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Monetary general equilibrium with transaction costs

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Abstract

Commodity money arises endogenously in a general equilibrium model with separate budget constraints for each transaction. Transaction costs imply differing bid and ask (selling and buying) prices. The most liquid good—with the smallest proportionate bid/ask spread—becomes commodity money. General equilibrium may not be Pareto efficient. If zero-transaction-cost money is available then the equilibrium allocation is Pareto efficient. Fiat money is an intrinsically worthless instrument. Its positive price comes from acceptability in paying taxes, and its use as a medium of exchange is based on low transaction cost.

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"[An] important and difficult question . . . [is] not answered by the approach taken here: the integration of money in the theory of value . . . "

Gerard Debreu, Theory of Value (1959)

1. Money in Walrasian general equilibrium

Two generations ago, Prof. Gerard Debreu suggested that the research agenda for mathematical general equilibrium theory should include a theory of money. Money is used to move purchasing power between markets and transactions, precisely the interaction between markets that general equilibrium emphasizes. Thus, general equilibrium modeling is an appropriate setting for microeconomic foundations of money. Nevertheless, an Arrow–Debreu model cannot successfully provide a role for money. The single budget constraint facing

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transactors in that model precludes a carrier of value between transactions. In order endogenously to derive the transactions role of money, multiple transactions—each with a budget constraint—and a motive for carrying value between them needs to be introduced. Though there has been progress on this head, e.g. Howitt (2000), Jones (1976), Kiyotaki and Wright (1989), Wallace (1980), full integration of money as a medium of exchange in the general equilibrium model has been incompletely successful.[1] The general equilibrium foundations of monetary theory should include parsimonious elementary economic conditions that allow commodity or fiat money to be sustained in an individually rational market equilibrium.

This paper's treatment seeks to provide—in a general equilibrium model with complete markets and complete information—weak sufficient conditions to derive a market equilibrium with the elementary properties of actual monetary economies. Under the conditions posited, trade is monetary in equilibrium; one side of almost all transactions is the economy's medium of exchange (Theorem 3). The key to the formalization is in Hahn (1971) and Foley (1970). Those papers remind us that transaction costs create a bid/ask spread between buying and selling prices. Menger (1892) recognized this price spread as a measure of liquidity and argued that the most liquid assets become endogenous commodity money, "goods [are]. . . *more or less saleable*, according to the. . . facility with which they can be disposed of. . . at current purchasing prices. . . with less or more diminution. . . men. . . exchange goods. . . for other goods. . . more saleable. . . [which] become *generally* acceptable media of exchange." A good is liquid if its bid and ask prices are close together. Thus, price theory implies a theory of liquidity. The most liquid good becomes 'money.' That is the outcome of the model below. Fiat money enters when government provides it (backed by the government's undertaking to accept fiat money in payment of taxes—a notion going back to Adam Smith). Then 'money' is government-issued fiat money, trading at a positive value though it conveys directly no utility or production (Theorem 4).

This essay proposes a parsimonious model of an economy where existence of a medium of exchange is an equilibrium result of the optimizing behavior of individual firms and households. The monetary character of trade, use of a medium of exchange, is shown to be an outcome of general equilibrium with transaction costs. Markets are assumed to be segmented;[2] there is a separate budget constraint at each transaction creating demand for a carrier of value between transactions. Commodity money arises endogenously as the most liquid (lowest transaction cost) asset. Government-issued fiat money sustains its function as a medium of exchange through low transaction cost. This essay presents a full information general equilibrium model with realistic modification of the Arrow–Debreu specification sufficient to derive this monetary structure as an outcome.[3]

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[1] The papers cited here are successful in bringing money into general equilibrium by introducing major frictions or market imperfections in transactions. The present paper seeks to be more parsimonious, using complete markets with minimal frictions sufficient to generate monetary equilibria.

[2] The notion of market segmentation is essential to monetization, Alchian (1977).

[3] A bibliography of the issues involved in this inquiry appears in Ostroy and Starr (1990). In addition, note particularly Banerjee and Maskin (1996), Carmona (2002), Hellwig (2000), Howitt (2000), Howitt and Clower (2000), Iwai (1996), Kiyotaki and Wright (1989), Kocherlakota and Wallace (1998), Monnet (2002), Rajeev (1999), Rey (2001), Ritter (1995), Townsend (1980), Trejos and Wright (1995), Wallace (2001), Young (1998), Clower (1995), and Marimon et al. (1990). The treatment of transaction costs in this essay (as opposed to the

The price system itself designates ‘money’ and guides transactors to trade using ‘money.’

It is useful to distinguish search/random matching models of money, e.g. Kiyotaki and Wright (1989), Trejos and Wright (1995), from general equilibrium models with transaction cost, e.g. Foley (1970), Hahn (1971), Starrett (1973), Ostroy and Starr (1974), Iwai (1996), Howitt (2000) and this essay. Search models emphasize very imperfect uncertain markets with limited ability of traders to locate desirable trades and with limited price flexibility. That approach is consistent with Smith (1776). General equilibrium with transaction cost models typically portray complete markets and a fully articulated price system. Using the complete markets approach allows us to pursue a parsimonious theory: What is a minimal set of market imperfections so that money arises endogenously? Starr (2003) and Starr (in press) provide elementary examples in a trading post model of the equilibria investigated here.

The random matching/search formalization of the friction in trade has a very classical implication: in the rare case where two agents have a double coincidence of wants and meet to trade, they will trade their goods or services directly for one another, Kiyotaki and Wright (1989), Trejos and Wright (1995). This is a distinctive feature, distinguishing the random matching/search models from complete market general equilibrium with transaction cost models. In actual monetary economies, in those comparatively rare instances where double coincidence of wants occurs, it is seldom resolved by barter exchange (a supermarket checkout clerk pays for groceries in money and an autoworker pays money to acquire a car).

This essay’s model is distinct from the overlapping generations model, Samuelson (1958), Wallace (1980, 2001), etc., emphasizing complete markets and including a transactions demand for money at a point in time, not only over time. In the overlapping generations model, demand for money cannot be sustained in the presence of other intertemporal assets carrying a positive rate of return. In the present model (with time dated goods), there may be a demand for money as the low transaction cost instrument even in the presence of assets whose yield dominates money’s.

The present model posits fully informed trade in many separate markets, with a separate budget constraint in each segmented market and transaction costs. The notion of multiple budget constraints is merely the formalization of the observation that budgets balance in each of many transactions separately, Hahn (1971), Ostroy (1973). A typical household will make many distinct transactions, with retailers, service providers, an employer, and so forth. In each of these transactions a budget constraint prevails. At prices prevailing in each transaction, budgets must balance; each party delivers value to the other equal to that he receives. Since there is a multiplicity of separate budget constraints, the market is said to be *segmented*. In addition, there are transaction costs in each market creating differing bid and ask prices. The notion of transactions as a resource using activity is embodied in market-making firms, Foley (1970), with a production technology transforming the ownership of goods between sellers and buyers.

recent focus in the literature on search and random matching equilibria) resembles the general equilibrium models with transaction cost developed in Foley (1970), Hahn (1971), Starrett (1973), and Kurz (1974). The structure of bilateral trade here however is more detailed, with a budget constraint enforced on each transaction separately, so that the Foley, Hahn, and Starrett models do not immediately translate to the present setting.

Multiple budget constraints create demand for media of exchange. Liquidity is priced: its price is the bid/ask spread. The most liquid asset, the instrument that provides liquidity at lowest cost, will be chosen as the medium of exchange. Thus, the choice of commodity money is the outcome of optimizing behavior of economic agents in market equilibrium. Fiat money—issued by government—derives its positive value from acceptability in payment of taxes, and it becomes the medium of exchange from its low transaction cost.

To prove existence of a general equilibrium in a segmented market with transaction cost this paper combines two available treatments. Foley (1970) provides a demonstration of existence of general equilibrium with bid and ask prices and transaction costs in a single unified market. Arrow and Hahn (1971, Chapter 6) demonstrates the existence of general equilibrium with externalities. The composite household model below then expands the commodity space and the population of households. Each commodity is treated as distinct depending on which market segment it trades in. Each household is treated as being many distinct counterparts depending on which market segment it trades in. The counterparts are then combined by formalizing an external effect (in the form of a common consumption and common maximand) among them. The general equilibrium of the composite household model with externalities is then a general equilibrium of the original segmented market economy.

2. Segmented market model

Hahn (1971) and Starrett (1973) introduce sequence economy models with a budget constraint at each of a succession of dates. The sequence economy models are a special case of the segmented market model. The present model is agnostic on the time structure, positing a multiplicity of budget constraints, each separately to be fulfilled. The segmented market model with its multiple budget constraints is intended to represent the requirement that at each of a variety of transactions, agents are required to pay for their purchases. Each household faces many budget constraints, not just one. The absence of a common monetary unit of account means that purchasing power in one market cannot be directly transferred to another—commodities or commodity money must be moved between them.

2.1. Markets

A market is the locus of transactions. Each household is expected to fulfill a budget constraint on each of many markets separately. In specifying the structure of markets the present paper breaks with most of the general equilibrium theory, and follows most closely Hahn (1971) and Starrett (1973). There is a finite set of markets M, each denoted $k \in M$. Multiple budget constraints, one at each $k \in M$, replace the single budget constraint of the Arrow–Debreu model. M also denotes the number of distinct markets.

2.2. Prices

Inasmuch as transactions are a resource using activity there will be a spread between selling and buying prices. Thus, on any single market, there are two prices for each good. The vector $p^{kS} \in \mathbb{R}^N_+$, represents the vector of selling (bid, wholesale) prices on market k

('selling' as viewed by the public). Similarly the vector $p^{kB} \in \mathbb{R}^N_+$, represents the vector of buying (ask, retail) prices on market k ('buying' as viewed by the public). It is convenient to suppress the notation p^{kB} and to concentrate instead on the vector of retail margins. The spread between buying and selling prices on market k is represented by $\pi^k = p^{kB} - p^{kS}$, assuming $\pi^k \geq 0$ (co-ordinatewise). This treatment follows Foley (1970) and facilitates the application of Debreu (1962). Meaningful prices for goods are the rates of exchange, price ratios, between goods on a typical market k. p^{kS} and π^k are quoted in pure numbers (there is no common monetary unit available). The resulting array of prices lies in $\mathbb{P} \subset \mathbb{R}^{2MN}_+$, the unit simplex in $\mathbb{R}^{2MN}_+$. A price vector is presented as $p = (p^{1S}, \pi^1; \dots ; p^{MS}, \pi^M)$, denoted $(p^{kS}, \pi^k)|_{k\in M} \in \mathbb{P}$.

2.3. Households

There is a finite set of households, H; the typical element is denoted $i \in H$. H will also denote the number of households. $x^i \in \mathbb{R}^{MN}$, denotes i's full transaction plan. $x^{ik} \in \mathbb{R}^N$ denotes i's transactions (not consumption) on market k, with positive co-ordinates denoting i's purchases on k, negative co-ordinates i's sales. $\{x^i\}$ denotes the complex of transaction plans for all $i \in H$. The emphasis on transactions rather than consumption simplifies the (abundant) notation and is consistent with Foley (1970). Define $x^{ikB} \in \mathbb{R}^N_+$ as $x^{ikB} \equiv (x^{ik})_+$, the vector of nonnegative co-ordinates of x^{ik} (zeros in place of the negatives).

$X^{ik} \subseteq \mathbb{R}^N$, represents household i's possible trade space in market k.

$X^i \subseteq \mathbb{R}^{MN}$, represents household i's possible trade space over the range of markets.

$X^i \equiv \prod_{k\in M} X^{ik}$, where $\prod$ denotes Cartesian product.[4]

$u^i(x^i) : X^i \to \mathbb{R}$, represents household i's utility function, again defined on transactions, rather than consumption. The treatment will eventually focus on the special case where u^i represents no preference on the location of purchases and only values the aggregate purchase (the case where $u^i(x^i)$ can be summarized as—adjusting for dimension—$u^i(\sum_{k\in M} x^{ik})$). Denote this useful special case as location indifferent utility.

In an interpretation consistent with Foley (1970) and Hahn (1971), households are not modeled as directly incurring transaction costs themselves. Rather they face the bid/ask spread presented by the market and see the transaction costs built into retail prices. Thus a household's action expending gasoline and time in going shopping is idealized as taking place in a firm. The determinants of the decision to do so are summarized in the posted bid and ask prices. An alternative treatment attributing the transaction activity to the household itself is in Kurz (1974).

2.4. Goods

There is a finite list of goods, $n = 1, \dots, N$. As in Debreu (1959), the list of commodities is subject to interpretation. They may simply all be spot goods. Alternatively, some may be spot and others for future delivery. Under uncertainty, they may be defined as well by the state of the world in which they are deliverable.

[4] The specification of X^i as a Cartesian product is mathematically convenient though restrictive.

2.5. Firms

There is a finite set of firms F, with the typical element $j \in F$. $Y^j \subseteq \mathbb{R}^{2N}$, represents firm j's technology set. The structure of markets leads to an awkward oversimplification on firms as market makers and intermediaries. So long as a firm is active only on a single market, the firm's profit as its maximand is well defined. A firm with actions on several markets does not have a well-defined concept of profit due to the differing prices across markets. This leads to the (unsatisfactory) usage that a typical firm is active on only one market.[5] Hahn (1971) treated this difficulty in precisely the same fashion. Each $j \in F$ is active on only one of the segmented markets, $k \in M$. $F(k) \subset F$ is the set of firms j active on market k (this may be only a single firm so that a 'market' and a firm are practically indistinguishable). The typical element of Y^j is (y^j, y^{jB}). y^j is j's net transaction; y^{jB} the portion of j's transaction undertaken at the higher retail (ask) prices, $p^{kB} = p^{kS} + \pi^k$. Both y^j and y^{jB} include both positive and negative co-ordinates. The value of this production plan is $p^{kS} \cdot y^j + \pi^k \cdot y^{jB}$. $0 \le \theta^{ij} \le 1$, is household i's share of firm j. Firms can perform conventional production activities, buying inputs and selling outputs. Following Foley (1970), one of the principal activities of a firm is to undertake transactions. Changing the ownership of a commodity—buying at the bid (wholesale) price and selling at the ask (retail) price—is treated as a production activity. Transactions are resource using; buying and selling take place at differing prices. The firm undertakes transactions to make a profit on the difference between buying and selling prices. Hence, the model here interprets the actions of wholesalers, retailers, brokers—any business that includes making a market—as a special case of production activity.

Though each firm is active on only one market k, a typical household can transact on a variety of markets. The household takes account of all prevailing prices in order to choose the best markets on which to transact. Transaction costs may differ across markets, so prevailing bid and ask prices may differ as well. Thus with N commodities and M market segments there are $2MN$ prevailing bid and ask prices. The reason for investigating this construct is to derive the monetary structure that it generates. The multiplicity of budget constraints implies a demand for a carrier of value between transactions. Based on prevailing bid and ask prices, a typical household might then decide to sell good 1 on one segmented market, acquiring there good 2, which it will then take to a second segmented market to trade for its desired purchase, good 3. Trade may occur in this fashion because prevailing transaction costs make it prohibitive to trade good 1 directly for good 3. Good 2 acts as a carrier of value from one market segment to another; it becomes a commodity money. The household decision-making that leads the household to choose good 2 to act in this fashion is based on the household's endowment, preferences, and prevailing prices. Menger (1892) argued that the choice of a commodity money will be based on liquidity. Theorem 3 below confirms this viewpoint.

Definition. Prices $(p^{*kS}, \pi^{*k})|_{k\in M} \in \mathbb{P}$ household plans $\{x^{*i}\}$, $x^{*i} \in X^i$, and firm plans $(y^{*j}, y^{*jB}) \in Y^j$ are said to constitute a *quasi-equilibrium if for each* $k \in M$,

[5] The arbitrage functions we might expect a firm to undertake are left to households.

$$(i) \quad (y^{*j}, y^{*jB}) \text{ maximizes } p^{*kS} \cdot y^j + \pi^{*k} \cdot y^{jb} \text{ subject to } (y^j, y^{jB}) \in Y^j, \text{ for each } j \in F(k), \tag{1}$$

(ii) for each $i \in H$, $x^{*ik} \in X^{ik}$ maximizes $u^i(x^i)$ subject to

$$p^{*kS} \cdot x^{ik} + \pi^{*k} \cdot x^{ikB} \leq \sum_{j \in F(k)} \theta^{ij}[p^{*kS} \cdot y^{*j} + \pi^{*k} \cdot y^{*jB}], \tag{2}$$

where i treats x^{*im} parametrically, for $m \in M, m \neq k$, or x^{*ik} minimizes $p^{*kS} \cdot x^k + \pi^{*k} \cdot x^{kB}$ subject to $u^i(x^i) \geq u^i(x^{*j})$, where i treats x^{*im} parametrically, for $m \in M, m \neq k$,

$$(iii) \quad \sum_{i \in H} x^{*ik} - \sum_{j \in F(k)} y^{*j} \leq 0, \tag{3}$$

and

$$\sum_{i \in H} x^{*ikB} - \sum_{j \in F(k)} y^{*jB} \leq 0, \quad \text{co-ordinatewise.} \tag{4}$$

The budget constraint (2) calls for special comment. Household i pays for his purchases on market k by delivering goods (negative co-ordinates in x^{*ik}) to k. In addition i may have some profit income from firm ownership in k. In the theorems below, firm technologies are convex cones fulfilling a zero profit condition in equilibrium, so profit income is not essential in the equilibria demonstrated. The segmented market structure necessitates that profit income on market k—which is merely an account balance on k—be spent only on market k. It can of course be used to acquire goods on k that can then be sold on another market k'. Those goods carrying value from one market to another act as media of exchange, commodity money.

3. Media of exchange

Consider the case of location indifferent utility. Recall that $x^{ikB} = (x^{ik})_+$. Let $(x^{ik})_-$ be the (nonpositive) vector of negative co-ordinates of x^{ik} (zeros in place of the positive co-ordinates). Household i's net trade consists of $\sum_{k \in M}(x^{ik})$. But i's gross trades may be much larger than his net trades. The gross trades (as positive values) are $\sum_{k \in M} x^{ikB} - \sum_{k \in M}(x^{ik})_-$. How do gross and net trades differ? If household i, in each market k, acquires (in payment for his sales) only goods that he eventually consumes, gross trades equal net trades. More generally, however, i may deliver most of his excess supplies on one market k' and acquire his consumption on a variety of other markets $k'', k''', \ldots$ In that case he will accept goods in k' in payment for his supplies that he will subsequently trade away on k'', $k''', \ldots$, in exchange for his planned consumption. Those goods temporarily held between markets are acting as media of exchange, commodity money.

The expression

$$e(x^i) \equiv \left[\sum_{k \in M} x^{ikB}\right] - \left[\sum_{k \in M}(x^{ik})\right]_+ - \sum_{k \in M}[x^{ik}]_- + \left[\sum_{k \in M}(x^{ik})\right]_- \tag{5}$$

represents i's flow of goods in trade acting as media of exchange. $e(x^i)$ is gross purchases minus net purchases, (algebraically) minus (negative) gross sales plus net sales. $e(x^i)$ is the flow of goods in excess of those minimally required physically to implement i's net trade. In equilibrium, $e(x^i)$ represents flows of goods acting as carriers of value, making sure that (2) is fulfilled at each $k \in M$.

4. Pareto efficiency

Definition. An allocation $\{x'^i\}$ is said to be Pareto superior to allocation $\{x^{oi}\}$, if for all $i \in H$, $u^i(x'^i) \geq u^i(x^{oi})$ with the strict inequality holding for some $i \in H$.

Definition. An allocation $\{x^i\}$ is said to be attainable if $x^i \in X^i$ for all $i \in H$, and there is $(y^j, y^{jB}) \in Y^j$ for each $j \in F$, so that $\sum_{i \in H}(x^{ik}, x^{ikB}) \leq \sum_{j \in F(k)}(y^j, y^{jB})$, for all $k \in M$.

Definition. An attainable allocation $\{x^{oi}\}$ is said to be Pareto efficient if there is no other attainable allocation $\{x'^i\}$, so that $\{x'^i\}$ is Pareto superior to $\{x^{oi}\}$.

Any reallocation may require incurring transaction costs. The presence of transaction costs and of the wedge between buying and selling prices are not in themselves indications of inefficient allocation. Technically necessary transaction costs incurred in moving to a preferred allocation are not inefficient, Hahn (1971), Starrett (1973). But transaction costs incurred merely in fulfilling budget constraints (2), are regarded as wasted resources. These are the transaction costs incurred in implementing media of exchange $e(x^i)$. An additional related source of inefficient allocation is preferable reallocations (net of technically necessary transaction costs) discouraged by the prospect of transaction costs to be incurred in fulfilling budget constraints.[6]

5. Assumptions

The following assumptions are familiar in conventional general equilibrium models and correspond essentially to those of Foley (1970). Assumptions H.1–H.4 apply to the households of the economy. Assumptions P.1–P.4 correspond to the production sector of the economy, including the transactions process as a resource using activity. These are sufficient to develop a model including an equilibrium with a commodity money in Theorems 1–3 below. An additional family of assumptions on taxation and fiat money issue, M.1–M.7, is developed later in the paper to characterize a fiat money equilibrium in Theorem 4.

[6] In conversation with Nobuhiro Kiyotaki, he argued that the notion of efficiency above is too restrictive. In the view he expresses, as I understand it, budget balance (2) is a technical necessity just as much as is a transaction technology, so the notion of Pareto efficiency should be subject to endowment, transaction technology, and budget balance.

H.1. $X^{ik} \subseteq \mathbb{R}^N$. X^{ik} has a lower bound. As above, $X^i \equiv \prod_{k\in M} X^{ik}$, where $\prod$ denotes Cartesian product. Note that under this definition, $X^i \subset \mathbb{R}^{MN}$, and X^i has a lower bound.

H.2. X^{ik} is closed and convex; $0 \in X^{ik}$. Note that under the definition embodied in H.1, X^i is closed and convex, and $0 \in X^i$.

H.3. $u^i : X^i \to \mathbb{R}$ is continuous, quasi-concave.

H.4. $x' \gg x^o$ implies $u^i(x') > u^i(x^o)$.

P.1. $0 \in Y^j$, $Y^j \subset \mathbb{R}^{2N}$.

P.2. *There is no* $(y^j, y^{jB}) \in Y^j$ *so that* $(y^j, y^{jB}) > 0$.

P.3. Y^j *is a convex cone with vertex at 0.*

P.4. $Y_k = \sum_{j\in F(k)} Y^j$, *is closed for each* $k \in M$.

6. A model of composite households with consumption externalities

We seek to establish the existence of a general (quasi-) equilibrium in the segmented market model. Rather than prove this directly, we take the approach of restating the model in a way that treats the model as a special case of Foley (1970) with externalities in consumption. That model's sufficient conditions are then adequate to ensure existence of equilibrium in the segmented market model. The strategy of proof is to expand the dimension of the commodity space by a factor of M, the number of distinct market segments. That is, there are MN formally distinct goods. Identical goods in distinct segments are then treated as different goods, with distinct prices, transacted by different firms, and consumed by formally distinct households. In the original segmented market model, each household is active in each of the M-segmented markets. We now restate this as each household $i \in H$ having M distinct counterparts, ik, $k \in M$, active on market segment k. The M households are linked in their preferences by an external effect. For each i, and each of the formally distinct households ik' and ik'' (for $k', k'' \in M$, $k' \neq k''$), ik'''s consumption plans enter as an external effect in ik''s utility, as though those consumptions were ik''s own. Hence, we can represent the M-segment complex of purchase plans of the typical household i, in the original segmented market model, as M distinct purchase plans of M distinct households linked by an external effect in the composite household model. There are then HM formally distinct households, each one with preferences linked by an external effect to $M-1$ counterparts. Since each household takes fully into account the consumptions of his $M-1$ counterparts, and since they share a common utility function, optimization for the complex of M distinct households ik, $k \in M$, in the composite household model is equivalent to that of household i in the segmented market model. We demonstrate the existence of equilibrium in the composite household model and then note that the conditions for equilibrium there are precisely equivalent to those of the segmented market model. Hence the segmented market model has a general equilibrium.

In the composite household economy we consider a revised population of households, $HM = \{ik | i \in H, k \in M\}$. For each $ik \in HM$, ik's buying and selling plans are restricted to segment k. $x^{ik} \in \mathbb{R}^{MN}$ with all co-ordinates for markets other than k set identically equal to 0. $x^{ik} = (0, 0, \ldots, 0, x^{ik}, 0, \ldots, 0, 0)$. Conversely, let $x^{i-k} \in \mathbb{R}^{MN}$ denote the MN-dimensional vector of external effects on ik coming from its counterparts active on markets $k' \neq k$. $x^{i-k} \equiv (x^{i1}, \ldots, x^{ik-1}, 0, x^{ik+1}, \ldots, x^{iM})$ setting at 0 the k-indexed co-ordinates. That is, each household $i \in H$ of the original segmented market model appears in the composite household model as M distinct households, one for each segmented market $k \in M$. The separate households are related by an external effect—each of the separate households appreciates fully the consumption decisions of its counterparts.

Household ik's utility function is characterized by strong external effects. ik's preferences are those of i in the segmented market model, applied to ik's net trades and those of im, $m \neq k$. That is,

$$u^{ik}(x^{ik}, x'^{i-k}) \equiv u^i(x'^{i1}, x'^{i2}, \ldots, x'^{ik-1}, x^{ik}, x'^{ik+1}, \ldots, x'^{iM}) \tag{6}$$

where x'^{im} for $m \neq k$ is treated parametrically. ik's income, to be spent on market k, comes from sales of goods on k and from ik's share of profits of firms active on k. We take ik's shares of firms $j \in F(k)$ to be identical to i's, θ^{ij} for $j \in F(k)$. Thus ik's budget constraint is

$$p^{kS} \cdot x^{ik} + \pi^k \cdot x^{ikB} \leq \sum_{j \in F(k)} \theta^{ij}[p^{kS} \cdot y^j + \pi^k \cdot y^{jB}]. \tag{7}$$

Definition. Prices $(p^{*kS}, \pi^k)|_{k \in M} \in \mathbb{P}$, household plans $x^{*ik} \in \mathbb{R}^{2MN}$, firm actions $(y^{*j}, y^{*jB}) \in Y^j$ are said to constitute a quasi-equilibrium in the composite household model if for each $k \in M$

$$(y^{*j}, y^{*jB}) \text{ maximizes } p^{*kS} \cdot y^j + \pi^{*k} \cdot y^{jB} \text{ subject to}$$
$$(y^j, y^{jB}) \in Y^j, \text{ for each } j \in F(k), \tag{8}$$

and for each $ik \in$ HM

$$x^{*ik} \text{ maximizes } u^{ik}(x^{ik}; x^{*i-k}) \text{ on } X^{ik} \text{ subject to}$$
$$p^{*kS} \cdot x^{ik} + \pi^{*k} \cdot x^{ikB} \leq \sum_{j \in F(k)} \theta^{ij}[p^{*ks} \cdot y^{*j} + \pi^{*k} \cdot y^{*jB}] \tag{9}$$

or

$$x^{*ik} \text{ minimizes } p^{*kS} \cdot x^{ik} + \pi^{*k} \cdot x^{ikB} \text{ subject to } u^{ik}(x^{ik}; x^{*i-k}) \geq u^i(x^{*i}),$$

and

$$\sum_{i \in H}(x^{*ik}) - \sum_{j \in F(k)} y^{*j} \leq 0, \text{ co-ordinatewise,} \tag{10}$$

for each $k \in M$, and

$$\sum_{i \in H}(x^{*ikB}) - \sum_{j \in F(k)} y^{*jB} \leq 0, \text{ co-ordinatewise, for each } k \in M. \tag{11}$$

7. Results

Theorems 1–3 below develop the model of commodity money equilibrium, using the existence properties for the composite household model in Appendix A. Theorem 1 merely states that the assumptions are sufficient to generate existence of a quasi-equilibrium.

Lemma 1 (Existence of a quasi-equilibrium in the composite household economy). *Assume* H.1–H.4, P.1–P.4. *Then the composite household economy has a quasi-equilibrium with prices* $(p^{*kS}, \pi^{*k})|_{k\in M} \in \mathbb{P}$.

Proof. See Appendix A. Foley (1970), Theorem 4.1. Arrow and Hahn's (1971) discussion on p. 135 of an economy with continuous external effects demonstrates the existence of a quasi-equilibrium (compensated equilibrium). □

Theorem 1 (Existence of a quasi-equilibrium in the segmented markets model). *Assume* H.1–H.4, P.1–P.4. *Then the segmented market economy has a quasi-equilibrium with prices* $(p^{*kS}, \pi^{*k})|_{k\in M} \in \mathbb{P}$.

Proof. Apply Lemma 1 to the composite household economy. For each $i \in H$, the conditions to maximize $u^{ik}(\cdot)$ subject to budget constraint in $k \in M$ in the composite household model are identical to those of maximizing $u^i(\cdot)$ subject to budget constraint in the segmented market model. □

Pareto inefficient allocation is possible in equilibrium of the segmented market economy. A special case of the segmented market model is a sequence economy, Hahn (1971). See the examples of inefficiency in a sequence economy in Starrett (1973) or Ostroy and Starr (1990). However, Theorem 2 below says that if there is a medium of exchange that operates with zero-transaction-cost ('money'), then common general equilibrium prices can be established. Then the allocation is Pareto efficient by the First Fundamental Theorem of Welfare Economics.

Theorem 2 (Efficiency of allocation with a transaction-costless medium of exchange). *Let* $(p^{*kS}, \pi^{*k})|_{k\in M} \in \mathbb{P}$ *be a quasi-equilibrium price vector and* $\{x^{*i}\}$, (y^{*j}, y^{*jB}) *be the corresponding equilibrium trading and production plans. Let* $x^{*i} \in$ [*interior* X^i], u^i *be differentiable at* x^{*i}, *and let utility be location indifferent, for all* $i \in H$. *Let there be good* n^* *so that* $\pi^{*k}_{n^*} = 0$, $p^{*kS}_{n^*} > 0$ *for all* $k \in M$. *Then the allocation* $\{x^{*i}\}$ *is Pareto efficient.*

Proof. The presence of good n^* overcomes the segmented structure of markets, allowing the M-segmented budget constraints to be equivalent to a single budget. Rescale the equilibrium price vector for clarity of exposition. The price vector that describes the single budget constraint is defined as

$$(p^{okS}, \pi^{ok})|_{k\in M} \equiv \left(\frac{1}{p^{*kS}_{n^*}}(p^{kS}, \pi^k)\right)|_{k\in M}. \tag{12}$$

$p_{n^*}^{okS} = 1$ for all k. Trade with some households buying n^* on one market and selling it on another results in the following marginal rates of substitution in equilibrium. The notation u_n^i indicates a marginal utility, with the subscript designating a partial derivative. For i a buyer of n,

$$\frac{u_n^i}{u_{n^*}^i} = \frac{\min_{k\in M}(p_n^{okS} + \pi_n^{ok})}{1},$$

and for i a seller of m

$$\frac{u_m^i}{u_{n^*}^i} = \frac{\max_{k\in M}(p_m^{okS})}{1},$$

and for i a seller of n and m

$$\frac{u_m^i}{u_n^i} = \frac{\max_{k\in M}(p_m^{okS})}{\max_{k\in M}(p_n^{okS})},$$

and for i a buyer of n and m

$$\frac{u_m^i}{u_n^i} = \frac{\min_{k\in M}(p_m^{okS} + \pi_m^{ok})}{\min_{k\in M}(p_n^{okS} + \pi_n^{ok})}$$

and for i a seller of n and buyer of m

$$\frac{u_m^i}{u_n^i} = \frac{\max_{k\in M}(p_m^{okS})}{\min_{k\in M}(p_n^{okS} + \pi_n^{okS})}.$$

Thus all households in a similar buying–selling position have the same marginal rates of substitution, equated to marginal rates of transformation (including transaction cost). This is a first order condition for Pareto efficiency.

Let us restate the standard notion of a market general equilibrium. For convenience prices will not be required to lie in the unit simplex. Prices $(p'^{kS}, \pi'^{k})|_{k\in M} \in \mathbb{R}_+^{2MN}$, household plans $\{x'^i\}$, $x'^i \in X^i$, and firm plans $(y'^j, y'^{jB}) \in Y^j$ are said to constitute a quasi-equilibrium in the unified market economy if

$$\text{(iv)} \quad (y'^j, y'^{jB}) \text{ maximizes } p'^{kS} \cdot y^j + \pi'^k \cdot y^{jB} \quad \text{subject to } (y'^j, y'^{jB}) \in Y^j, \text{ for each } j \in F, \tag{13}$$

and

(v) for each $i \in H$, $x'^{ik} \in X^{ik}$ maximizes $u^i(x^i)$ subject to

$$\sum_{k\in M} [p'^k \cdot x^{ik} + \pi'^k \cdot x^{ikB}] \le \sum_{k\in M} \sum_{j\in F(k)} \theta^{ij} [p'^{kS} \cdot y'^j + \pi'^k \cdot y'^{jB}], \tag{14}$$

or

$$x'^i \text{ minimizes } \sum_{k\in M} [p'^{kS} \cdot x^k + \pi'^k \cdot x^{kB}] \text{ subject to } u^i(x^i) \ge u^i(x'^i),$$

and

$$\text{(vi)} \quad \sum_{k\in M}\sum_{i\in H} x'^{ik} - \sum_{j\in F} y'^{j} \leq 0 \tag{15}$$

and

$$\sum_{k\in M}\sum_{i\in H} x'^{ikB} - \sum_{j\in F} y'^{jB} \leq 0, \text{ co-ordinatewise.} \tag{16}$$

This is identical to the quasi-equilibrium of Foley (1970), and in the setting of transacting firms, to that of Debreu (1962). Thus, the rescaled price vector (p^{oS}, π^{o}) and the allocation of the quasi-equilibrium in the segmented market economy, $\{x^{*i}\}$, $\{y^{*j}, y^{*jB}\}$, under the conditions of the theorem, is a quasi-equilibrium in the unified market economy. Then the allocation is Pareto efficient by the First Fundamental Theorem of Welfare Economics (e.g. Arrow and Hahn, Theorem 5.3). □

Theorem 3 (Demand for media of exchange). *Let* $(p^{kS}, \pi^{k})|_{k\in M} \in \mathbb{P}$ *be a quasi-equilibrium price vector, and* $\{x^{*i}\}$, (y^{*j}, y^{*jB}) *be the corresponding equilibrium trading and production plans. Let* $x^{*i} \in$ *[interior* X^{i}*], i's utility be location indifferent and differentiable at* x^{*i}. *For some* $n^{*} = 1, \ldots, N$, let $p^{kS}_{n^*} > 0$, *for all* $k \in M$. *Further, let*

$$\frac{\pi^{k}_{n^*}}{p^{kS}_{n^*} + \pi^{k}_{n^*}} < \frac{\pi^{k}_{n}}{p^{kS}_{n} + \pi^{k}_{n}} \tag{17}$$

and

$$\frac{\pi^{k}_{n^*}}{p^{kS}_{n^*}} < \frac{\pi^{k}_{n}}{p^{kS}_{n}} \tag{18}$$

for all $k \in M$, for all $n \neq n^{*}$ with $p^{kS}_{n} > 0$. That is, on all markets, both on the buying and selling side, suppose good n^{*} has the narrowest proportionate bid/ask spread of any good. Let

$$\frac{p^{kS}_{n} + \pi^{k}_{n}}{p^{kS}_{m}} > \frac{p^{k'S}_{n}}{p^{k'S}_{m} + \pi^{k'}_{m}} \tag{19}$$

for every $m, n = 1, 2, \ldots N$, $m \neq n$, and for every distinct $k, k' \in M$. Then the only nonull co-ordinates of $e(x^{*i})$ are in good n^{*}.

Proof. Note that (19) implies there are no profitable arbitrage opportunities open to households at prices $(p^{kS}, \pi^{k})|_{k\in M}$. Proof by contradiction. Suppose not. Then $e_{n}(x^{*i}) > 0$ for some $n \neq n^{*}$. But then we will show that i's transaction plan x^{*i} is not optimizing. There is an alternative x'^{i} fulfilling (2) with utility higher than x^{*i}. x'^{i} can be formulated using more n^{*} as medium of exchange, less n. The strict inequalities in (17) and (18) imply—under H.4—that x'^{i} is preferable. To demonstrate this more precisely, suppose for example that $\left[\left[\sum_{k\in M} x^{*ikB}\right] - \left[\sum_{k\in M}(x^{*ik})\right]_{+}\right]_{n} > 0$ (this is one of several cases where $e(x^{*i})_{n} > 0$; other cases are similar). Consider k' so that $x^{*ik'B}_{n} > 0$ and k'' so that $x^{*ik''}_{n} < 0$. Choose small $\varepsilon > 0$. Formulate x'^{ikB} as equal to x^{*ikB} in all co-ordinates except $x'^{ik'}_{n^*}, x'^{ik'}_{n}, x'^{ik''}_{n^*}, x'^{ik'}_{n}$. Set $x'^{ik'}_{n} = x^{*ik'}_{n} - \varepsilon$; set $x'^{ik'}_{n^*} = x^{*ik'}_{n^*} + \varepsilon((p^{kS}_n + \pi^k_n)/(p^{kS}_{n^*} + \pi^k_{n^*}))$; set $x'^{ik''}_{n} = x^{*ik''}_{n} + \varepsilon$; and set $x'^{ik''}_{n^*} = x^{*ik''}_{n^*} - \varepsilon(p^{k''S}_n / p^{k''S}_{n^*})$. Note $\varepsilon((p^{kS}_n + \pi^k_n)/(p^{kS}_{n^*} + \pi^k_{n^*})) - \varepsilon(p^{k''S}_n / p^{k''S}_{n^*}) > 0$ Then x'^i fulfills (2), but $u^i(x'^i) > u^i(x^{*i})$. This demonstrates the contradiction. □

Theorem 3 embodies Menger's (1892) argument that market equilibrium designation of commodity money is based on liquidity. Theorem 3 proposes that there be a single good n^* with narrowest proportional bid/ask spread at prevailing prices. Then n^* will be the unique medium of exchange. Theorem 3 poses a simplified case—there could be several goods tied for narrowest bid/ask spread or the good with a narrow bid/ask spread could vary across markets (in which case there would be no common medium of exchange). Nevertheless, the underlying principle is clear. Liquidity is priced in the bid/ask spread and the most liquid good(s) will be the medium(a) of exchange.

When a household engages in trade, its sales from endowment or its income from business may be concentrated on one market k' but its purchases for consumption may center on another market k''. The two values, sales (plus profits) and purchases, must balance on each market separately, (2). Hence the household uses a carrier of value, commodity money $e(x^i)$, to shift purchasing power among markets. It seeks to do so in the most advantageous fashion possible, losing as little purchasing power as possible in the process. That is how the household forms its optimizing choice of $e(x^i)$. For arbitrary $(p^{kS}, \pi^k)|_{k\in M} \in \mathbb{P}$, there is no simple general characterization of $e(x^i)$. But Theorem 3 describes the most interesting special case. Suppose—at prevailing prices—there is a natural money, n^*, a good with such low transaction costs that the prevailing bid/ask spread makes it the least costly way to move purchasing power across all markets. Then n^* is the only good that will be used as $e(x^i)$. All transactions will either be for directly useful trades—delivering supplies, fulfilling demands—or they will be in n^* acting as a medium of exchange.

8. Fiat money and government

To incorporate fiat money there are two issues to be addressed: How does an intrinsically worthless instrument become positively valued in equilibrium? How does this instrument become the common medium of exchange? The answer to the first question is "taxes."[7] The answer to the second is "low transaction costs." Introduce a government in the model with the powers to issue fiat money and to collect taxes. Fiat money is intrinsically worthless; it enters no one's utility function. But the government is uniquely capable of issuing it and of declaring it acceptable in payment of taxes. Adam Smith (1776) notes "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money" (vol. I, Book II, Chapter 2).

[7] The currently prevailing (alternative) treatment of the value of fiat money in the literature, Wallace (1980), Kiyotaki and Wright (1989), Samuelson (1958), etc. is to treat money as a bubble in an infinite horizon model. In this approach, money is completely unbacked, its value sustained by the expectation of its future value. There are typically multiple equilibria then, including a nonmonetary equilibrium where in the absence of this expectation, fiat money has zero value.

Abba Lerner (1947) comments "The modern state can make anything it chooses generally acceptable as money. . . a simple declaration. . . will not do. . . But if the state is willing to accept the proposed money in payment of taxes . . . the trick is done." Taxation—and fiat money's guaranteed value in payment of taxes—explains the positive equilibrium value of fiat money.[8] Assume the fiat money also to have very low transaction costs. Then the conclusion follows. Fiat money becomes the common medium of exchange.[9] For an example of how this logic works see Starr (2003). To formalize these views we extend the model of Sections 1–7 by introducing government and fiat money.

Define taxes, fiat money and government in the following way. Government, denoted G, is formalized as another household with distinctive properties for its trade set, X^G. Tax receipt certificates are good $N-1$. Every household $h \in H$ has a tax quota $\tau^h > 0$, so that there is positive marginal utility from acquiring additional x^h_{N-1}up to the level τ^h. No firm $j \in F$ can produce $N-1$ and no household in H can achieve a net disbursement of $N-1$ (that is, no household is endowed with $N-1$). Government, denoted G, is the unique source of N and is endowed with $N-1$ (more formally, X^G admits the possibility of a net disbursement of $N-1$, tax receipts), but not so much that it becomes a drug on the market. Good N will be treated as fiat money. We assume that no household gets positive utility from net acquisition of good N. Government G, declares its willingness to accept N (which nobody wants) in exchange for $N-1$ (which everybody wants). This amounts merely to defining u^G, G's utility function, with strictly positive marginal utility for N and for $N-1$. State these notions as:[10]

M.1. For all $h \in H, u^h$ is location indifferent. For each $x^h \in X^h$, each $n = 1, 2, \ldots, N-2$, there is $\tau^h > 0$ so that if $\sum_{k\in M} x^{hk}_{N-1} \leq \tau^h$, then

$$+\infty > \frac{(\partial u^h(x^h))/(\partial x^h_{N-1})}{(\partial u^h(x^h))/(\partial x^h_n)} \gg 0.$$

M.2. $X^G \equiv \prod_{k\in M}[\mathbb{R}^N_+ - \{(0, 0, \ldots, \gamma^k, \tau'^k\}]$, where $\gamma^k \geq 0$, $\sum_{k\in M}\gamma^k = \sum_{h\in H}\tau^h$, $\gamma^h > 0$ only for k so that $j^o \in F(k)$, for j^o described in M.7; where $\tau'^k \geq 0$ and $\sum_{k\in M}\tau'^k = \sum_{h\in H}\tau^h$. For all $h \in H$, all $x \in X^h$, $x^{hk}_{N-1} \geq 0$.

M.3. For all $x \in X^G$, $\partial u^G(x^G)/\partial x^G_{N-1} = \partial u^G(x^G)/\partial x^G_N > 0$.

M.4. For all $h \in H$, for $x^h_N > 0$, $\partial u^h(x^h)/\partial x^h_N = 0$; for $x^h_N < 0$, $\partial u^h(x^h)/\partial x^h_N > \partial u^h(x^h)/\partial x^h_{N-1} > 0$.

[8] See also Dubey and Geanakoplos (2001), Li and Wright (1998) and Starr (1974).

[9] A more complex argument involves a scale economy, ruled out by the present paper's convexity assumption. If there is a scale economy in transaction costs and if government is a large conomic agent, then government transactions in fiat money ensure sufficient scale to result inlow transaction costs. Hence fiat money becomes the unique common medium of exchange, Starr (2003), Starr and Stinchcombe (1999), Tobin (1959, 1980).

[10] The partial derivatives representing marginal utilities in the assumptions below are assumed to exist. The assumptions can be restated without differentiability, but the notion of marginal utility is convenient here.

M.5. For all $j \in F$, all $(y^j, y^{jB}) \in Y^j$, $y^j_{N-1} = y^j_n = 0$.

M.6. For all $j \in F$, let $(y'^j, y'^B) \in Y^j$, let $y'^j = y'^j$, $y'^{jB}_n = y''^{jB}_n$ for $n = 1, 2, \ldots, N-1$. Let $y'^{jB}_N \geq y'^{jB}_N$. Then $(y'^j, y'^{jB}) \in Y^j$. [M.6 creates an exception to P.2; trade in money is not resource using.]

M.7. Let $(y'^j, y'^{jB}) \in Y^j$, let $y'^j = y'^j$, $y'^{jB}_n = y''^{jB}_n$ for $n = 1, 2, \ldots, N-2$. Let $y''^{jB}_{N-1} \geq y'^{jB}_{N-1}$, $y''^{jB}_N \geq y'^{jB}_N$. Then for at least one $j^o \in F$, $(y''^{jo}, y''^{joB}) \in Y^{jo}$. [M.7 creates an exception to P.2. For at least one $j^o \in F$, trade in money and taxes is not resource using. Under M.5 note that money and taxes are pure exchange goods; they are not produced.]

M.6 and M.7 allow a modest free lunch—no transaction costs in money and taxes. Hence, Theorem 1 above cannot directly be applied. This difficulty is treated in the proof of Theorem 4.

Assumptions M.1–M.7 define the notions of fiat money and taxation. M.1 says that households try to arrange their affairs to pay their taxes and that the marginal rate of substitution of tax payment for other goods is bounded away from zero when taxes have not fully been paid. Since fiat money is acceptable in payment of taxes, M.1 guarantees a finite price level in terms of N. That is, M.1 puts a floor on the value of fiat money. M.2, M.5and M.6 say that government, G, is the unique source of money, good N, and of tax receipt certificates, good $N-1$. Government sells tax receipt certificates only on markets where they incur no transaction cost and does not flood the market with money, good N. M.3 says that government, G, is willing to accept money, good N, one for one, in exchange for tax receipt certificates, $N-1$. M.4 says there is no utility to holding money, good N, for any household; only government G behaves as though money, N, is desirable. A household may be a net seller of money on some markets, but it will never be a net seller in aggregate (preferably, this consideration would be embodied in a constraint, but stating it in the household utility function is the most convenient formalization available here). M.7 says that money, good N, carries low transaction costs. M.7 says that there is at least one market where both money and tax receipts, goods $N-1$ and N, carry low transaction costs.

Theorem 4 (Existence of a fiat money quasi-equilibrium). *Assume H.1–H.4, P.1–P.4, M.1–M.7. Then the economy has a quasi-equilibrium with prices* $(p^{*kS}, \pi^{*k})|_{k\in M} \in \mathbb{P}$. *Further,* $p^{*kS}_N > 0$, *for all* $k \in M$.

Proof. We cannot directly apply Theorem 1 because of free transactions in $N-1$ and N under M.6 and M.7, violating P.2. Let $K^q \subset \mathbb{R}^{2N}$ be a cube centered at the origin of side $q = 1, 2, 3, \ldots$ Consider the truncated economy characterized by firms with production technologies $Y^j \cap K^q$. Apply the Theorem of Debreu (1962) to the composite household economy with truncated technology. Let the (truncated) economy's quasi-equilibrium prices and firm actions be $(p^{*kS}, \pi^{*k})|_{k\in M}$, $(y^{qj}, y^{qjB}) \in Y^j$. Under M.6, $y^{qj}_{N-1} = y^{qj}_N = 0$ for all j. Firm j's retail actions in $N-1$ and N, y^{qjB}_{N-1}, y^{qjB}_N, may increase without bound as q becomes large. This can occur in equilibrium of the qth truncated economy as q becomes large only in the case of wash sales of $N - 1$ and N, which have no effect on household utility (since they wash) or on firm inputs or profits (by M.6 and M.7). Real equilibrium activity, in goods $1, 2, 3, \ldots, N-2$, is necessarily bounded by P.1–P.4. Then household and firm actions are set-valued and there is also bounded equilibrium household and firm action. Take a convergent subsequence in prices and actions. Its limit is market equilibrium prices and household and firm actions, $(p^{*kS}, \pi^{*k})|_{k \in M} \in \mathbb{P}$, $(y^{*j}, y^{*jB}) \in Y^j$, $\{x^{*i}\}$. $p^{*kS}_{N-1} > 0$ by M.1, M.5, and M.7. Then $p^{*kS}_N > 0$ by arbitrage under M.3. Thus there is a monetary equilibrium for the composite household economy. Then apply the same argument as in the proof of Theorem 1: equilibrium prices of the composite household economy are equilibrium prices of the segmented market economy. □

In Theorem 4, positivity of the price of fiat money, $p^{*kS}_N > 0$ for each k, comes from the structure developed in M.1–M.7. Good N is desirable since it is desired by G on the same basis as $N - 1$ (M.3) and all households want $N - 1$ (M.1). These statements hold net of transaction costs since these are small (M.6, M.7). Further the scarcity value of N and $N - 1$ is ensured by limitations on supply (M.2, M.4, M.5).

Corollary 1 (To Theorems 2 and 4). *Assume* H.1–H.4, P.1–P.4, M.1–M.7. *Let the segmented market economy have a quasi-equilibrium with prices* $(p^{*kS}, \pi^{*k})|_{k \in M} \in \mathbb{P}$, $p^{*kS}_N > 0$, *and trading plans* $\{x^{*i}\}$. *Let* u^i *be location indifferent and differentiable at* x^{*i} *for all* $i \in H$. *For all* $i \in H$, *let* $x^{*i} \in$ [*interior* X^i] *and be equilibrium trading plans. Then* $\pi^{*k}_N = 0$, *for all* $k \in M$, *and the equilibrium allocation is Pareto efficient.*

Proof. Theorem 4 tells us that there is a quasi-equilibrium with $p^{*kS}_N > 0$. By M.7 and marginal cost pricing we have $\pi^{*k}_N = 0$. Then Theorem 2 implies a Pareto efficient allocation. □

Corollary 1 restates Theorem 2 in the case of fiat money. A zero-transactions-cost medium of exchange assures Pareto efficiency of equilibrium allocation. In cases where good N, fiat money, is the unique zero-transaction-cost instrument, then Theorem 3 implies that fiat money is the only medium of exchange in equilibrium.

9. Conclusion

An Arrow–Debreu general equilibrium model modified to include transaction costs and multiple budget constraints implies monetary trade as a consequence of the equilibrium. Commodity (and fiat) money flows are endogenously determined as part of the equilibrium actions of firms and households. Liquidity is priced in the bid/ask spread; prices provide a direct incentive to concentrate the medium of exchange function in goods with the narrowest bid/ask spread. Fiat money's positive value is supported by acceptability in payment of taxes and it becomes the common medium of exchange because of low transaction cost. Commodity or fiat money with a zero-transaction-cost leads to Pareto efficient equilibrium allocation.

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Appendix A. Foley's transaction cost model and Arrow and Hahn's treatment of external effects

Foley (1970) noted the formal equivalence of the existence of a quasi-equilibrium in Debreu (1962) to its existence in a model of transaction cost featuring a doubled dimension of the commodity space and convex transaction technology. Arrow and Hahn (1971, Chapter 6) showed that the results demonstrating existence of quasi-equilibrium (compensated equilibrium) could be generalized to a model including continuous external effects among households. The combined result below notes that the same logic means that the Foley (1970) result holds in the presence of continuous external effects among households. All of these results apply in a single market. Now consider M separate markets where each household and each firm is allowed to be active in only one market and each household has ownership shares only of the firms in its separate market. The adapted result below extends the combined result to this separated market model: continuous external effects are consistent with a quasi-equilibrium (compensated equilibrium) in the separated market model. Equilibrium of the composite household economy is a special case of this result.

A.1. Published results

(Foley (1970)): Let $M = 1$, and assume H.1–H.4, P.1–P.4, with no external effects. Then there is a quasi-equilibrium $(p^{*S}, \pi^*) \in \mathbb{R}_+^{2N}$.

(Arrow and Hahn (1971)): Let M=1, and assume H.1–H.4, P.1–P.4, with continuous external effects (each household's utility function is continuous in the consumptions of other households). Let transaction costs be nil. Then there is a compensated equilibrium (quasi-equilibrium) $p^* \in \mathbb{R}_+^N$.

A.2. Combined result

Assume H.1–H.4, P.1–P.4, with continuous external effects (each household's utility function is continuous in the consumptions of other households). Let M=1. Then there is a quasi-equilibrium $(p^{*S}, \pi^*) \in \mathbb{R}_+^{2N}$.

A.3. Adapted result

Assume H.1–H.4, P.1–P.4, with continuous external effects. Let $M > 1$ with each firm and household active on only one $k \in M$. Then there is a quasi-equilibrium $(p^{*kS}, \pi^{*k})|_{k\in M} \in \mathbb{R}_+^{2MN}$.

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