FINAL EXAMINATION

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Ross or Jong — who promise to be clueless), until the examination is collected.

The exam is due by 2:00 PM, Wednesday, March 17, 2010. Submit your exam to Ms. Sydney Sprung in 245 Sequoyah. Office hours at 245 Sequoyah are 7:30 AM to noon, and 1:00 PM to 3:30.

Answer all 5 (five) questions. .

All notation not otherwise defined is taken from Starr's General Equilibrium Theory, draft second edition. If you need to make additional assumptions to answer a question, that's OK. Do state the additional assumptions clearly.

1. Recall in Starr's General Equilibrium Theory, that in defining household demand behavior we used the truncated budget set (where the length of the consumption vector is limited to a maximum value of c),

$$\tilde{B}^i(p) = \{x \mid x \in \mathbf{R}^N, p \cdot x \leq \tilde{M}^i(p)\} \cap \{x||x| \leq c\}.$$

We defined demand behavior as

$$\tilde{D}^i(p) \equiv \{x \mid x \in \tilde{B}^i(p) \cap X^i, x \succeq_i y \text{ for all } y \in \tilde{B}^i(p) \cap X^i\}$$

$$\equiv \{x \mid x \in \tilde{B}^i(p) \cap X^i, x \text{ maximizes } u^i(y) \text{ for all } y \in \tilde{B}^i(p) \cap X^i\}.$$

We then established in Theorem 12.2, under additional assumptions, that $\tilde{D}^i(p)$ is well defined ($\tilde{D}^i(p)$ is non-empty).

(i) Show that this result depends on the truncation of $\tilde{B}^i(p)$. That is, define

$$B^i(p) \equiv \{x \mid x \in \mathbf{R}^N, p \cdot x \leq M^i(p)\}.$$

and

$$D^i(p) \equiv \{y \mid y \in B^i(p) \cap X^i, y \succeq_i x \text{ for all } x \in B^i(p) \cap X^i\}$$

$$\equiv \{y \mid y \in B^i(p) \cap X^i, u^i(y) \geq u^i(x) \text{ for all } x \in B^i(p) \cap X^i\}$$

(these functions do not have the tilde, $\tilde{}$, superscript). Show that for some prices (where $p_k = 0$ for some goods k) and preferences, $D^i(p)$ may not be well defined under the same situation where $\tilde{D}^i(p)$ will be well-defined. **(Question 1 continues next page)**

(ii) Explain why it is unsound economic analysis to restrict the description of the household opportunity set by $\{x||x| \leq c\}$. Note Lemma 14.1 and Theorem 16.1(b). Is it helpful that the restriction is not binding in equilibrium ? Explain.

2. **(i)** The First Fundamental Theorem of Welfare Economics, Theorem 19.1, uses the property of local non-satiation (nearby to every consumption plan of household i there is another that is preferable). This property follows from weak monotonicity of preferences, C.IV* or the combination of nonsatiation C.IV and convexity of preferences C.VI(C). Show that the theorem is false without local non-satiation.

(ii) Review the proof of Theorem 19.1 in Starr's General Equilibrium Theory, draft second edition. There must be a first step or first equation in the proof that is false without the above assumptions. Where does the proof first go wrong without them? Explain.

3. Consider a voting plan for a group of voters to choose the best one of ten possibilities: A, B, C, D, E, F, G, H, I, J. Each voter submits a ballot ranking the possibilities. The voting procedure then gives his first place choice a weight of 10; the second place choice is given a weight of 9; ...; the tenth place choice is given a weight of 1. For each possibility, the weighted votes of all the voters are then added up. The possibility achieving the highest total of weighted votes is declared the winner.

(i) Evaluate the weighted voting procedure in terms of the Sen version of the Arrow axioms. Does the procedure result in a transitive ranking of the alternatives (perhaps including ties)? Does the procedure fulfill: Pareto Principle? Independence of Irrelevant Alternatives? Non-Dictatorship? Unrestricted Domain? Explain.

(Question 3 continues next page)

(ii) Consider the following example to demonstrate whether voters find it advantageous to misstate their true preferences to influence the outcome. Let there be three voters with the following rankings. Topmost proposition is weighted 10, bottom is weighted 1:

Larry	Moe	Curly
A	D	G
B	E	H
C	F	I
D	G	J
E	H	A
F	I	B
G	J	C
H	A	D
I	B	E
J	C	F

Given this ranking G gets 21 points and looks like a winner (Ross can't do all these sums in his head — he thinks that's right). Can Moe restate his preferences to make D a winner? How?

4. (Starr's General Equilibrium Theory, draft second edition, problem 20.10)
In discussing the relationship of saving to consumption in a monetary economy, Keynes writes

"An act of individual saving means — so to speak — a decision not to have dinner to-day. But it does not necessitate a decision to have dinner or to buy a pair of boots a week hence or a year hence or to consume any specified thing at any specified date. Thus it depresses the business of preparing to-day's dinner without stimulating the business of making ready for some future act of consumption...If saving consisted not merely in abstaining from present consumption but in placing simultaneously a specific order for future consumption, the effect might indeed be different." — J. M. Keynes, *The General Theory...*, chap. 16.

Can the difficulty Keynes notes ("depresses the business of preparing to-day's consumption without stimulating ... some future act of consumption") occur in an Arrow-Debreu economy with a full set of futures markets in equilibrium? In particular, in an Arrow-Debreu economy with a full set of futures markets, is it true that (paraphrasing Keynes) *saving consists merely in abstaining from present consumption but not in placing simultaneously a specific order for future consumption?* Explain.

5. In addition to demonstrating core convergence in a pure exchange economy, Debreu & Scarf (1963) demonstrate that the same results hold for a simple production economy, where all households and coalitions have access to the same linear convex technologies. Assume this property in Question 5. You should not need to consult Debreu & Scarf (1963).

Consider the following two-commodity competitive economy. The two commodities are x and y. Each household i is endowed with $r^i = (x^{\circ i}, 0)$. There is a linear technology that converts x into y one-for-one. All households have the same utility function:

$$u^i(x^i, y^i) = x^i + 2y^i - \sum_{h \neq i, h \in H} y^h$$

Each household prefers to consume y rather than x, but there is a negative externality. All households are annoyed by others' consumption of y. That is true even within a coalition; for each i in a coalition S, i incurs the same negative external effect from $y^h, h \in S, h \neq i$, as i incurs from $y^h, h \in H \setminus S, h \neq i$. Assume that the number of elements in H is large and that $\sum_{h \neq i, h \in H} x^{\circ h}$ is much larger than $x^{\circ i}$ for each $i \in H$.

(i) Does this economy have a competitive equilibrium? You may assume that there are perfectly competitive firms with access to the linear technology, achieving zero profits in equilibrium (if it exists), and rebating any profits to the household owners. Explain.

(ii) Assuming there exists a competitive equilibrium, is the equilibrium allocation Pareto efficient? Does the First Fundamental Theorem of Welfare Economics (Theorem 19.1) apply? Explain.

(iii) Assuming there exists a competitive equilibrium, is the equilibrium allocation in the Core? Assume that the weakly convex preferences here do not prevent Theorem 21.1 from applying. Explain.

(iv) Is the core nonempty? Explain.