfires where you are actually going to be waiting.) Cao Cao did think the campfires were meant to fool him into taking the Main Road, and so took the Huarong Road. (*Three Kingdoms* also gives Cao Cao’s rationale: “Don’t you know what the military texts say? ‘A show of force is best where you are weak. Where strong, feign weakness.’” In other words, put the campfires where you are *not* going to be waiting. Cao Cao must have bought a used, outdated edition of the textbook, one level of thinking behind Kongming’s....)

(The story nonetheless had a happy ending, because Kongming, after capturing Cao Cao, released him.)

These and related questions involving deceptive communication are analyzed using level- $k$  models in the paper at <http://dss.ucsd.edu/~vcrawfor/CrawAER03.pdf> and the lecture slides at <http://dss.ucsd.edu/~vcrawfor/SMUPubLecSlides.pdf>.]

6. Consider an Intersection game like Alphonse and Gaston, in which two drivers meet at the intersection of two roads, with one on each road and no way to distinguish between their roles. The payoffs are:

		Go	Stop
Go		0	1
Stop		1	0

(a) Letting  $p$  be the probability that each driver plays Go, find the mixed-strategy Nash equilibrium and explain why it is an equilibrium. Compute players’ equilibrium expected payoffs.

**(a)  $p = \Pr\{\text{Go}\}$  for Row =  $1/2 = \Pr\{\text{Go}\}$  for Column. Neither can do better with another strategy. Both players have equilibrium expected payoff  $1/2$ .**

(b) If the players have no way to distinguish between their roles, would you expect them to be able to coordinate on one of the Pareto-efficient pure-strategy equilibria? Why or why not?

**(b) No. Coordination on one of the pure-strategy equilibria requires the players to choose asymmetric strategies, but there are no observable differences between them on which to base differences in their strategies. To put it another way, for any rationale for the equilibrium (Go, Stop) there is an equally convincing rationale for the equilibrium (Stop, Go). The only symmetric equilibrium, from (a), is inefficient.**

For part (c), assume that players are repeatedly paired at random from a single large population to play the Intersection game, with no way to distinguish their roles once paired; and that they adjust their strategies over time in a way that increases the population frequency of a pure strategy that has higher expected payoff, given the current mix of strategies in the population.

(c) How would you expect the mixture of strategies, Go or Stop, to evolve over time? Explain.

**(c) Whenever the frequency of Go is low, Stop has higher expected payoff, so the frequency of Stop will increase. Whenever the frequency of Stop is low, Go has higher expected payoff, so the frequency of Go will increase. The population will converge to a mixture in which both have equal expected payoffs, with  $\Pr\{\text{Go}\} = \Pr\{\text{Stop}\} = 1/2$ .**

Return to considering a single two-person interaction as in parts (a)-(b). But for parts (d)-(f), imagine that a stoplight is installed at the intersection, which both players can see before they decide whether to Go or Stop. The stoplight is Green for one driver if and only if it is Red for the other driver, and at any given time when they meet, it is equally likely to be Green for Row and Red for Column or Red for Row and Green for Column.

(d) Show in a new payoff matrix how the stoplight changes the game and its set of equilibria. (Hint: The payoffs for the various combinations of Stop and Go are still as in the above payoff matrix, but now players have more strategies because they can make their decision to Go or Stop depend on whether the light is Red or Green, for example choosing strategies such as “Stop on Green, Go on Red” (there are no traffic laws in the game, so this is just as possible as “Go on Green, Stop on Red”). Further, to evaluate the consequences of their strategies, you now have to make expected-payoff calculations that take into account their uncertainty about whether the light will be Green or Red for them and the effect this has on the final outcome, given their strategies. For example, if they both choose the strategy “Stop on Green, Go on Red”, half the time the light will be Green for the Row player and therefore Red for the Column player, and the outcome will be that the Row player Stops and the Column player Goes, and half the time the light will be Red for the Row player and Green for the Column player, and the outcome will be that the Row player Goes and the Column player Stops. Players’ expected payoffs will be a 50-50 average of their payoffs in the first case and their payoffs in the second case.)

(d) The new matrix has four pure strategies for each player: G, G for Go on Red, Go on Green; S, S for Stop on Green, Stop on Red; etc. The expected payoffs of each strategy combination are half the payoff their strategies yield when the light is Red for Row and Green for Column, and half the payoff their strategies yield when the light is Green for Row and Red for Column.

	G, G	S, S	G, S	S, G
G, G	0, 0	1, 1	$\frac{1}{2}$ , $\frac{1}{2}$	$\frac{1}{2}$ , $\frac{1}{2}$
S, S	1, 1	0, 0	$\frac{1}{2}$ , $\frac{1}{2}$	$\frac{1}{2}$ , $\frac{1}{2}$
G, S	$\frac{1}{2}$ , $\frac{1}{2}$	$\frac{1}{2}$ , $\frac{1}{2}$	1, 0	0, 1
S, G	$\frac{1}{2}$ , $\frac{1}{2}$	$\frac{1}{2}$ , $\frac{1}{2}$	0, 1	1, 0

(e) Would you expect this game to yield players Pareto-efficient payoffs if they have no way to distinguish between their roles? Explain.

(e) **Maybe. There are now Pareto-efficient symmetric equilibria, which the players could coordinate on. But there are two of them (Stop on Red, Go on Green; and Go on Red, Stop on Green), so this is not a foregone conclusion.**

For part (f), assume as for part (c) that players are repeatedly paired at random from a single large population to play the Intersection game with a stoplight as described above, with no way other than the stoplight to distinguish their roles. Further assume that they adjust their strategies over time in a way that always increases the population frequency of a pure strategy that has higher expected payoff, given the current mixture of strategies in the population.

(f) How would you expect the mixture of strategies, G, G; S, S; G, S; and S, G, to evolve over time? Explain.

(f) **(The analysis is close to the dynamic analysis of Battle of the Sexes at pp. 59-60 of the Behavioral Game Theory lecture slides.) Depending on whether the initial conditions have more or less drivers playing G, S than S, G, the population is likely to converge to “all-G, S” or “all-S, G”. These two strategies are neutral against the other strategies G, G and S, S, and so which has higher expected payoff initially depends on which is more frequent. This preponderance will be increased by the dynamics until either G, S or S, G takes over the entire population. Unless there is some cultural or psychological difference that distinguishes G, S from S, G, both outcomes are ex ante equally likely.**