Economics 200B
FINAL EXAMINATION

This examination is open-book, open-notes. You may consult any published source (cite your references). Other people are **closed**. Do not discuss with classmates, friends, professors (except with Ross or Aislinn --- who promise to be clueless), until the examination is collected. Notation not defined here is taken from Starr's *General Equilibrium Theory*.

Make sure your exam has your name on it. Please place your exam in an envelope with Ross's name on it. Turn the exam in at the desk in **room 245 Sequoyah, by 3:00 PM, Friday, March 21, 2008 or e-mail to abohren@weber.ucsd.edu.**

Answer any FIVE (5) questions. They count equally. An exam with more than five answers will be randomized to choose which answers to mark. Please try to keep your answers brief, clear, and legible. State clearly any additional assumptions you need.

1. Consider majority voting over pairwise alternatives subject to agenda manipulation. Use the following voting rules and preference profiles. There are three propositions to choose among, A, B, and C. There are three voters, 1, 2, 3. The notation > indicates strict preference.

Rules: There is a chairman who sets the agenda, the order of voting. He announces two propositions to choose between; the winner of that vote faces a runoff against the remaining alternative.

Profile I:

Voter 1: A > B > C
Voter 2: B > C > A
Voter 3: C > A > B

Profile II:

Voter 1: A > B > C
Voter 2: B > C, B > A, (C vs. A preference is unspecified)
Voter 3: C > B > A

Claim: Under Profile I the chair can arrange that any one of the three propositions be the winner by the chair's choice of the order of voting. Under Profile II, the choice is independent of the order of the agenda.

(a) Demonstrate the claim.

(b) Discuss with regard to Black's Single Peaked Preferences Theorem (Thm 1, Class notes for February 21) and the definition of an Arrow Social Welfare Function (Class notes for February 19) as a mapping into the space of transitive preference orderings.

2. A conventional definition of competitive equilibrium is

Defintion: $\{p^o, x^{oh}, y^{oj}\}$ with $p^o \in R^N_+$, $x^{oh} \in R^N_+$, $y^{oj} \in R^N$, $x^{oh} \in X^h$, $y^{oj} \in Y^j$, is a competitive equilibrium if

(i) y^{oj} maximizes $p \cdot y$ for all $y \in Y^j$,

(ii) x^{oh} maximizes $u^h(x)$ subject to $p \cdot x \leq p \cdot r^h + \Sigma_j \alpha^{ij}(p \cdot y^{oj})$

(iii) $\Sigma_h x^{oh} \leq \Sigma_h r^h + \Sigma_j y^{oj}$ with $p^o_k = 0$ for k so that the strict inequality holds.

(a) The concept of competitive equilibrium is supposed to reflect decentralization of economic behavior. Explain how this definition embodies the concept of decentralization.

(b) The concept of competitive equilibrium is supposed to reflect market clearing. Explain how this definition includes market clearing.

(c) The concept of competitive equilibrium is supposed to represent competitive price-taking behavior by firms and households. Explain how this definition implies price-taking. Is price-taking part of this definition or a conclusion from it?

3. Consider the existence and efficiency of general competitive equilibrium in an economy subject to each of two alternative tax schemes:

(I) Income tax on endowment,

(II) Income tax on net sales of endowment,

All taxes are rebated as lump sums, denoted T, equally to all households. Each household i treats T parametrically (as fixed and independent of his own consumption choices). We use the following notation:

p is the N-dimensional nonnegative price vector,

x^i ($=D^i(p)$) is the N-dimensional vector of i's consumption as a function of p, based on i's budget which is denoted $M^i(p)$. x^i is a decision for household i.

r^i is the N-dimensional nonnegative vector of i's endowment

#H is the finite integer number of households in the economy consisting of the set H.

(I) We characterize case I, with a 33% tax rate as

$M^i(p) = (1- 0.33)\ p\cdot r^i + T$ where $T = (1/\#H)\Sigma_{h\in H}\ (0.33\ \ p\cdot r^h)$.

The budget constraint is $p\cdot x^i \leq M^i(p)$.

(II) We characterize case II, with a 33% tax rate as

$M^i(p) = p\cdot r^i - (0.33)p\cdot(r^i - x^i)_+ + T$

where $T = (1/\#H)\Sigma_{h\in H}[0.33p\cdot(r^h - x^h)_+]$

The notation $(\ )_+$ indicates the vector consisting of the nonnegative co-ordinates of ( ) with zeroes replacing the negative co-ordinates of ( ). The budget constraint is $p\cdot x^i \leq M^i(p)$.

Please make the usual assumptions about continuity, convexity, monotonicity of preference, and adequacy of income.

(a) In each of cases I and II, will a Walrasian competitive equilibrium exist generally? Explain why or why not. State any additional assumptions you need. Feel free to cite well known results.

(b) The First Fundamental Theorem of Welfare Economics (stated and proved in a setting without taxation) says that a competitive equilibrium allocation is Pareto efficient. In each of cases I and II --- assuming you can find a competitive equilibrium --- does that result still hold? Explain why or why not. If not, explain how the proof of the theorem fails [Recall the proof of 1stFTWE in a pure exchange economy: A superior allocation is more expensive. If it's more expensive then it needs an endowment (for the economy as a whole) bigger in at least one co-ordinate; therefore it is not attainable.] State any additional assumptions you need. Feel free to cite well known results.

4. Consider two firms on a river, U (upstream) and D (downstream). They sell their outputs on perfectly competitive output markets at prices p^U and p^D respectively. The upstream firm U emits effluent into the river in the amount h^U increasing the downstream firm D's costs. A Lindahl pricing scheme is introduced so that D can sell emission permits h^D to U. U is then required to limit h^U so that $h^U \leq h^D$. The Lindahl auctioneer maintains an auction on h^D , sold at p^h by D, purchased by U. The market clearing condition is $h^U \leq h^D$ with equality if $p^h > 0$. Cost functions are $C^U(q^U, h^U)$, $C^D(q^D, h^D)$. Profit functions including the expense and revenue from h^D are

$$\Pi^U = p^U q^U - C^U(q^U, h^U) - p^h h^U$$
$$\Pi^D = p^D q^D - C^D(q^D, h^D) + p^h h^D$$

Assume an interior solution.

(a) Find the first order conditions for U and D's profit maximization. Find the conditions determining the quantity q^U, q^D, h^U, h^D.

(b) Assume a Lindahl equilibrium and an interior solution. Show that the Lindahl equilibrium corresponds to a choice of $h^U = h^D$ maximizing $(\Pi^U + \Pi^D)$ for the given prices and cost functions. Explain the efficiency properties of this solution.

5. In the following question there are at least three dates: 0 (the market date), 1 (immediate future), 2 (next future), ...

(a) Consider an economy over time without uncertainty and a full set of Arrow-Debreu futures markets (this is the setting of Starr's *General Equilibrium Theory* section 15.2). Equilibrium prices and allocations are established at date 0 (the market date). Markets then reopen one period later, at date 1.

How do prices adjust at date 1? Describe the transactions that take place at date 1. Explain.

(b) Consider an economy over time with uncertainty and a full set of Arrow-Debreu contingent commodity (futures) markets (this is the setting of Starr's *General Equilibrium Theory* section 15.4). Equilibrium prices and portfolio allocations among contingent commodities are established at date 0 (the market date). Markets then reopen one period later, at date 1. Of the several events agents considered at date 0 to be possible at date 1, one has occurred. Subjective probabilities of date 2 events (contingent on the event at date 1) are unchanged. Portfolio preferences and attitudes to risk are unchanged.

How do prices adjust at date 1? Describe the transactions that take place at date 1. Explain.

(c) Consider an economy over time with uncertainty and a full set of Arrow-Debreu contingent commodity (futures) markets (this is the setting of Starr's *General Equilibrium Theory* section 15.4). Equilibrium prices and portfolio allocations among contingent commodities are established at date 0 (the market date). Markets then reopen one period later, at date 1. Of the several events agents considered at date 0 to be possible at date 1, one has occurred. But subjective probabilities of date 2 events (contingent on the event at date 1) have changed.

How do prices and portfolios adjust at date 1? Describe the transactions that take place at date 1. Explain.

6. Consider an economy with private goods, n=1, 2, ..., N, and public goods N+1, N+2, ...N+K. There is a set of households H comprising the economy. For each $i \in H$, i's N dimensional private consumption vector is denoted x^i. The prevailing array of public goods is the K dimensional vector Π, experienced in common by all households. Household i's utility can be characterized as $u^i(x^i, \Pi)$.

A Bergson-Samuelson social welfare function for this economy with public goods is characterized as

$$W(u^1(x^1, \Pi), u^2(x^2, \Pi), u^3(x^3, \Pi), ..., u^{\#H}(x^{\#H}, \Pi))$$

Let u^i be continuous, (weakly) concave, differentiable, with positive first derivatives in the co-ordinates of x^i for all $i \in H$. The arguments of W are the #H scalar valued variables u^i . Let W be continuous, (weakly) concave, differentiable, with positive first derivatives in its #H arguments.

Resource constraints in the economy are characterized by the restriction

(*) $(\Sigma_{i \in H} x^i , \Pi) \in \Omega \subseteq R^{N+K}$, where Ω is a closed, convex, non-empty attainable output set.

(a) Define a Pareto efficient allocation in this economy.

(b) Comment on opinions (i) and (ii) below.

Dr. Arthur Optimand, Vice-President of Accenture, is retained to advise the economy on finding an optimal allocation of public goods. He recommends that the allocation of x^{o1} , x^{o2} , ..., $x^{o\#H}$ be left to competitive market mechanisms, but he identifies a choice of

Π^o that maximizes W subject to (*) and to the market allocation of x^{o1} , x^{o2} , ..., $x^{o\#H}$. Dr. Optimand's recommendation meets with two responses:

(i) Excellent choice!! The maximum of W subject to constraint is Pareto efficient. It fulfills the first order condition for Pareto efficiency with a public good, $\sum_{h \in H} MRS^h_{\pi,k} = \sum_{h \in H} \frac{u^h_\pi}{u^h_k} = \frac{\partial k}{\partial \pi} = MRT_{\pi,k}$

where π denotes a typical public good and k denotes a typical private good and derivatives are evaluated at (x^o, Π^o) .

(ii) Nonsense!! By the Arrow Theorem the choice of W is completely arbitrary; any choice necessarily favors some households over others. Further the specification of W depends on different households' utilties being comparable, which is a vacuous untestable assumption. This is simple meaningless garbage. Dr.Optimand's choice of Π^o is arbitrary. There is no reason to expect it to be Pareto efficient.

7. Let N=2. Denote the two goods x and y. Consider a representative agent economy, with a large number of households. All households h, have the same utility function and the same endowment and income:

$$u^h(x, y) = x^{(1/2)}y^{(1/2)} + 2[x]$$

where [x] denotes the greatest integer ≤ x. u^h is discontinuous at many points in R^2_+. [Hint: It may help to think about demand behavior in the neighborhood of $(p_x,p_y) = (^1/_2, ^1/_2)$ with household income equal to 1. As the price vector moves from (.51, .49) to (.50, .50), note the change in the demand for x.]

(a) Assume the usual other assumptions of the general equilibrium model (continuity and convexity of production technologies, convexity of possible consumption sets, adequacy of income, monotonicity and so forth). Can we be sure that there is a competitive equilibrium in this economy?
Explain fully.

(b) When there exists a competitive equilibrium, will the equilibrium allocation be Pareto efficient? Explain fully.