

Mengerian saleableness and commodity money in a Walrasian trading post example[☆]

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Abstract

In an economy with commodity-pairwise trading posts and transaction costs, commodity money is endogenously determined in general equilibrium. Absent double coincidence of wants, the low-transaction cost commodity (with the narrowest proportional bid/ask price spread) becomes the common medium of exchange.

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“The false definitions of money divide up into two main groups: those that consider it to be something more, and those that consider it to be something less, than the most saleable commodity.”

— William Roscher, *Principles of Political Economy* (1878)

1. A price theory of money: commodity money as the most liquid good

This paper presents a simple example augmenting an Arrow–Debreu–Walras general equilibrium model sufficiently to allow monetary structure to appear as a result, not an assumption, of the model. It is well known that the Arrow–Debreu model cannot support money; for households it uses a single budget constraint and for firms a single profit expression, both summarizing all buying and selling transactions in a single equation. The most elementary function of money — the medium of exchange — is as a carrier of value held between successive

transactions. In order to model the function of a carrier of value the model will need to distinguish transactions individually. That notion is formalized below as a trading post model, where the budget constraint is fulfilled at each trading post transaction separately. The trading post model derives, from elementary initial conditions, a competitive general equilibrium with a unique commodity money, the common medium of exchange.

The distinguishing feature of the endogenous commodity money is liquidity. This notion goes back over a century. Carl Menger (1892) wrote:

why...is...economic man...ready to accept a certain kind of commodity, *even if he does not need it*,...in exchange for all the goods he has brought to market[?].... *The theory of money necessarily pre-supposes a theory of the saleableness [Absatzfähigkeit] of goods...* [Call] goods...*more or less saleable*, according to the...facility with which they can be disposed of...at current purchasing prices or with less or more diminution... Men...exchange goods...for other goods...*more saleable*...[which] become generally acceptable media of exchange. [emphasis in original]¹

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¹ See Radford (1945) on the evolution of a cigarette currency and Newhouse (2004) on convergence to monetary equilibrium in a 3-commodity model. Banerjee and Maskin (1996) focus on the ease or difficulty of assessing quality — a form of saleableness — as the rationale for a common medium of exchange.

“Saleableness” is liquidity. Though Menger notes many dimensions to liquidity (delay, uncertainty, search,...), a simple characterization is the difference between the bid (wholesale) price and the ask (retail) price. A commodity that acts as a medium of exchange is necessarily repeatedly bought (accepted in trade) and sold (delivered in trade). Therefore a good with a narrow spread between bid and ask price is priced to encourage households to use it as a carrier of value between trades, as a medium of exchange with relatively low cost. This paper formalizes Menger’s remark in a simple example, demonstrating that a commodity money equilibrium is sustained by competitive equilibrium bid and ask prices and transaction costs in a trading post model.

Trade takes place at commodity-pairwise trading posts — with a budget constraint enforced at each transaction, so that there is a role for a carrier of value between trading posts. Walras (1874) forms the picture this way (assuming m distinct commodities): “we shall imagine that the place which serves as a market for the exchange of all the commodities (A), (B), (C), (D)...for one another is divided into as many sectors as there are pairs of commodities exchanged. We should then have $\frac{m(m-1)}{2}$ special markets each identified by a signboard indicating the names of the two commodities exchanged there as well as their...rates of exchange...”²

The pattern of trade across trading posts is determined endogenously. A barter equilibrium occurs when most trading posts are active in equilibrium, one for each pair of distinct goods, trading for one another. Conversely, if most trading posts are inactive in equilibrium, active trade concentrating on the small number of posts trading a single good pairwise against all others, then the equilibrium will be described as *monetary*, with the single commonly traded good as *commodity money*.

2. Households

Consider a pure exchange trading post economy with N commodities, $N \geq 3$. Γ denotes the greatest integer $\leq (N-1)/2$.

Let $[i, j]$ denote a household endowed with good i who prefers good j ; $i \neq j$, $i, j = 1, 2, \dots, N$. Household $[i, j]$ ’s endowment is 1 unit of commodity i . Denote the endowment of $[i, j]$ as $r_i^{[i, j]} = 1$. $[i, j]$ ’s utility function is $u^{[i, j]}(x_1, x_2, x_3, \dots, x_N) = \sum_{k \neq j} x_k + Ax_j$, $A \gg 1$. That is, household $[i, j]$ values goods 1, 2, 3, ..., N as linear substitutes, with good j being many times more desirable than any other.

Consider a population denoted \mathcal{A} of households including Γ households endowed with each good and each household desiring a good different from its endowment. There are Γ households endowed with good 1, preferring respectively, goods 2, 3, 4, ..., $\Gamma+1$: $[1, 2], [1, 3], [1, 4], \dots, [1, \Gamma+1]$. There are Γ households endowed with good 2, preferring respectively goods 3, 4, 5, ..., $\Gamma+2$: $[2, 3], [2, 4], [2, 5], \dots, [2, \Gamma+2]$. The roll call of households proceeds so forth, through $[N, 1], [N, 2], [N, 3], \dots, [N, \Gamma]$.

Population \mathcal{A} displays absence of double coincidence of wants.³ For each household endowed with good i and desiring good j , $[i, j]$, there is no precise mirror image, $[j, i]$. Nevertheless, there are Γ households endowed with one unit of commodity 1, and Γ households strongly preferring commodity 1 to all others. That is true for each good. Thus gross supplies equal gross demands, though there is no immediate opportunity for any two households to make a mutually advantageous trade. Jevons (1875) tells us that this is precisely the setting where money is suitable to facilitate trade.

3. Trading posts

For each pair of distinct commodities, there is a trading post where those two may be traded for one another. The notation $\{i, j\}$ represents the trading post where good i is traded for good j and (vice versa) good j is traded for i . Operating the trading post is a resource-using activity. For ease of notation, costs will be compensated unit-for-unit by goods traded at the post. Denote this cost of operating trading post $\{i, j\}$ as $C^{\{i, j\}}$.

With N commodities, there are $N(N-1)/2$ trading posts. Most will be inactive (but priced) in a monetary equilibrium. Using this model as a basis for deriving the use of a common medium of exchange represents: (i) that a meaningful discussion of means of payment depends on the notion that goods do not trade for all other goods in a single transaction; segmentation of the market is part of monetization, Alchian (1977); (ii) monetary trade is an equilibrium outcome based on individual optimization and market clearing, where barter could be chosen as an alternative.

Budgets must balance at each trading post — that is, you pay for what you get not only over the course of all trade (as in the Arrow–Debreu model) but at each trading post separately. A household delivers good i to trading post $\{i, j\}$ and the delivery is evaluated at the post’s bid price determining how much good j the household receives. Budget balance requires that the values be equal.

4. Transaction costs and prices

Consider trading posts with a linear transaction cost structure. The trading post buys goods from households and resells them or retains them to cover transaction costs. Let the cost structure of trading post $\{i, j\}$, $i, j = 1, 2, \dots, N-1$, $i \neq j$, be:

$$C^{\{i, j\}} = .1 \times (\text{volume of goods } i \text{ and } j \text{ purchased by the post}).$$

Marginal cost of trading i for j is 0.1 times the gross quantity traded. The trading post expects to cover its transaction costs through the bid/ask spread.

³ “Double coincidence of wants”, Jevons (1875) posits that barter is the outcome in the rare event where traders can directly, without an intermediary good, arrange pairwise mutually improving trades. An exchange of good i for good j then includes one trader with an excess supply of i and an excess demand for j , and a second trader with the opposite unsatisfied supply and demand.

² Shapley and Shubik (1977) and Starr (2003) also treat the trading post model. See also Banerjee and Maskin (1996) and Howitt (2005).

Trading good N is assumed to be costless. Thus,

$$C^{(N,j)} = .1 \times (\text{volume of good } j \text{ purchased by the post}), \text{ for } j = 1, 2, \dots, N - 1.$$

Trading post $\{1,2\}$ accepts good 1 in exchange for good 2 and accepts good 2 in exchange for good 1. Prices are expressed as a rate of exchange between goods 1 and 2. That is, good 1 is priced in units of good 2 and good 2 is priced in units of good 1. In order to cover the post’s operating costs, the prices at which the public buys (ask or retail prices) are higher than those at which the public sells (bid or wholesale prices). The difference between buying and selling prices covers operating costs.

At trading post $\{i,j\}$, the ask price of j (denominated in i per unit j) is the inverse of the bid price of i (denominated in j per unit i). Denote the bid price of good i at $\{i,j\}$ as $q_i^{\{i,j\}}$. Then the ask price of j is $[q_i^{\{i,j\}}]^{-1}$. Denote the purchase of i by a typical household h at $\{i,j\}$ as $b_i^h\{i,j\}$, sale of j as $s_j^h\{i,j\}$. Then the budget constraint facing household h at $\{i,j\}$ is $b_i^h\{i,j\} = s_j^h\{i,j\} q_j^{\{i,j\}}$. Household h ’s consumption of good i then is $x_i^h \equiv r_i^h + \sum_{j=1}^N [b_i^h\{i,j\} - s_i^h\{i,j\}]$.

In an economy of N commodities there are $N(N-1)/2$ trading posts each with two posted prices (bid for one good in terms of a second, and bid price of the second in units of the first) totaling $N(N-1)$ pairwise price ratios. Prices are posted at all trading posts — including those without active trade.

The price system here must answer the question: which trading posts operate at positive trading volume? In actual economies, most conceivable pairwise commodity trades do not occur. A trading post becomes unattractive in equilibrium, and will have zero trading volume (a corner solution), when its bid/ask spread is wide enough to discourage trade.

5. Marginal cost pricing equilibrium

An array of prices $q_i^{\{i,j\}}$ and trades $b_i^h\{i,j\}$, $s_j^h\{i,j\}$ for $h \in A$ is said to be a *marginal cost pricing equilibrium* if each household $h \in A$ optimizes utility subject to budget at prevailing prices, each trading post clears, and trading posts cover marginal costs through bid/ask spreads.

More formally, a marginal cost pricing equilibrium under the transaction cost function above consists of $q_i^{\{i,j\}}$, $q_j^{\{i,j\}}$, $1 \leq i, j \leq N$, $i \neq j$, so that:

For each household $h \in A$, there is a utility optimizing plan $b_n^{oh\{i,j\}}$, $s_n^{oh\{i,j\}}$ so that

$$b_i^{oh\{i,j\}} = s_j^{oh\{i,j\}} q_j^{\{i,j\}} \text{ (budget balance),}$$

For each $i, j, i \neq j$,

$$\sum_h b_n^{oh\{i,j\}} \leq \sum_h s_n^{oh\{i,j\}}, \quad n = i, j \text{ (market clearing),}$$

For $i=1, \dots, N-1; j=1, 2, \dots, N-1; i \neq j$,

$$0.1 \times \sum_{h \in A} \left[s_i^{oh\{i,j\}} + s_j^{oh\{i,j\}} \right] = \sum_{h \in A} \left(\left[s_i^{oh\{i,j\}} - b_i^{oh\{i,j\}} \right] + \left[s_j^{oh\{i,j\}} - b_j^{oh\{i,j\}} \right] \right)$$

For $i=1, \dots, N-1; j=N$,

$$0.1 \times \sum_{h \in A} \left[s_i^{oh\{i,N\}} \right] = \sum_{h \in A} \left(\left[s_i^{oh\{i,N\}} - b_i^{oh\{i,N\}} \right] + \left[s_N^{oh\{i,N\}} - b_N^{oh\{i,N\}} \right] \right) \text{ (transaction cost coverage).}$$

The concluding expressions are (linear) marginal cost pricing conditions; each trading post should cover its costs through the difference in goods bought (at bid price) and sold (at ask price).

The budget balance requirement applies at each transaction at each trading post. Thus, a household acquiring good j for i at $\{i,j\}$ and retrading j at $\{j,k\}$ is acquiring j at its ask price (in terms of i) at $\{i,j\}$ and delivering j at its bid price (in terms of k) at $\{j,k\}$. In that sequence of trades, the trader experiences — and pays — j ’s bid/ask spread.

6. Monetary equilibrium

Market clearing bid prices appear in Table 1. Each entry represents the bid price of the column good in units of the row good. In this array, good N — with the narrowest prevailing bid/ask spread — is the most liquid (saleable) good, Menger’s candidate for commodity money.

The array of equilibrium trades follows:

For $i=1, 2, 3, 4, \dots, N-1; j \neq N$,

$$s_i^{o\{i,j\}\{i,N\}} = 1, b_N^{o\{i,j\}\{i,N\}} = 1, s_N^{o\{i,j\}\{j,N\}} = 1, b_j^{o\{i,j\}\{j,N\}} = .9.$$

For $i=1, \dots, N-1; j=N$,

$$s_i^{o\{i,j\}\{i,N\}} = 1, b_N^{o\{i,j\}\{i,N\}} = 1.$$

For $i=N, j=1, 2, \dots, 1$,

$$s_N^{o\{N,j\}\{N,j\}} = 1, b_j^{o\{N,j\}\{N,j\}} = .9.$$

The arrangement is a market clearing equilibrium with all trade going through good N . Good N acts as medium of exchange,

Table 1
Monetary equilibrium marginal cost pricing-market clearing bid prices at trading posts

| Selling: | 1 | 2 | 3 | ... | ... | ... | N-1 | N |
|----------|-----|----|----|-----|-----|-----|-----|----|
| Buying: | 1 | X | .8 | .8 | .8 | .8 | .8 | .9 |
| | 2 | .8 | X | .8 | .8 | .8 | .8 | .9 |
| | 3 | .8 | .8 | X | .8 | .8 | .8 | .9 |
| | ... | .8 | .8 | .8 | X | .8 | .8 | .9 |
| | ... | .8 | .8 | .8 | .8 | X | .8 | .9 |
| | ... | .8 | .8 | .8 | .8 | .8 | X | .9 |
| | N-1 | .8 | .8 | .8 | .8 | .8 | .8 | X |
| | N | 1 | 1 | 1 | 1 | 1 | 1 | X |

commodity money. The trading posts dealing in good N , $\{N,1\}$, $\{N,2\}$, $\{N,3\}$, ..., $\{N,N-1\}$, cover their operating costs. For each good $n=1,2,3,\dots,N-1$, they find Γ sellers coming to the post delivering one unit of n in exchange for N , and Γ buyers coming to the post, exchanging good N for good n . The trading post clears.

Household $\{3,4\}$ for example, wants to trade good 3 for good 4. He considers trading the goods directly at $\{3,4\}$. Pricing at $\{3,4\}$ means that household $\{3,4\}$ could deliver good 3 to $\{3,4\}$ and receive good 4 after incurring a 20% discount covering the bid/ask spread, using direct trade. Alternatively, $\{3,4\}$ can trade at $\{3,N\}$ and at $\{4,N\}$. He sells 3 at $\{3,N\}$ in exchange for N and sells N at $\{4,N\}$ in exchange for the 4 he really wants. In this indirect trade, he incurs a 10% discount, saving 10% compared to direct trade, by using monetary trade with good N as ‘money’. Indirect monetary trade is more attractive because it is less expensive. The lower expense reflects lower resource costs due to the low-transaction cost of good N and the matching of suppliers and demanders of each good $n=1, 2,\dots,N-1$, at the trading posts $\{N,n\}$ where good N is traded. As *Jevons (1875)* reminds us, the common medium of exchange overcomes the absence of a double coincidence of wants. Thus each household needs to incur the transaction cost on only one side of the monetary trade he enters.

In equilibrium, all trading posts $\{i,j\}$, $i, j \neq N$, except those dealing in good N become inactive. All trading posts are priced, but trade is transacted only at the $N-1$ posts dealing in N . The trading posts clear. Good N has become the common medium of exchange, commodity money.

6.1. Demand for money

This is a single period flow equilibrium model. Hence this model of commodity money does not imply a demand for a stock of money — or for an inventory of any good. There are no stocks to account for. Flows clear the markets.

There is no time structure and no cash-in-advance (or inventory-in-advance) restriction on trade. Those restrictions would create a demand for inventories — including money stock — but require a time structure and an equilibrium notion that includes both stocks and flows (e.g. *Kurz, 1974; Heller and Starr, 1976*).

6.2. Fiat money

The obvious interpretation of good N in the treatment here is that it is commodity money. To interpret good N as fiat money, the model would need to provide an explanation for why households $[\Gamma+1,N]$, $[\dots,N]$, $[N-1,N]$, desire acquisition of an unbacked currency. It would be sufficient that they plan to retrade fiat money for some other desired good in a succeeding period and that they expect it to be valuable then, *Grandmont (1977), Wallace (1980), Kiyotaki and Wright (1989)*.

7. Conclusion

There is a surprise here. *Tobin (1961, 1980)* and *Hahn (1982)* despaired of achieving a general equilibrium model based on elementary price theory resulting in a common medium of exchange. But the price array in *Table 1* leads directly to a monetary equilibrium. Monetary trade is the result of decentralized optimizing decisions of households guided by prices without government, central direction, or fiat. The price system provides all the coordination required to maintain a common medium of exchange. Of course, we expect successful decentralized coordination in an Arrow–Debreu–Walras general equilibrium model, *Debreu (1959)*. But the Arrow–Debreu model is framed for a non-monetary economy. The example here demonstrates — as *Menger (1892)* argued — that the price system can generate a monetary equilibrium with a single common medium of exchange.

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