The Structure of Exchange in Barter and Monetary Economies
Author(s): Ross M. Starr
Source: *The Quarterly Journal of Economics*, Vol. 86, No. 2 (May, 1972), pp. 290-302
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/1880564
Accessed: 19/07/2011 15:01

THE STRUCTURE OF EXCHANGE IN BARTER AND MONETARY ECONOMIES *

ROSS M. STARR

I. Transactions and money in general equilibrium models, 290. — II. Representation of equilibrium and exchange, 292. — III. The barter economy, 293. — IV. The money economy, 297. — V. Relation of monetary to barter exchange, 299. — VI. Behavior of money balances, 300. — VII. Conclusion, 301.

I. TRANSACTIONS AND MONEY IN GENERAL EQUILIBRIUM MODELS

In a Walrasian pure exchange general equilibrium model,[1] trade takes place between individual households and "the market." Households do not trade directly with each other. This aspect makes it difficult to study transactions and money in this family of models. The analysis below sets up a framework with a general equilibrium viewpoint in which transactions and money enter essentially. The focus is on the structure — and presumed awkwardness — of barter and the resultant superiority of monetary exchange. Ineffectiveness of barter is supposed to arise inasmuch as, unlike monetary exchange, barter requires "double coincidence of wants." [2]

The most precise statement of the problem can be found in Jevons:

> The earliest form of exchange must have consisted in giving what was not wanted directly for that which was wanted. This simple traffic we call *barter* . . . , and distinguish it from sale and purchase in which one of the articles exchanged is intended to be held only for a short time, until it is parted with a second act of exchange. The object which thus temporarily intervenes in sale and purchase is money.
>
> The first difficulty of barter is to find two persons whose disposable possessions mutually suit each other's wants. There may be many people wanting, and many possessed those things wanted; but to allow an act of barter, there must be a double coincidence which will rarely happen.[3]

* It is a pleasure to acknowledge the advice and criticism of K. J. Arrow. I have had helpful discussions and correspondence with D. Foley, W. P. Heller, J. Ostroy, E. Thompson, and J. Tobin. Any errors are the author's responsibility. The research described in this paper was carried out under grants from the National Science Foundation and from the Ford Foundation.

1. G. Debreu, *Theory of Value* (New York: John Wiley and Sons, 1959).
2. W. S. Jevons, *Money and the Mechanism of Exchange* (London: D. Appleton and Co., 1875), and N. Wallace, "An Approach to the Study of Barter and Money Market Structures," unpublished manuscript, University of Minnesota.
3. Jevons, *op. cit.*, p. 3.

Thus, for Jevons, barter is not merely the exchange of goods against goods, but rather the exchange of reciprocally desired goods. A barter transaction is one in which, for each trader, excess demand is not increased for any commodity and excess supply is not increased for any commodity.[4]

The concept of double coincidence has two parts. The first is that all trade in a barter economy satisfies some ultimate want. When goats are traded for apples it is because the owner of the goats has an excess supply of goats and an excess demand for apples; the owner of the apples has an excess supply of apples and an excess demand for goats. The second part of the double coincidence condition is the idea that the only compensation a trader receives for supplying a second trader's wants is received from the second trader. One would suppose that this condition is obvious except that it is somewhat at variance with the spirit and form of most general equilibrium models.

The double coincidence of wants requirement is a severe restriction on the trades that can take place. Indeed, it is very easy to generate examples of economies where trade is necessary to reach efficient allocations and yet in which no trade can take place because there is no trade satisfying the "double coincidence of wants" condition. This is the substance of Theorem 1 and the three-man, three-good example, below.

Some considerable prestidigitation is required to make difficulties arising from the absence of a double coincidence of wants a reason for the introduction of money. If we agree that to operate a system of exchange under such restrictive rules is awkward or ineffective, that hardly seems reason to complicate the system further by the introduction of another commodity. However, when money is introduced to this family of models, it is defined to be *the* commodity to which the standard restrictions on desirability of commodities traded do not apply. Money is the only commodity that can be accepted in trade though the recipient has no excess demand for it; money is the only commodity that can be given in trade though the donor has no excess supply of it. The effect of introducing money is seen in Theorem 2.

But instead of introducing a single extra commodity for which the double coincidence condition need not hold, why not simply eliminate the double coincidence condition? This would allow all commodities to change hands without necessarily satisfying ultimate

4. An alternative interpretation is that in barter each trade (weakly) increases each trader's utility.

wants. All goods would act as "money." This is the argument of Lemma 1. The answer is not clear. There is a definite feeling in the monetary literature that the number of media of exchange, "moneys," should be small. In particular, not every commodity should be accepted in exchange, like money, only soon to be traded again.

It is very difficult in a general equilibrium model to discover why any commodity should be unacceptable in trade as a medium of exchange. In a general equilibrium model all prices are known to all traders, thus eliminating price uncertainty as a rationale for unacceptability. We generally abstract from transactions costs, which if they differed among commodities might make one commodity preferred over another as a medium of exchange. And in general equilibrium models all commodities have those other properties that are supposed to make them peculiarly suited to function as media of exchange: divisibility and cognizability. I do not think that the analysis below resolves this problem, but it should serve to put it in relief.

II. Representation of Equilibrium and Exchange

I will consider a model of two closely related economies. The focus is not on the existence and determination of equilibrium prices, the initial concern of most general equilibrium analysis, but rather on the nature of the transactions that take place once the prices have been determined and are taken as given. One economy is a traditional pure exchange barter economy. The second is an identical economy except that an additional commodity is introduced. This $N+1^{\text{st}}$ good is thought to behave like "money." The intention is to compare the two economies, and in some cases to see to what extent quantities determined in one economy can be adequately substituted into the other.

Such substitution is designed as a use of the concept of the "classical dichotomy" between money and value theory. Working on the assumption that meaningful relative price determination is the result solely of real variables, we can take a price vector p determined as an equilibrium for the barter economy and attach an arbitrary price of money p^m so that $p^M = (p, p^m)$ is an equilibrium for the corresponding monetary economy. Notations will be defined as needed. Generally, a notation of the form x^B indicates that x is a monetary quantity and x^B is its barter counterpart. A notation of the form x^M indicates that x is a barter quantity and x^M

is the monetary counterpart of x. The process of converting a quantity to its barter or monetary counterpart will usually consist simply in the deletion or insertion, respectively, of an $N+1^{st}$ coordinate.

Trades are described as a quantity of goods going from trader j to trader i, a_{ij}. In the barter economy a_{ij} will be an N dimensional vector; in the monetary economy a_{ij} will be an $N+1$ dimensional vector. a_{ij}^n then denotes the amount of commodity n going from trader j to trader i. An array of a_{ij} for all possible pairs of traders i, j, then describes all trades taking place.

III. The Barter Economy

The economy consists of a finite set of traders T. A commodity bundle is an element of the nonnegative orthant of E^N. A transaction is an element of E^N; a transaction is not generally nonnegative. A price system, usually denoted p, is an element of the nonnegative orthant of E^N. For each $t \epsilon T$, there is an excess demand correspondence $d_t(p)$. Note that for any $x \epsilon d_t(p)$, $p \cdot x = 0$.

The notation $|T|$ denotes the number of elements in the set T. A complex of transactions in the economy is represented as a rectangular array, a $|T|^2 \times N$ matrix.[5] Each row of the matrix corresponds to a pair of traders. The N column entries of each row represent amounts of various goods being exchanged between the two traders. Each row of the matrix will be denoted by two indices. Each index indicates a trader. Thus we write that A is an exchange, $A = \| a_{ij} \|$, where i, $j \epsilon T$, and a_{ij} is the transaction between i and j, an N dimensional vector.

Definition. *An exchange,* $\mathrm{A} = \| \mathrm{a}_{ij}{}^{k} \|$, *is a* $|T|^2 \times N$ *matrix such that* $\mathrm{a}_{ij} = -\mathrm{a}_{ji} \cdot \mathrm{a}_{ij}$ *is called transaction* ij.

The restriction that $a_{ij} = -a_{ji}$ ensures that goods sent from i to j are received by j and understood to be from i. The sign convention indicates the direction in which the goods are going. A commodity whose component in a_{ij} is positive is going from j to i; commodities with negative entries in a_{ij} are going from i to j.

Definition. *An exchange* A *is said to be price consistent at (price vector)* p, *if for each row of* A, a_{ij}, $\mathrm{p} \cdot \mathrm{a}_{ij} = 0$.

Price consistency is a concept fundamental to the transactions analysis of a monetary economy. Because it applies to transactions and not directly to excess demands or consumptions, it is a condi-

5. Since the number of pairs of traders in the economy is $|T|(|T|-1)/2$, this is a larger array than we need, but it makes for easier bookkeeping.

tion that does not appear in the general equilibrium literature. What price consistency requires is that all goods acquired must be paid for by sending goods of equal value from the trader acquiring the purchased goods to the trader supplying them. Price consistency is fulfilled whenever an exchange of goods involves a quid pro quo of equal value at the prices quoted. This is, of course, a considerably more stringent requirement than the usual condition on demand functions that the value of goods supplied to the market should equal the value of goods demanded from the market. Price consistency requires that the value of goods supplied to another trader equal the value of goods received from him. Without some requirement of this sort there is no point in discussing media of exchange, inasmuch as there is no need to pay the seller for goods purchased.

The price consistency condition is merely the abstraction of the fact verified by casual empiricism that when one buys something, one pays the seller for it. Payment for goods purchased seems a concept almost absent from general equilibrium theory. It is required there that the value of goods demanded equal the value of goods supplied, but there is no requirement that the supplier of goods demanded be the recipient of goods supplied. If transactions are actually supposed to take place in a general equilibrium model, then one might conclude that when a trader seeks to purchase goods from their owner he says to the owner, "I wish to acquire from you k units of good n, of which I understand you have an excess supply. I assure you that this acquisition will not cause a violation of my budget constraint at prevailing prices. You may of course consider that by supplying me with k units of n, your budget is enhanced by kp^n." Exchanges consisting of transactions like this are studied in Lemma 2. Since the world of general equilibrium theory is one of certainty, of honest men making binding contracts in good faith with no possibility of default, the seller agrees to the above sale and delivery is made. The only payment for the goods consists in an addition to the seller's budget and a subtraction from the buyer's. These budgets seem to exist mainly in the memories or records of the agents in question. Such a system is unsatisfactory in a world of deceit, forgetfulness, and (honest) mistakes in arithmetic.

The following definition seeks to formulate part of the concept of double coincidence of wants in a market economy.

DEFINITION. *Let* A *be an exchange, and let* p *be a price vector.* A *is said to be monotonically excess demand diminishing at prices* p *if for each* $i \epsilon T$ *there is* $w_i \epsilon d_i(p)$ *so that*

(i) *sign* $a_{ij}^k = $ *sign* w_i^k *or* $a_{ij}^k = 0$, *for* $j \epsilon T$, $k = 1, \ldots, N$,

and

(ii) $| \sum_{j \epsilon T} a_{ij}^k | \leqslant | w_i^k |$.

The sign restriction (i) says that each transaction of an exchange satisfying the definition reduces, or does not increase, the magnitude of excess demands and supplies of each commodity for both parties to the transaction. Condition (ii) ensures that a trader does not overfulfill his excess demands, acquiring more than his demand for some good, delivering more than his excess supply.

One should note that monotone excess demand diminution is only half of Jevons' "double coincidence" of wants. Fulfillment of the former implies that goods are supplied by traders with excess supplies to traders with excess demands. It does not imply that the latter have excess supplies of goods for which the former have excess demands. If an exchange is price consistent and monotonically excess demand diminishing, then I think it fulfills Jevons' concept of "double coincidence" of wants. In such a case each trader supplies others with goods of which he has an excess supply and receives from them each individually goods of an equal value of which they have an excess supply and for which he has an excess demand.

Definition. *Let* A *be an exchange.* A *is said to be excess demand fulfilling at prices* p *if, for each* $t \epsilon T$, $(\sum_{i \epsilon T} a_{ti}) \epsilon d_t(p)$.

Definition. *Let* $p \epsilon E^N$, $p \geqslant 0$. p *is said to be an equilibrium price vector if for each* $t \epsilon T$ *there is* $x_t \epsilon d_t(p)$ *so that* $\sum_{t \epsilon T} x_t = 0$.

Lemma 1. *Let* p *be an equilibrium price vector. There is an exchange* A *that is price consistent and excess demand fulfilling.*

Proof. Choose $x_t \epsilon d_t(p)$ for each $t \epsilon T$ so that $\sum_{t \epsilon T} x_t = 0$. Let $a_{1i} = -x_i$, $i \epsilon T$, $i \neq 1$, $a_{ij} = 0$ for $i \neq 1 \neq j$. Then we have $p \cdot a_{ij} = 0$ all i, j. $\sum_{j \epsilon T} a_{ij} = x_i$, all $i \neq 1$. $\sum_{j \epsilon T} a_{1j} = \sum_{i \epsilon T,\, i \neq 1} -x_i = \sum_{i \epsilon T} -x_i + x_1 = 0 + x_1 = x_1$. Thus A is price consistent and excess demand fulfilling. QED

Lemma 1 makes the reasonably obvious statement that at equilibrium prices there is an exchange that fulfills all traders' excess demands and relieves them of their excess supplies. Further, the exchange is price consistent; for every delivery of goods there is a quid pro quo of equal value. How is this achieved? In the proof, this is achieved by having all traders give their excess supplies to trader 1 and accept from trader 1 their excess demands. A single trader performs the function of a market clearinghouse familiar from general equilibrium theory; we might just as well have several such traders. There is a clearinghouse function that will usually have to be performed. Clearly such an exchange will usually lack the monotone excess demand diminution property; there are large flows of goods through traders with neither excess demands nor supplies for them.

LEMMA 2. *Let* p *be an equilibrium price vector. There is an exchange* A *that is monotonically excess demand diminishing and excess demand fulfilling at* p.

Proof. For each $t \epsilon T$ choose $x_t \epsilon d_t(p)$ so that $\sum_{t \epsilon T} x_t = 0$. The proof proceeds by distributing excess supplies of a commodity among traders with excess demands for the commodity. Such an operation performed over all traders and all commodities yields an exchange satisfying the two conditions. Without loss of generality let $x_1{}^i < 0$. That is, trader 1 has an excess supply of commodity i. Survey traders 2, 3, . . . , $|T|$ in order; if $x_2{}^i > 0$ let $a_{21}{}^i = \min(|x_1{}^i|, |x_2{}^i|)$; if not, let $a_{21}{}^i = 0$. If $x_3{}^i > 0$, let $a_{31}{}^i = \min(|x_3{}^i|, |x_1{}^i - x_{21}{}^i|)$; if not, let $a_{31}{}^i = 0$ and so on for all commodities i and all trading pairs $(1, t)$, $t \epsilon T$. For all i so that $x_1{}^i < 0$, $a_{t1}{}^i = \min(|x_t{}^i|, |x_1{}^i - \sum_{r \epsilon T, r<t} a_{r1}{}^i|)$ if $x_t{}^i > 0$, and $a_{t1}{}^i = 0$ if $x_t{}^i \leqslant 0$. Let $x_2{}^i < 0$ some i. Then if $x_1{}^i > 0$ set $a_{12}{}^i = \min(|x_2{}^i|, |x_1{}^i|)$; if not, set $a_{12}{}^i = 0$. If $x_{3i} > 0$ set $a_{32}{}^i = \min(|x_2{}^i - x_{12}{}^i|, |x_3{}^i - x_{31}{}^i|)$, $a_{32}{}^i = 0$ otherwise . . . . Since $\sum_{t \epsilon T} x_t = 0$ this distribution will exhaust all excess supplies and fill all excess demands. QED

According to Lemma 2, for any equilibrium price vector there is an exchange that satisfies all traders' excess demands, involves them in no transaction that would increase the magnitude of any excess demand or supply, but does not involve payment directly to the supplier by the recipient for goods received. I think it is just such exchanges that are at the back of one's mind in most general equilibrium analysis.

LEMMA 3. *Let* p *be a price vector (not necessarily an equilibrium price vector). There is an exchange* A *that is price consistent and monotonically excess demand diminishing.*

Proof. Let A be the exchange all of whose elements are zero.

QED

THEOREM 1. *Let* p *be an equilibrium price vector. For any two of the three conditions:*

(i) *price consistency,*

(ii) *monotone excess demand diminution,*

(iii) *excess demand fulfillment,*

there is an exchange satisfying those two conditions at p.

Proof. Lemmas 1, 2, 3. QED

The three conditions of Theorem 1 cannot generally all be satisfied by the same exchange. A useful example of this is the case of three goods and three traders. Let prices be (1, 1, 1) and suppose $d_1(p) = (1, 0, -1)$, $d_2(p) = (-1, 1, 0)$, $d_3(p) = (0, -1, 1)$. This is typical of the cases where, though equilibrating trades are obvious, there is no transaction between any pair of traders that diminishes excess demands, increases no excess supplies, and gives payment of equal value for all goods received.

The relation of the three concepts adduced to the double coincidence of wants now becomes clear. Double coincidence holds at equilibrium prices p if there is an exchange A such that:

(i) Goods delivered to trader i from trader j are paid for with goods of equal value sent from i to j. That is, the exchange is price consistent.

(ii) Only goods for which trader i has an excess demand and of which trader j has an excess supply are sent from j to i. That is, the exchange is monotone excess demand diminishing.

(iii) Trade proceeds to equilibrium; all excess demands are satisfied. Thus, the exchange is excess demand fulfilling.

(i) is implicit in Jevons. If (i) were not required there would be no point to the insistence on a double coincidence; a single coincidence of demand and supply would be sufficient for trade to take place. (ii) is explicit. (iii) brings us into a meaningful general equilibrium framework.

IV. The Money Economy

I am about to perform a bit of sleight of hand that has unfortunately fallen into disrepute of late, the trick of converting a barter economy to a monetary economy by the introduction of an $N+1^{st}$ good. The difference between the monetary and barter economies is the interpretation of excess demand diminution. In the monetary economy, the constraints of that definition are not applied to the $N+1^{st}$ good.

Definition. *A monetary exchange is a* $|T|^2 \times (N+1)$ *matrix,* $\|a_{ij}^k\|$, *such that* $a_{ij} = -a_{ji}$, a_{ij} *is called transaction* ij.

In keeping with the classical dichotomy approach to monetary economies, we arbitrarily set the price of money, $p^{N+1} \equiv 1$. Also, all traders' excess demands and supplies of the $N+1^{st}$ good are taken to be zero.

DEFINITION. *Let* $A = \| a_{ij}^k \|$, $k = 1, \ldots, N+1$, *be a monetary exchange. The real counterpart of* A, *denoted* A^B, *is* $\| a_{ij}^k \|$, $k = 1, \ldots, N$. *That is, the real counterpart of a monetary exchange is the same exchange with all* $N+1^{st}$ *elements of the monetary exchange deleted.*

Let p be a price vector for the barter economy. Then $p^M \equiv (p, 1)$ is a price vector for the monetary economy. Let $p = (p^1, p^2, \ldots, p^{N-1}, p^N, 1)$ be a price vector for the monetary economy. Then $p^B = (p^1, p^2, \ldots, p^{N-1}, p^N)$ is a price vector for the barter economy.

The following definition embodies the special status of the $N+1^{st}$ good.

DEFINITION. *Let* A *be a monetary exchange and* p *be a monetary price vector.* A *is said to be monotone excess demand diminishing at* p *if* A^B *is monotone excess demand diminishing at* p^B.

The implication here is that, unlike most goods, money will be accepted in exchange whether it is desired or not.

DEFINITION. *Let* A *be a monetary exchange and* p *be a price system for the monetary economy.* A *is said to be excess demand fulfilling at* p *if* A^B *is excess demand fulfilling at* p^B.

The following theorem, Theorem 2, constitutes the fundamental reason for the introduction of money in this family of models. Theorem 2 asserts the existence in the monetary economy of exchanges having characteristics discussed as desirable earlier in this essay. As shown in the three-man, three-good example, such exchanges do not generally exist for the barter economy.

THEOREM 2. *Let* p *be an equilibrium price vector for the monetary economy. There is a monetary exchange* A *that, at* p, *is price consistent, monotonically excess demand diminishing, and excess demand fulfilling.*

Proof. Choose $x_t \epsilon d_t(p)$ for each $t \epsilon T$ so that

$$(1) \qquad \sum_{t \epsilon T} x_t = 0.$$

For $k = 1, \ldots, N$, choose a_{ij}^k so that sign $a_{ij}^k =$ sign $x_i^k = -$sign x_j^k or $a_{ij}^k = 0$ and so that

$$(2) \qquad \sum_{j \epsilon T} a_{ij}^k = x_i^k \text{ all } i \epsilon T, \; k = 1, \ldots, N.$$

(1) ensures the existence of such a_{ij}^k. Let

$$(3) \qquad a_{ij}^{N+1} = -\sum_{k=1}^{N} p^k a_{ij}^k.$$

(3) gives price consistency. Sign restrictions on a_{ij}^k imply excess demand diminution. (2) implies excess demand fulfillment. QED

Theorem 2 reiterates the fundamental point discussed earlier. In a monetary economy all excess demands can be fulfilled by trades each of which satisfies some excess demand of the trader accepting goods, alleviates an excess supply of the trader furnishing same, and includes direct payment in full to the supplier for goods received. This is not generally true of a barter economy.

V. Relation of Monetary to Barter Exchange

In a classical dichotomy world, money may facilitate commerce, and certainly does not impede it. One can show this by the ingenuous approach of describing a barter exchange and simply noting that a monetary exchange identical to the barter exchange except that there is an appropriate $N+1^{st}$ element in each row is a monetary exchange that has all the qualities (e.g., price consistency, excess demand fulfillment) of the barter exchange from which it was derived.[6] Since for every acceptable barter exchange there is a corresponding acceptable monetary exchange and the converse is false, there are more acceptable monetary exchanges. This suggests that if one is seeking an extremum of some function over exchanges — minimizing search or transactions costs, for example — the extremum over the monetary exchanges will be at least as good as that over barter exchanges.

Theorem 3. *Let* A *be a barter exchange that is monotonically excess demand diminishing and excess demand fulfilling at prices* p. *Then there is a monetary exchange* B *that is price consistent, monotonically excess demand diminishing, and excess demand fulfilling at* p^M *such that* $B^B = A$.

Proof. Let $B = \| b_{ij}^k \|$. For $k=1, 2, \ldots, N$ let $a_{ij}^k \equiv b_{ij}^k$. Let $b_{ij}^{N+1} = -p \cdot a_{ij}$.

Then

$$p^M \cdot b_{ij} = p \cdot b^B{}_{ij} + b_{ij}^{N+1} = p \cdot a_{ij} - p \cdot a_{ij} = 0.$$

Thus, B is price consistent. Since A is excess demand fulfilling and monotonically excess demand diminishing, so is B. QED

Corollary 1 to Theorem 3. *Let* A *be a barter exchange that is monotonically excess demand diminishing, excess demand fulfilling, and price consistent at prices* p. *Then there is a monetary exchange* B *with the same properties at* p^M *so that* $b_{ij}^{N+1} = 0$ *for all* $i, j \epsilon T$.

6. This argument enters essentially in F. H. Hahn, "On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy," in F. H. Hahn and F. P. R. Brechling, eds., *The Theory of Interest Rates* (London: Macmillan; New York: St. Martin's Press, 1965).

Proof. Let B be as in the proof of the theorem. $b_{ij}^{N+1} = -p \cdot a_{ij}$. By price consistency of A, $b_{ij}^{N+1} = -p \cdot a_{ij} = 0$. QED

COROLLARY 2 TO THEOREM 3. *Let* M(p) *be the class of all barter exchanges,* A, *that are monotonically excess demand diminishing, excess demand fulfilling, and price consistent at prices* p. *Let* N(p) *be the family of* B^B *where* B *is a monetary exchange having those properties at* p^M. *Then* $M(p) \subseteq N(p)$.

Proof. Let $B = A^M$ be as constructed in the proof of Corollary 1, then $A \epsilon M(p)$ implies $A \epsilon N(p)$. QED

COROLLARY 3 TO THEOREM 3. *Let* g *be a real valued function defined on barter exchanges. Then*

$$\min_{A\epsilon N(p)} g(A) \leqslant \min_{A\epsilon M(p)} g(A).$$
$$\max_{A\epsilon N(p)} g(A) \geqslant \max_{A\epsilon M(p)} g(A).$$

Proof. Follows directly from Corollary 2. QED

One might note that Corollaries 2 and 3 are of somewhat limited interest inasmuch as $M(p)$ is nonempty only if the economy fulfills double coincidence of wants at p.

VI. BEHAVIOR OF MONEY BALANCES

Returning now to a starting point of this essay, we can analyze part of the classical dichotomy. As an assumption the thesis has been built into the analysis mainly by assuming that demand for goods depends only on the relative prices of goods (definition of excess demand fulfillment in the monetary economy). What does the classical dichotomy mean in this family of models? Clearly it does not mean that transactions are unaffected by the introduction of money. The emphasis of this study is money's effect on transactions. Rather, the classical dichotomy means that the introduction of money does not affect the total net trade (i.e., final consumption) achieved by any trader. That is,

DEFINITION. *Let* A *be a monetary exchange.* A *is said to fulfill the classical dichotomy at prices* p, *if*

$$\sum_{j\epsilon T} a_{tj}^{B} \epsilon d_t(p^B) \quad \textit{for all} \quad t\epsilon T.$$

DEFINITION. *Let* $A = \| a_{ij}^k \|$ *be a monetary exchange.* A *is said to be money clear if for each* $t\epsilon T$ $\sum_{i\epsilon T} a_{ti}^{N+1} = 0$.

LEMMA 4. *Let* A *be a monetary exchange. At prices* p, A *fulfills the classical dichotomy if and only if* A *is excess demand fulfilling.*

Proof. Compare definitions of excess demand fulfillment in the monetary economy and classical dichotomy. QED

THEOREM 4. *Let* p *be an equilibrium price vector for the monetary economy. Let* A *be a monetary exchange that is price consistent and excess demand fulfilling at* p. *Then* A *is money clear.*

COROLLARY 1. *Let* p *be an equilibrium price vector for the monetary economy. Let* A *be a monetary exchange that is price consistent and fulfills the classical dichotomy. Then* A *is money clear.*

COROLLARY 2. *Let* p *be an equilibrium price vector for the monetary economy. There is a monetary exchange that, at* p, *is price consistent, monotonically excess demand diminishing, excess demand fulfilling, classical dichotomy fulfilling, and money clear.*

Proof of Theorem 4 and Corollaries. Corollary 1 follows from the theorem immediately by application of Lemma 4. Corollary 2 follows by applying the theorem and Lemma 4 to the exchange shown to exist in Theorem 2.

Price consistency gives

(1) $p \cdot a_{ij}=0$, all i, $j\epsilon T$.

By the definition of excess demand fulfillment in the monetary economy,

(2) $(\sum_{j\epsilon T} a_{ij}{}^B)\epsilon d_i(p^B)$ all $i\epsilon T$.

By (1) $\sum_{j\epsilon T} p \cdot a_{ij}=0$ all i, but

(3) $\sum_{j\epsilon T} p \cdot a_{ij}=p \cdot \sum_{j\epsilon T} a_{ij}=p^B \cdot \sum_{j\epsilon T} a_{ij}{}^B+\sum_{j\epsilon T} a_{ij}{}^{N+1}$.

By (2) $p^B \cdot \sum_{j\epsilon T} a_{ij}{}^B=0$ so by (1) and (3)

$$0=\sum_{j\epsilon T} p \cdot a_{ij}=p^B \cdot \sum_{j\epsilon T} a_{ij}{}^B+\sum_{j\epsilon T} a_{ij}{}^{N+1}=0+\sum_{j\epsilon T} a_{ij}{}^{N+1}$$

so $\sum_{j\epsilon T} a_{ij}{}^{N+1}=0$ for all $i\epsilon T$. QED

Theorem 4 makes the reasonably elementary point that in an economy where no trader has an excess supply or demand for money holdings, exchanges that fulfill excess demands and are consistent with prices will make no change in money holdings.

VII. CONCLUSION

This essay seeks to analyze the structure of transactions and the use of money in an economy with emphasis on coincidence of wants as a condition for barter exchange. Stating this family of questions in a form susceptible of a rigorous abstract analysis is itself a substantial innovation. Theorems 1 and 2 and the discussion surrounding them emphasize that three conditions on exchange are closely related to the desirability of money in the economy. Of the conditions on exchange — monotone excess demand diminution, price consistency, excess demand fulfillment — there is always a barter exchange satisfying any two, but only if there is double coincidence of all wants will there be a barter exchange satisfying all three. Theorem 2 makes the fundamental point that in a monetary economy all three conditions can always be satisfied. Theorem 3 and its corollaries assert — roughly — that anything a barter economy can do a monetary economy can do better (or as well), at least in the case where the monetary system itself is costless.

The broader intention of the essay is to help make a start at filling Hicks's prescription for making a rigorous microeconomic theory of money.[7] As such it joins a small but growing literature.[8]

YALE UNIVERSITY

7. J. Hicks, "A Suggestion for Simplifying the Theory of Money," *Economica*, 1935; reprinted in Hicks's *Critical Essays in Monetary Theory* (Oxford: Oxford University Press, 1967).

8. Some of the elements of the literature are R. Clower, "Micro-Foundations of Monetary Theory," *Western Economic Journal*, V (1967); D. K. Foley, "Economic Equilibrium with Costly Marketing," Working Paper No. 52, Department of Economics, Massachusetts Institute of Technology, Feb. 1970; F. Hahn, "Equilibrium with Transactions Costs," Walras-Bowley Lecture, Econometric Society Meeting, Dec. 1969, mimeographed; J. Niehans, "Money and Barter in General Equilibrium with Transactions Costs," unpublished manuscript; J. Ostroy, "Exchange as an Economic Activity," Ph. D. thesis, Northwestern University, 1970; E. C. H. Veendorp, "General Equilibrium Theory for a Barter Economy," *Western Economic Journal*, VIII (March 1970); and Wallace, *op. cit.*