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Transactions Costs, Technological Choice, and Endogenous Growth*

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Hicks ("A Theory of Economic History," Clarendon Press, Oxford, 1969) argues that an important aspect of industrial development is the adoption of technologies requiring highly illiquid capital investments. The adoption of such technologies becomes economically viable in the presence of low-cost financial markets that provide liquidity to investors. This observation provides a mechanism by which the costs of transacting in financial markets affect the equilibrium choice of technology, productive efficiency, and, by implication, growth. We analyze how the costs of financial market transactions affect the set of technologies in use and the equilibrium growth rate. Transactions cost reductions may, depending on the capital structure, enhance or reduce growth. *Journal of Economic Literature* Classification Numbers: D90, E13, G14.

1. INTRODUCTION

Different economies have financial systems that function with widely varying degrees of technical efficiency. Participants in the financial markets of various economies, or even the same economy at different points in time, face substantially different costs of undertaking financial transactions. But

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what implications does the technical efficiency of an economy's financial system have for its efficiency in physical production and, in particular, for its level of real activity or its rate of growth? A number of heuristic arguments [5, 10, 12, 17, 18, 22] exist suggesting that there are important connections between an economy's financial system and its level (or rate of growth of) real development. This essay seeks to investigate the nature of these connections.

Hicks [15] emphasized the importance of the financial sector while discussing the historical question: what made the industrial revolution revolutionary? He argued that the industrial revolution was not associated with the implementation of any important newly discovered technologies. Indeed, Hicks suggested that most of the technical innovations associated with the industrial revolution had been made some time before its onset, but without being fully implemented. Hicks proposed that this was because their adoption on an economical scale required that large-scale investments be committed for long periods in relatively illiquid capital. Such investments would not have appeared attractive in the absence of financial arrangements that could provide investors in these technologies with liquidity.

> What happened in the Industrial Revolution ... is that the range of fixed capital goods that were used in production ... began noticeably to increase ... But fixed capital is sunk; it is embodied in a particular form, from which it can only gradually ... be released. In order that people should be willing ... to sink large amounts of capital, ... it is the availability of liquid funds which is crucial. This condition was satisfied in England ... by the first half of the eighteenth century ... The liquid asset was there, as it would not have been even a few years earlier. [Hicks [15], pp. 143–145]

Thus, according to Hicks, the choice of production technologies is intimately linked to the costs of trading in (and access to) liquid financial markets. It was therefore necessary that the financial revolution[1] occur before the industrial revolution in order to provide the liquid capital markets necessary for the adoption of technologies requiring inherently illiquid capital investments.

Hicks' argument provides a specific mechanism by which the liquidity of financial markets (which we take to be inversely related to the costs of transacting in them), the choice of technology (or technologies) in use, and real activity are intimately related. In addition, Hicks' observations have the following important implication: increases in the liquidity of financial

[1] The financial revolution is a term applied in [8] to the rapid development of British financial markets in the first half of the 18th century.

markets favor the adoption of technologies which rely relatively heavily on supporting financial market transactions. We seek to explore the theoretical implications of these observations.

We consider an overlapping generations model with production, like that of Diamond [6], but modified to allow for endogenous growth. In order to capture Hicks' notion that the use of technologies requiring illiquid capital investments is important, we modify Diamond's model by allowing for the existence of several different technologies for converting current output into future capital. These technologies vary both by their productivity, and by their gestation period (or time to maturity). In particular, we can imagine that the most productive capital investment technologies are also those with relatively long gestation periods. Thus, their use requires that capital be committed for long periods, and hence the use of these technologies requires that inherently illiquid investments be made.[2]

Given the two-period overlapping generations structure of our model, the use of long-gestation capital production technologies requires that ownership of capital-in-process (CIP) be transferred through a sequence of owners in secondary capital markets. Thus, the use of these technologies requires the support of financial market trading structures. We measure the liquidity of these markets by their transactions costs. Liquid markets are low transactions cost markets; illiquid markets have high transactions costs. For simplicity, we assume a proportional transactions cost structure.[3] Then, following Hicks, our interest is in how the costs of transacting in these markets affect the equilibrium choice of capital production technology, and the equilibrium rate of growth of the economy. In considering these issues we will also examine how the liquidity of capital resale markets affects the equilibrium rate of return on savings and the volume of activity in secondary capital markets.

The results we obtain are as follows. We begin by stating conditions under which the economy we describe will have a unique, non-trivial constant growth rate equilibrium. Conditions under which the equilibrium interest rate will exceed the (endogenous) rate of growth are also derived. We then consider the comparative dynamic consequences of a reduction in the costs of transacting in secondary capital markets. It is possible to show that reductions in transactions costs always favor reliance on longer gestation—and consequently more transactions intensive—capital production

[2] The importance of lengthy gestation periods for capital investment has a long tradition in both capital theory [3] and the literature on economic development [4, 26, 27].

[3] Transactions costs are intended to represent any wedge between buying and selling prices of assets. This includes comissions, fees, taxes, and the bid/ask spread between buying and selling prices. We could also include the time required to arrange a sale or purchase as a component of transactions costs.

technologies. In addition, lower transactions costs necessarily raise the equilibrium rate of return on savings. However, an increase in the technical efficiency of capital resale markets can lead to either an increase or a reduction in the economy's long-run equilibrium rate of growth.[4] Moreover, as transactions costs in these markets are reduced, the effect on the equilibrium rate of growth can easily be non-monotonic. Thus, the relationship between financial market efficiency and growth is an inherently complex one.

Why is this the case? In the economy we consider the equilibrium rate of growth depends on five factors: (a) the capital production technology in use, (b) its productivity, net of transactions costs, (c) the savings rate, (d) the composition of savings, and (e) the ratio of labor's share to capital's share in output. Financial market conditions influence the first four of these aspects of the economy. First, the capital production technology in use will simply be the one that maximizes the internal rate of return on investments, net of transactions costs. Under a mild (and, we argue, empirically plausible) technical condition, reductions in transactions costs have a greater effect on the internal rate of return for longer than for shorter maturity investments, as the former are more transactions intensive. Thus, reductions in transactions costs favor the use of longer gestation capital production technologies. If these are also the more efficient capital production technologies, this effect is conducive to higher rates of growth. In addition, lower transactions costs raise the net of transactions cost productivity of all investment technologies, which also tends to raise the growth rate. Moreover, this same observation implies that as transactions costs fall, the internal rate of return on all investments—and hence the equilibrium rate of return on savings—rises. Under our assumptions this acts to increase the savings rate, which again tends to enhance growth.

The subtlety associated with the increasing technical efficiency of financial markets is that the increased liquidity of these markets has general equilibrium consequences for the composition of savings. As lower transactions costs lead to the employment of longer maturity capital investments there will be an increased reliance on transactions in capital resale markets. This can cause a shift in the composition of savings away from the initiation of new capital investment and toward the purchase of existing CIP in secondary capital markets. Since CIP is purchased from old agents, who consume the proceeds of the sales, this change in the composition of savings tends, ceteris paribus, to reduce the equilibrium rate of growth.[5]

[4] Related results are obtained for economies with steady-state equilibria in [1, 2].

[5] This effect is exactly analogous to the consequences of introducing money into growth models: the introduction of a financial asset diverts savings away from new capital investment. See for instance [19, 24, 25]. Note that this effect can occur only when changes in the structure of transactions costs cause a change in the equilibrium choice of capital production technology. This point is discussed in more detail in Section 5.

When this effect is large enough, improvements in financial market efficiency are growth reducing. Transactions cost reductions will be conducive to growth, on the other hand, when they imply relatively unimportant changes in the composition of savings. Such compositional effects will be of most limited significance when there are relatively large intrinsic differences in the productivities of different capital production technologies, and when there are fairly large reductions in transactions costs.

The possibility that reductions in transactions costs can lead to reductions in the equilibrium rate of growth is more than simply a theoretical curiosity. In practice, there have been many attempts to stimulate financial market development in developing countries by reducing the perceived costs of transacting in these markets. Often these attempts have been counterproductive.[6] Our analysis suggests how and when this phenomenon will occur.

The remainder of the paper proceeds as follows. Section 2 describes the underlying environment and the nature of trade in our model. Section 3 states the conditions that must be satisfied by a competitive equilibrium. Section 4 establishes the existence of a unique, non-trivial constant growth rate equilibrium and states conditions under which the equilibrium growth rate will not exceed the equilibrium rate of interest. Section 5 examines the consequences of improvements in financial market efficiency, while Section 6 provides a completely worked out example. Section 7 is a conclusion.

2. The Model

2.1. *The Environment*

We consider a two-period lived, overlapping generations model with production. We let $t = 0, 1, \ldots$ index time, and at each date t a new young generation appears with $N > 1$ members. All young agents are identical and are endowed with one unit of labor when young, which is supplied inelastically. All agents are retired when old, and no agents other than the initial old have any endowment of capital or consumption goods at any date.

There is a single final good in each period, which is produced using N intermediate goods (one per young agent) as inputs. Let $i = 1, \ldots, N$ index young agents. Then young agent i at t produces a quantity $x_t(i)$ of the type i intermediate good. Intermediate goods themselves are produced using capital and labor as inputs. Let $k_t(i)$ and $L_t(i)$ denote the quantity of capital and labor employed by young agent i at t. Then

$$x_t(i) = Ak_t(i)\, L_t(i)^{1-\delta}; \qquad \delta \in (0, 1). \tag{1}$$

[6] See, for example, [4, 7, 11, 16, 26, 28, 29].

The linearity of the intermediate goods production technology in $k_t(i)$ allows for the existence of an equilibrium with a constant rate of growth of real output. In order to prevent difficulties associated with the allocation of profits accruing from the technology in (1), we assume that labor is a non-traded factor of production. Thus, each young agent uses only his own labor to produce intermediate goods, and $L_t(i) = 1$.[7]

We let Y_t denote aggregate production of the single final good at t, and use $y_t \equiv Y_t/N$ to denote per capita output. We assume that this good is produced according to a standard constant returns to scale production function using the N intermediate goods as inputs, and in particular that

$$Y_t = \left[N^{(\theta-1)} \sum_{i=1}^{N} x_t(i)^{\theta} \right]^{1/\theta}; \qquad \theta < 1. \tag{2}$$

The final good either can be consumed or can be converted into future capital.

We assume that there are $J > 1$ technologies for converting final goods into capital; these technologies are indexed by $j = 1, ..., J$. One unit of the final good invested in technology j at t returns $R_j > 0$ units of capital (gross of transactions costs) at $t + j$. Thus, capital production technologies vary by gestation period (j) and productivity (R_j). Capital, once produced, is used in the production process and then depreciates completely. All types of capital, however produced, are assumed to be perfectly substitutable in the intermediate goods production process.[8]

For technology j then, j describes the maturity of the particular capital investment. Our capital production technology has an "Austrian" flavor, in that capital production technologies vary purely by productivity and time to maturity, with capital investments being unproductive until they mature. At some cost in added complexity it would be possible to allow all capital to mature after one period, but to have capital produced via different technologies have different productive lifetimes. Here we retain the simpler Austrian specification.

[7] This particular specification of the production structure is drawn from [13, 14]. It is designed to allow for the existence of a "steady-state equilibrium" displaying a constant (endogenous) rate of growth. It would be straightforward to use other standard devices for accomplishing this such as the "externalities" model of [21, 23], or the two-sector model of [20]. (See [9] for an application of the latter model in an overlapping generations context.)

[8] The possibility that capital produced by different technologies is not perfectly substitutable as an input in production is considered in [2]. When these different types of capital are imperfectly substitutable, all capital production technologies can be in use simultaneously. A comparison of the results here with those in [2] will indicate that our conclusions do not depend on changes in transactions costs causing potentially discrete changes in the set of technologies in use.

Since agents are two-period lived, a young agent who initiates a capital investment in any technology $j > 1$ will seek to sell his "immature" capital in a secondary capital market. In order to capture the notion of the "liquidity" of these markets, we assume that there are costs associated with transacting in them. For simplicity we assume a proportional transactions cost structure. Then if ownership of one unit of CIP in technology j that is h periods old ($j-h$ periods from maturity) is transferred, a fraction $\alpha^{j,h} \in [0, 1)$ of the CIP is consumed in the transactions process. Our interest is in how the transactions cost structure affects the choice of capital production technologies in use. Finally, we note that CIP in technology j becomes usable capital only after it has been in place for j periods.

Let C_{1t} (C_{2t}) denote age 1 (2) consumption of the final good by a representative member of generation t. Members of this generation are assumed to have the homothetic utility function

$$u(C_{1t}, C_{2t}) = (C_{1t}^{1-\eta} + \beta C_{2t}^{1-\eta})/(1-\eta); \qquad \eta > 0. \tag{3}$$

Finally, there is a set of initial old agents at $t=0$. These agents are endowed with the initial per capita capital stock $k_0 > 0$.

2.2. *Trade in Goods and Factors*

We assume that trade in goods markets and rental markets in capital is frictionless; that is, transactions in them are costless. This assumption makes our economy as close to an endogenous growth version of the Diamond [6] model as possible.

Only young agent i can produce intermediate goods of type i. Let $p_t(i)$ denote the price of these goods at t (in units of time t consumption). It is natural to model each intermediate goods producer as being imperfectly competitive, so agent i does *not* take $p_t(i)$ as given.

It is therefore necessary to derive a demand curve for intermediate goods of type i. We assume that final goods producers do take $p_t(i)$ as given; they choose a vector of intermediate inputs $[x_t(1), ..., x_t(N)]$ to maximize

$$Y_t - \sum_{i=1}^{N} p_t(i)\, x_t(i)$$

subject to (2). The first-order condition for this problem has the form

$$p_t(i) = y_t^{1-\theta} x_t(i)^{\theta-1}, \tag{4}$$

which is an inverse demand function for intermediate goods of type i.

Young producers of intermediate goods use their own labor and must rent capital from old agents in competitive rental markets. Let r_t denote the time t rental rate. Then young agent i chooses a value $x_t(i)$ and a value $k_t(i)$ to maximize $p_t(i)\,x_t(i) - r_t k_t(i)$, subject to (1), (4), and $L_t(i) = 1$. This problem can be reduced to

$$\max_{k_t(i)} \{ y_t^{1-\theta}[Ak_t(i)]^{\theta} - r_t k_t(i) \}.$$

The first-order condition for this problem sets

$$\theta y_t^{1-\theta} A^{\theta} k_t(i)^{\theta-1} = r_t. \tag{5}$$

Equilibrium Factor Prices

Given the symmetric nature of the model, we seek an equilibrium where $x_t(i)$ and $k_t(i)$ are independent of i. Moreover, if k_t is the per person capital stock at t, rental market clearing then requires that $k_t(i) = k_t$, $\forall i$. Therefore, from (1), $x_t(i) = x_t = Ak_t$, $\forall i$. Equation (5) then reduces to

$$r_t = \theta A \equiv \rho; \qquad \forall t. \tag{5'}$$

Thus ρ denotes the (time invariant) marginal product of capital, which here is simply a parameter. Finally, define $w_t(i)$ to be the real income of young agent i at t. Using (5) and $k_t(i) = k_t$, we have that

$$w_t(i) = w_t = y_t^{1-\theta}(Ak_t)^{\theta}(1-\theta) = (1-\theta)\,Ak_t; \qquad \forall t. \tag{6}$$

From (6) it follows that the income of young agents grows at the same rate as the per capita capital stock.

2.3. *Savings/Portfolio Decisions*

After earning the income w_t at t, young agents must make a savings/portfolio decision. We represent the choices available to these agents as follows. Let $S_t^{j,h}$ denote the amount of type j capital that is h periods old acquired by a young agent at t, where $S_t^{j,h}$ is measured in units of CIP. Thus, for example, $S_t^{j,0}$ denotes the amount of newly initiated investment in technology j, while $S_t^{j,j-1}$ is the investment in type j capital that will mature in one period. Let $P_t^{j,h}$ denote the price, in units of current consumption, of one unit of technology j CIP that has been in place h periods. We assume, without loss of generality, that transactions costs are born by sellers of CIP. Then a young agent at t faces the budget constraints

$$C_{1t} + \sum_{j=1}^{J} \sum_{h=0}^{j-1} P_t^{j,h} S_t^{j,h} \leqslant w_t \tag{7}$$

$$C_{2t} \leqslant \sum_{j=1}^{J} \sum_{h=0}^{j-1} P_{t+1}^{j,h+1} S_t^{j,h}(1-\alpha^{j,h+1}). \tag{8}$$

This agent must choose a vector of consumption levels (C_{1t}, C_{2t}), and a matrix of capital investments $(S_t^{j,h})$, to maximize $[C_{1t}^{1-\eta} + \beta C_{2t}^{1-\eta}]/(1-\eta)$, subject to (7), (8), and non-negativity.

The solution to the problem of a young agent is straightforward. Since any (non-negative) investment choices are possible, the agent will only make investments earning the highest rate of return, net of transactions costs. Thus if $S_t^{j,h} > 0$ and $S_t^{l,m} > 0$ for some (j, h) and (l, m), it follows that

$$(1-\alpha^{j,h+1})\, P_{t+1}^{j,h+1}/P_t^{j,h} = (1-\alpha^{l,m+1})\, P_{t+1}^{l,m+1}/P_t^{l,m} \geqslant (1-\alpha^{q,p+1})\, P_{t+1}^{q,p+1}/P_t^{q,p} \tag{9}$$

holds for all (q, p). Any investments that are actually made must have equal returns, net of transactions costs, and these returns must exceed those available on any investments that are not made.

Let γ_t denote the common (gross) rate of return on assets held in positive amounts between t and $t+1$. Then, if $S_t^{j,h} > 0$ for some young agent at t,

$$\gamma_t = (1-\alpha^{j,h+1})\, P_{t+1}^{j,h+1}/P_t^{j,h}. \tag{10}$$

Since rates of return on all capital investments in use are equated, obviously each young agent at t is individually indifferent regarding the composition of his portfolio. Thus, for each individual, only the real value of savings $\tilde{S}_t \equiv \sum_j \sum_h S_t^{j,h}$ is determinate. Clearly then,

$$\begin{aligned} \tilde{S}_t &\equiv \arg\max\{[(w_t - \tilde{S}_t)^{1-\eta} + \beta(\gamma_t \tilde{S}_t)^{1-\eta}]/(1-\eta)\} \\ &\equiv \{\beta^{1/\eta}\gamma_t^{(1-\eta)/\eta}/[1+\beta^{1/\eta}\gamma_t^{(1-\eta)/\eta}]\}\; w_t \\ &\equiv \psi(\gamma_t)\, w_t. \end{aligned} \tag{11}$$

In particular, young agents at t have a savings rate, $\psi(\gamma_t)$, which is a function of the rate of return γ_t alone. We henceforth assume that

$$\psi'(\gamma_t) \geqslant 0 \tag{a.1}$$

holds, so that savings are non-decreasing in rate of return. Of course (a.1) holds iff $\eta \leqslant 1$.

It will be useful for future reference to have a notation for the fraction of real savings at t that is held in type j CIP that is h periods old. This fraction—which in the aggregate is determinate—is denoted by $\theta_t^{j,h}$, and it obviously is given by

$$\theta_t^{j,h} \equiv P_t^{j,h} S_t^{j,h}/\tilde{S}_t.$$

Evidently $\theta_t^{j,h} \geqslant 0$ holds $\forall j, h, t$, as does

$$\sum_{j=1}^{J} \sum_{h=1}^{j-1} \theta_t^{j,h} = 1. \tag{12}$$

3. Constant Growth Equilibria

3.1. *Asset Market Equilibrium*

In this section, we examine "steady-state" (that is, constant growth rate) equilibria in which the same capital production technology (j) is in use permanently. In such equilibria, the equilibrium choice of capital production technology, j, the steady-state equilibrium rate of return γ, and the equilibrium asset prices $P_t^{j,h}$ are determined as follows.

First, since one unit of the final good invested in technology j at t initiates one unit of technology j CIP (by assumption), $P_t^{j,0} \equiv 1$. In addition, one unit of mature technology j CIP yields R_j units of rentable capital, and the capital rental rate is ρ. Hence, $P_t^{j,j} \equiv \rho R_j$. The remainder of the prices $P_t^{j,h}$ are determined below.

Since technology j is in use at all dates, it follows that $S_t^{j,h} > 0$ holds for some agent, for all $h = 0, 1, \ldots, j-1$, for all t. Therefore,

$$\gamma = (1 - \alpha^{j,h+1})\, P_{t+1}^{j,h+1} / P_t^{j,h} \tag{10'}$$

holds for all h and t. Then γ can be determined from the following observation. We have that

$$\rho R_j \equiv P_{t+j}^{j,j} = (P_{t+j}^{j,j} / P_{t+j-1}^{j,j-1})(P_{t+j-1}^{j,j-1} / P_{t+j-2}^{j,j-2}) \cdots (P_{t+1}^{j,1} / P_t^{j,0})\, P_t^{j,0}. \tag{13}$$

Substituting (10′) into (13) yields

$$\rho R_j = \prod_{h=0}^{j-1} \gamma / (1 - \alpha^{j,h+1}). \tag{14}$$

If we define

$$\tilde{R}_j \equiv R_j \prod_{h=0}^{j-1} (1 - \alpha^{j,h+1})$$

to be the net of transactions costs productivity of technology j, we can rewrite (14) as

$$\gamma^j = \prod_{h=0}^{j-1} \gamma = \rho \tilde{R}_j. \tag{15}$$

Thus $\gamma = (\rho \tilde{R}_j)^{1/j}$ holds, so that the steady-state equilibrium rate of return on savings is simply the internal rate of return—net of transactions costs—on any investment technology in use in equilibrium.

From this observation it is easy to determine which capital production technologies will be employed. In particular, for any technology l, it must be the case that

$$\rho R_l \equiv P_{t+l}^{l,l} \equiv (P_{t+l}^{l,l}/P_{t+l-1}^{l,l-1})(P_{t+l-1}^{l,l-1}/P_{t+l-2}^{l,l-2}) \cdots (P_{t+1}^{l,1}/P_t^{l,0})\, P_t^{l,0}$$
$$\leqslant \gamma^l \Big/ \prod_{h=0}^{l-1} (1 - \alpha^{l,h+1}). \tag{16}$$

From (15) and (16) one obtains

$$(\rho \tilde{R}_j)^{1/j} = \gamma \geqslant (\rho \tilde{R}_l)^{1/l}; \qquad \forall l. \tag{17}$$

Thus, if j is an equilibrium choice of capital production technology, technology j generates the highest internal rate of return on investment, net of transactions costs, or, in other words,

$$j = \arg\max_l \,[(\rho \tilde{R}_l)^{1/l}]. \tag{18}$$

We henceforth assume that there is a unique choice of technology that maximizes this internal rate of return.[9] Then, in the steady state, $\theta_t^{l,h} = 0\ \forall l \neq j,\ \forall h = 0, ..., l-1$.

It remains to determine the values $P_t^{j,h}$. For the technology satisfying (18), some new capital investment will be undertaken at all dates, and in addition immature CIP of all vintages will be traded in secondary capital markets. In order for these markets to clear, of course, it is necessary that the time t demand for technology j CIP that is h periods old equals its supply. The demand for type j CIP that is h periods old at t is simply $\theta_t^{j,h}\psi(\gamma)\, w_t/P_t^{j,h}$, measured in units of CIP. In particular, young agents at t save $\psi(\gamma)\, w_t$ in real terms, and allocate a fraction $\theta_t^{j,h}$ of this savings to type (j, h) CIP. Dividing by $P_t^{j,h}$ then converts this quantity into units of CIP. Similarly, the supply of type j CIP of vintage h at t is $[\theta_{t-h}^{j,0}\psi(\gamma)\, w_{t-h}/P_{t-h}^{j,0}] \prod_{l=0}^{h-1} (1 - \alpha^{j,l+1})$. This is the case since $\theta_{t-h}^{j,0}\psi(\gamma)\, w_{t-h}/P_{t-h}^{j,0}$ units of new investment in technology j were initiated h periods ago, and $1 - \prod_{l=0}^{h-1} (1 - \alpha^{j,l+1})$ of this CIP has been consumed in the transactions

[9] This will obviously be the case generically. If the marginal product of capital were not independent of the capital–labor ratio, however, then the capital investment technology that maximized the internal rate of return on investment would depend on the per capita capital stock. In this situation, there would be some values of the capital–labor ratio where more than one capital production technology maximizes the internal rate of return. This possibility, which introduces some additional complications, is considered in [1].

process. Therefore, since $P^{j,0}_{t-h}=1$, market clearing in type (j,h) CIP requires that

$$\theta^{j,h}_t \psi(\gamma)\, A(1-\theta)\, k_t = P^{j,h}_t \theta^{j,0}_{t-h} \psi(\gamma)\, A(1-\theta)\, k_{t-h} \times \prod_{l=0}^{h-1} (1-\alpha^{j,l+1}) \tag{19}$$

hold for all $h=1, ..., j-1$, for all t.

We now observe that

$$\begin{aligned} P^{j,h}_t &\equiv (P^{j,h}_t/P^{j,h-1}_{t-1})(P^{j,h-1}_{t-1}/P^{j,h-2}_{t-2}) \cdots (P^{j,1}_{t-h+1}/P^{j,0}_{t-h})\, P^{j,0}_{t-h} \\ &= \gamma^h \Big/ \prod_{l=0}^{h-1} (1-\alpha^{j,l+1}) \end{aligned}$$

holds, for all $h=1, ..., j-1$, for all t. If we then define $\sigma_t \equiv k_{t+1}/k_t$ to be the real growth factor between t and $t+1$, and we impose our focus on constant growth equilibria, we can rewrite (19) as

$$\theta^{j,h}_t = \theta^{j,0}_{t-h}(\gamma/\sigma)^h; \qquad h=1, ..., j-1. \tag{20}$$

If in addition we impose constancy of portfolio weights through time,[10] it follows that

$$\theta^{j,h} = \theta^{j,0}(\gamma/\sigma), \tag{20'}$$

for all $h=1, ..., j-1$.

3.2. *Savings Equals Investment*

Since only technology j will be in use in a steady-state equilibrium, the time t per capita capital stock must equal $\tilde{R}_j$ (the net of transactions cost productivity of technology j) times the initiation of new capital investment in that technology j periods earlier. Thus,

$$k_t = \tilde{R}_j \theta^{j,0} \psi(\gamma)\, w_t = \tilde{R}_j \theta^{j,0} \psi(\gamma)(1-\theta)\, A k_{t-j}. \tag{21}$$

Using the assumption that $\sigma_t \equiv k_{t+1}/k_t$ is constant in (21) yields

$$\sigma^j = \tilde{R}_j \psi(\gamma)(1-\theta)\, A\theta^{j,0}. \tag{22}$$

[10] It is easy to show that constancy of σ_t implies that portfolio weights are also independent of t.

Defining

$$\mu_j \equiv \tilde{R}_j \psi(\gamma)(1-\theta) A, \tag{23}$$

(22) can be written more compactly as

$$\sigma^j = \mu_j \theta^{j,0}. \tag{24}$$

Equations (12), (15), (18), (20′), and (24) constitute the steady-state equilibrium conditions for this economy. Equation (15) asserts that the equilibrium rate of return on savings is simply the internal rate of return on any capital production technology in use. Equation (18) asserts that only capital production technologies that maximize the internal rate of return on capital investments are in use in equilibrium. Thus, j and γ are determined purely by technology, as is μ_j. Equations (12), (20′), and (24) then jointly determine equilibrium portfolio weights and the long-run rate of growth for the economy. As is apparent from (24), the portfolio composition of savings is a fundamental determinant of the economy's asymptotic growth rate. We now turn to a characterization of this growth rate in equilibrium.

4. Characterization of Constant Growth Equilibria

Since $\theta_t^{j,h}$ is independent of t in any constant growth rate equilibrium, and since $\theta_t^{l,h} \equiv 0$ for any $l \neq j$, Eqs. (12) and (20′) imply that

$$\theta^{j,0} \sum_{h=0}^{j-1} (\gamma/\sigma)^h = 1. \tag{25}$$

Defining the functions $f_j: \Re_+ \to \Re_+$, $j = 1, \ldots, J$, by

$$f_j(x) \equiv \sum_{h=0}^{j-1} x^h = (1-x^j)/(1-x),$$

we may rewrite Eq. (25) as

$$\theta^{j,0} f_j(\gamma/\sigma) = 1. \tag{26}$$

If we then substitute (24) into (26), we obtain the condition that determines the steady-state equilibrium rate of growth:

$$\sigma^j f_j(\gamma/\sigma) = \mu_j. \tag{27}$$

In order to characterize solutions to (27), and to describe how they vary with changes in transactions costs, it is useful to define a new set of functions, $F_j: \Re_+ \to \Re_+$, $j = 1, ..., J$, by

$$F_j(x) \equiv f_j(x)/x^j.$$

In addition, it is useful to have a compact notation for the internal rate of return, net of transactions costs, on investments in technology j. This rate of return is henceforth denoted by

$$\gamma_j \equiv (\rho \tilde{R}_j)^{1/j}. \tag{28}$$

Then the steady-state equilibrium rate of return on savings is simply

$$\gamma = \max[\gamma_1, \gamma_2, ..., \gamma_J]. \tag{29}$$

Finally, we define the scalars ϕ_j, $j = 1, ..., J$ by

$$\phi_j \equiv \psi(\gamma_j)(1-\theta)/\theta \equiv \mu_j/(\gamma_j)^j. \tag{30}$$

ϕ_j is simply the savings rate at the rate of return γ_j, multiplied by the ratio of labor's share to capital's share in output. Then if we divide both sides of Eq. (27) by $(\gamma_j)^j$, we obtain an alternative representation of that equilibrium condition:

$$F_j(\gamma_j/\sigma) = \phi_j \equiv \mu_j/(\gamma_j)^j. \tag{31}$$

Equation (31) and

$$j = \arg\max_l\, [\gamma_l] \tag{32}$$

constitute the conditions that determine a steady-state equilibrium.

It will now be useful to state some properties of the functions F_j.

LEMMA 1. *For $j = 1, ..., J$, F_j has the following properties*:

(a) $F_j(x) \geqslant 0, \quad \forall x \geqslant 0.$

(b) $\lim_{x \to 0} F_j(x) = \infty.$

(c) $\lim_{x \to \infty} F_j(x) = 0.$

(d) $F_j(1) = j.$

(e) $F_j'(x) < 0, \quad \forall x \geqslant 0.$

The proof of Lemma 1 appears in the Appendix.

Equation (31), Lemma 1, and the assumption that (32) has a unique solution immediately imply the following result.

PROPOSITION 1. *There is a unique steady-state equilibrium growth rate,* σ^*. *It satisfies* $F_j(\gamma_j/\sigma^*) = \phi_j$, *with* j *given by* (32).

A natural question in any overlapping generations economy concerns when the equilibrium rate of interest [γ_j, with j given by (32)] exceeds the equilibrium rate of growth, σ. Conditions under which this occurs are stated in Proposition 2.

PROPOSITION 2. $\gamma_j \geqslant \sigma^*$ *holds iff* $j \geqslant \phi_j$.

Proof. Since $F_j' < 0$, $\gamma_j/\sigma^* \geqslant 1$ holds iff $j = F_j(1) \geqslant \phi_j$. This establishes the proposition. ▌

Since $\phi_j \equiv \psi(\gamma_j)(1-\theta)/\theta$, since a reasonable upper bound for $(1-\theta)/\theta$ empirically is about three, and since ψ is a savings rate, Proposition 2 implies that the equilibrium rate of interest will exceed the equilibrium rate of growth whenever the equilibrium choice of gestation length is sufficiently long.

In addition to characterizing the equilibrium growth rate, we are also interested in describing the volume of financial market activity relative to total wealthholding. The value of secondary capital market transactions at t, measured in units of current consumption, is simply savings by young agents at t, less the value of new investment which they initiate. In per capita terms this quantity is given by $(1-\theta^{j,0})\,\psi(\gamma_j)\,w_t$. If we take the ratio of this value to total savings (wealth), this ratio—which measures the amount of wealth consumed by purchases in secondary capital markets—is given by

$$Q_j(\sigma^*;\mu_j) \equiv 1 - (\sigma^*)^j/\mu_j \equiv 1 - \theta^{j,0}. \tag{33}$$

Thus Q_j gives the (scaled) volume of secondary capital market activity when technology j is in use.

5. THE EQUILIBRIUM CONSEQUENCES OF INCREASED LIQUIDITY

As we have indicated, we are interested in how the liquidity of secondary capital markets (that is, the costs of transacting in these markets, or their technical efficiency) affects the equilibrium choice of technology (productive efficiency) and, through this mechanism, the asymptotic growth rate. In the process we will also describe how changes in market liquidity affect the equilibrium return on savings (γ), and the equilibrium level of financial market activity. As we will show, changes in the efficiency of various technologies and their utilization are only part of the story in analyzing how changes in transactions costs affect growth. It is also necessary to consider how these changes affect the composition of saving.

In order to examine the relationship between market liquidity and various other aspects of equilibrium, it is useful to have a convenient parameterization of transactions costs. A reduction in transactions costs, of course, acts to raise $\tilde{R}_j$, for all $j > 1$. We now assume that there is a single parameter, z, which influences all of the net of transactions costs returns to the various capital investment technologies. In particular, we let the net of transactions costs productivity of technology j be given by

$$\tilde{R}_j = \tilde{R}_j(z); \qquad j = 1, ..., J. \tag{34}$$

To be concrete, let increases in z represent reductions in transactions costs, so that $\tilde{R}_j'(z) \geqslant 0\ \forall j$. In addition, since there are no transactions associated with one-period investments ($\tilde{R}_1 = R_1$), we impose that $\tilde{R}_1'(z) \equiv 0$, and we assume that $\tilde{R}_j'(z) > 0$ holds $\forall j > 1$. Finally, unambiguous results along certain dimensions require that we place some structure on how changes in z affect the net of transactions cost productivities of particular investments. Our specific technical assumption is that

$$\tilde{R}_j'(z)/j\tilde{R}_j(z) \geqslant \tilde{R}_l'(z)/l\tilde{R}_l(z) \tag{a.2}$$

whenever $j > l$. Assumption (a.2) asserts that reductions in transactions costs have a larger proportional effect on the internal rate of return for long-gestation than for short-gestation investments. This seems like a natural assumption, since longer lived investments are more transactions intensive than shorter lived investments.[11]

[11] Assumption (a.2) is satisfied by some obvious transactions cost structures. We have, of course, set $\alpha^{j,j} \equiv 0$, since we have assumed that there are no transactions costs in capital rental markets. In addition, $\alpha^{j,0} \equiv 0$, since there are no transactions associated with initiating a new capital investment. Suppose we then assume that $\alpha^{j,h} \equiv \alpha$, with $\alpha \in [0, 1)$, for all $h \neq 0, j$. It is easy to verify that this constant proportional transactions costs structure satisfies (a.2). Also, under our assumptions, (a.2) is necessarily satisfied if $J = 2$.

We also note that the assumption that transactions costs are most significant for assets of longest maturity is an empirically plausible one. For example, according to the Wall Street Journal of July 23, 1993, the bid/ask spread on a three-month Treasury bill of the previous day was 0.005% of the price. The bid/ask spread on a 30-year Treasury bond was 0.062% of price, and the bid/ask spread on a 30-year Treasury strip (a pure discount instrument, equivalent to a long-term bill) was 0.7% of price. Thus, these bid/ask spreads vary by a factor of 100 with maturity alone. This effect is, of course, compounded as a longer term instrument is rolled-over many more times in its lifetime.

The capital production technology that maximizes the internal rate of return on investment will now depend on the transactions cost parameter z. To denote this dependence let

$$j(z) \equiv \arg\max_{l} \left[\gamma_1(z), \gamma_2(z), ..., \gamma_J(z)\right], \tag{35}$$

where

$$\gamma_l(z) \equiv [\rho \tilde{R}_l(z)]^{1/l}$$

is the internal rate of return on technology l (net of transactions costs), as before. Our first result is that, under (a.2), a reduction in transactions costs cannot reduce the equilibrium maturity of the capital production technology.

PROPOSITION 3. *Suppose that* (a.2) *holds. Then* $z_1 > z_2$ *implies that* $j(z_1) \geq j(z_2)$.

Proposition 3 is proved in the Appendix. It asserts that as the liquidity of secondary capital markets increases, this is conducive to the use of longer gestation (more transactions intensive) investments. We now explore the implications of this observation.

As will become apparent, the consequences of changes in transactions costs for both the equilibrium growth rate and the level of financial market activity depend very heavily on whether they change the equilibrium choice of capital production technology. We now consider each possibility.

5.1. *Unchanged Capital Production Technology*

In this section, we imagine that $z_1 > z_2$, and that $j(z_1) = j(z_2)$. We are thus considering changes in transactions costs that do not affect the choice of the investment technology in use.

Obviously if $j(z) = 1$, a change in z of this type has no effect on R_1, and hence no effect on γ, σ^*, or the volume of financial market transactions (which is zero). Thus, for the remainder of this section, we assume that $j(z) > 1$. Also, since in this section we hold $j(z)$ fixed, we will (temporarily) suppress the notational dependence of j on z.

When $j = j(z)$ is unchanged by a change in z, the equilibrium rate of growth σ^* is given by Eq. (31); that is,

$$F_j\{[\rho \tilde{R}_j(z)]^{1/j}/\sigma^*\} = \psi\{[\rho \tilde{R}_j(z)]^{1/j}\}(1-\theta)/\theta, \tag{31'}$$

where we have used the definitions of $\gamma_j(z)$ and ϕ_j to obtain (31′). Moreover, the affect of changes in z on σ^* can be obtained simply by differentiating (31′). Doing so yields

$$(z/\sigma^*)\, d\sigma^*/dz = [z\tilde{R}_j'(z)/\, j\tilde{R}_j(z)][F_j'(-) - \sigma\psi'(-)(1-\theta)/\theta]/F_j'(-). \quad (36)$$

The following proposition follows immediately from (a.1) and Lemma 1.

PROPOSITION 4. *If changes in z do not affect j [that is, if $z \in (z_1, z_2)$ with $j(z_1) = j(z) = j(z_2)$], then $d\sigma^*/dz > 0$.*

Thus, when there is no change in the equilibrium choice of investment technology, reductions in transactions costs necessarily raise the equilibrium growth rate. This occurs for two reasons; the improvement in the transactions technology effectively makes investment more efficient, and at the same time the equilibrium rate of return—$\gamma_j(z)$—necessarily increases. If $\psi' > 0$, the consequence is a higher savings rate, which also acts to increase the rate of growth.

When $\psi'(\gamma) \equiv 0$ (that is, when the savings rate is constant, as would be the case with logarithmic utility), the following proposition is also immediate from (36).

PROPOSITION 5. *Suppose that $\psi'(\gamma) \equiv 0$. Then $(\sigma^*/z)\, d\sigma^*/dz$ is larger the larger is j.*

Proposition 5 follows from (36), $\psi'(\gamma) \equiv 0$, and (a.2). The proposition asserts that the more illiquid is the capital production technology in use (the longer its maturity), the greater is the proportional effect on the growth rate of a given reduction in transactions costs. Thus, transactions costs assume greater importance as increasingly illiquid capital production technologies are used.

It remains to consider the effects of a reduction in transactions costs for the equilibrium level of secondary capital market activity, $Q_j(\sigma^*; \mu_j) \equiv 1 - (\sigma^*)^j/\mu_j$. These effects are described in Proposition 6.

PROPOSITION 6. *If changes in transactions costs do not affect j, then $d[(\sigma^*)^j/\mu_j]/dz \geqslant 0$. In other words, a reduction in transactions costs does not increase the ratio of financial market transactions to total wealth.*

Proposition 6 is proved in the Appendix. It asserts that a reduction in transactions costs [that does not alter $j(z)$] will cause a higher fraction of savings to be invested in new capital initiation. This is an effect that also is conducive to higher rates of capital formation and growth.

When transactions cost reductions do not affect the equilibrium choice of capital investment technology, these reductions have unambiguous consequences for equilibrium growth rates, for rates of return on savings, and for the level of secondary capital market activity. This is not the case when there is an effect on the equilibrium maturity of capital investments. We now consider that situation.

5.2. *Changes in the Equilibrium Choice of Capital Production Technology*

Suppose that $z_1 > z_2$ and that $j(z_1) > j(z_2)$. We now analyze how, in this case, reduced transactions costs impact on steady-state equilibrium rates of return on savings, on rates of growth, and on the level of activity in secondary capital markets.

Our first result is that lower transactions costs necessarily lead to higher steady-state rates of return on savings. This observation follows from the fact that, by definition,

$$\begin{aligned}\gamma_{j(z_1)}(z_1) &\equiv [\rho \tilde{R}_{j(z_1)}(z_1)]^{1/j(z_1)} \\ &> [\rho \tilde{R}_{j(z_2)}(z_1)]^{1/j(z_2)} \\ &> [\rho \tilde{R}_{j(z_2)}(z_2)]^{1/j(z_2)} \equiv \gamma_{j(z_2)}(z_2).\end{aligned}$$

Thus, whether or not there is a change in the choice of capital production technology, a reduction in transactions costs raises the rate of return on savings. This finding mirrors standard heuristic arguments in the literature on economic development [17, 22].

As far as the equilibrium rate of growth is concerned, however, matters are substantially more complicated. To make the dependence of the equilibrium growth rate on z more explicit, we now denote this rate by $\sigma^*(z)$. Then, from (31), we have that

$$F_{j(z_1)}[\gamma_{j(z_1)}(z_1)/\sigma^*(z_1)] = \psi[\gamma_{j(z_1)}(z_1)](1-\theta)/\theta \tag{37}$$

$$F_{j(z_2)}[\gamma_{j(z_2)}(z_2)/\sigma^*(z_2)] = \psi[\gamma_{j(z_2)}(z_2)](1-\theta)/\theta. \tag{38}$$

Since $\psi'(\gamma) \geqslant 0$, apparently $\psi[\gamma_{j(z_1)}(z_1)] \geqslant \psi[\gamma_{j(z_2)}(z_2)]$. In addition, we have

LEMMA 2. $F_{j(z_1)}(x) > F_{j(z_2)}(x)$ *holds, for all* $x > 0$.

The proof of Lemma 2 appears in the Appendix. It follows from these observations that

$$\gamma_{j(z_1)}(z_1)/\sigma^*(z_1) \gtreqless \gamma_{j(z_2)}(z_2)/\sigma^*(z_2)$$

can hold. Evidently, if $\gamma_{j(z_1)}(z_1)/\sigma^*(z_1) \leqslant \gamma_{j(z_2)}(z_2)/\sigma^*(z_2)$ is satisfied, then $\sigma^*(z_1) > \sigma^*(z_2)$, and a reduction in transactions costs necessarily raises the economy's asymptotic growth rate. However, if $\gamma_{j(z_1)}(z_1)/\sigma^*(z_1) > \gamma_{j(z_2)}(z_2)/\sigma^*(z_2)$ holds, $\sigma^*(z_1) \gtreqless \sigma^*(z_2)$ is possible. Then it is possible in turn that an increase in the liquidity of capital resale markets can reduce the rate of growth.

In the next section we illustrate by example that $\sigma^*(z_1) \gtreqless \sigma^*(z_2)$ can hold. Why is it that an improvement in the liquidity of financial markets can reduce the economy's long-run growth rate? The answer is that an improvement in the functioning of secondary capital markets has three consequences: it raises the net of transactions costs productivity of each capital production technology with $j > 1$, it raises the steady-state equilibrium rate of return on savings, and it affects the composition of savings. The first effect is conducive to higher equilibrium rates of growth, and so is the second if $\psi' > 0$. The third effect need not be and, when it is large enough, greater technical efficiency in financial markets can be detrimental to growth. We now pursue this observation in more detail.

Evidently, $\sigma^*(z_1) < \sigma^*(z_2)$ can hold only if $\gamma_{j(z_1)}(z_1)/\sigma^*(z_1) > \gamma_{j(z_2)}(z_2)/\sigma^*(z_2)$. When this occurs, as is apparent from Eq. (25), $\theta^{j,0}$ must fall. In other words, a reduction in transactions costs leads to a greater volume of activity (relative to total savings) in secondary capital markets in this case. The implied increase in activity in these markets diverts savings away from the initiation of new capital investments. This change in the composition of savings in favor of financial assets (claims to existing CIP) acts to reduce the rate of growth.[12] When it is large enough, $\sigma^*(z_1) < \sigma^*(z_2)$ will hold.

To summarize, a reduction in transactions costs raises $\tilde{R}_j$ (if $j > 1$), raises γ, and hence possibly raises $\psi(\gamma)$. As is apparent from Eq. (22), all of these effects operate to increase the rate of growth. However, when a reduction in transactions costs increases j, $\theta^{j,0}$ can fall. Inspection of (22) will indicate that the relative importance of the decline in $\theta^{j,0}$ determines whether transactions cost reductions are growth enhancing.

6. An Example

In this section we present a fully worked out example to illustrate the preceding discussion. The example has $J = 2$, so that there are only two

[12] This result is analogous to the finding in money and growth models that the introduction of financial assets diverts savings away from capital investments and hence tends to reduce the steady-state capital stock [19, 24, 25]. Here, of course, this effect may be counteracted by the increase in both the productivity of capital investments, net of transactions costs, and the potentially higher savings rate.

capital production technologies. In addition, young agents have the logarithmic utility functions $u(C_{1t}, C_{2t}) = (1-\lambda)\ln C_{1t} + \lambda \ln C_{2t}$, with $\lambda \in (0, 1]$. Then $\psi(\gamma) \equiv \lambda$. In addition, the internal rates of return on the two technologies, net of transactions costs, are given by $\gamma_1 = \rho R_1$ and $\gamma_2 = (\rho \tilde{R}_2)^{0.5}$. Thus the equilibrium choice of capital production technology is $j = 2$ iff $\rho \tilde{R}_2 > (\rho R_1)^2$, or equivalently, iff

$$\tilde{R}_2/R_1 > \rho R_1. \tag{39}$$

When (39) fails, $j = 1$. Equation (39) will fail when transactions costs are sufficiently high ($\tilde{R}_2$ is sufficiently low relative to R_1).

For this example, $\phi_1 = \phi_2 = \lambda(1-\theta)/\theta$. In addition, $F_1(x) \equiv x^{-1}$ and $F_2(x) \equiv (1-x^2)/x^2(1-x) = (1+x)/x^2$ hold. Then, when (39) fails (so that technology 1 maximizes the internal rate of return on investment), (31) reduces to

$$\sigma^* = \gamma_1 \lambda(1-\theta)/\theta \equiv \sigma_1. \tag{40}$$

Equation (40) gives the equilibrium rate of growth in this case, and of course when $j = 1$, the value of secondary capital market transactions will be zero.

When (39) holds, technology 2 maximizes the internal rate of return on investment, $j = 2$, and $\gamma = \gamma_2 > \gamma_1$. In this case, Eq. (31) becomes

$$F_2(\gamma_2/\sigma) \equiv [1 + (\gamma_2/\sigma)]/(\gamma_2/\sigma)^2 = \lambda(1-\theta)/\theta. \tag{41}$$

Solving (41) for the equilibrium growth rate yields

$$\sigma^* = \gamma_2\{[1 + 4\lambda(1-\theta)/\theta]^{0.5} - 1\}/2 \equiv \sigma_2, \tag{41$'$}$$

which is the equilibrium rate of growth when $j = 2$.

Before comparing σ_1 and σ_2, we make a few observations about the equilibrium that obtains in each of these cases. When $j = 1$, $\sigma_1 \leqslant \gamma_1$ holds iff $\lambda(1-\theta)/\theta \leqslant 1$. When $j = 2$, $\sigma_2 \leqslant \gamma_2$ holds iff $\lambda(1-\theta)/\theta \leqslant 2$. If $1 < \lambda(1-\theta)/\theta \leqslant 2$ holds, then a reduction in transactions costs that raises the equilibrium maturity of capital investments can move the economy from a situation where the rate of growth exceeds the rate of interest to the opposite state of affairs. In addition, if $j = 2$, a reduction in transactions costs cannot alter the equilibrium choice of capital investment technology. Since this same reduction raises γ_2, it necessarily raises the equilibrium rate of growth.

When will $\sigma_2 > \sigma_1$ hold, and hence when will reductions in transactions costs that change the equilibrium capital gestation length be growth enhancing? A comparison of (40) and (41$'$) indicates that $\sigma_2 > \sigma_1$ holds iff

$$\gamma_2/\gamma_1 > [2\lambda(1-\theta)/\theta]/\{[1 + 4\lambda(1-\theta)/\theta]^{0.5} - 1\}. \tag{42}$$

For high enough transactions costs (low enough values of $\tilde{R}_2$), $\gamma_2/\gamma_1 < 1$ will hold, as will $j = 1$, and $\sigma^* = \sigma_1$. Reductions in transactions costs (increases in $\tilde{R}_2$) that result in $\gamma_2/\gamma_1 > 1$ obtaining will cause the economy to shift to long-gestation capital investments. However, it is straightforward to show that

$$[2\lambda(1-\theta)/\theta]/\{[1+4\lambda(1-\theta)/\theta]^{0.5}-1\} > 1.$$

Thus if γ_2/γ_1 is too close to one, this same reduction in transactions costs will *reduce* the equilibrium rate of growth. However, a sufficiently large increase in $\tilde{R}_2$ will result in (42) being satisfied. This is the case where large enough reductions in transactions costs (that change the equilibrium choice of capital production technology) will be growth enhancing.

It is easy to produce parameter values that result in

$$1 < \gamma_2/\gamma_1 < [2\lambda(1-\theta)/\theta]/\{[1+4\lambda(1-\theta)/\theta]^{0.5}-1\}. \tag{43}$$

For example, if $\lambda(1-\theta)/\theta = 1$ holds, (43) reduces to $\gamma_2/\gamma_1 \in (1, 1.61)$. Thus a reduction in transactions costs can easily result in a reduction in the equilibrium rate of growth.

7. Conclusion

The notion that more productive capital has a relatively long time to payout (a long gestation period) has a rich tradition in both capital theory [3] and development economics [4, 26, 27]. We have presented a model where the employment of long-gestation capital production technologies requires the existence of supporting financial markets. The costs of transacting in financial markets therefore influence the choice of technology to be adopted. Through this mechanism, these costs have potentially complicated consequences for the equilibrium rate of growth that an economy experiences.

Economies with high transactions costs will rely on short-gestation capital production technologies which economize on financial market activity. Reductions in transactions costs can lead to the use of longer gestation (more transactions intensive) technologies, and such reductions necessarily raise the real return on savings. However, they do not necessarily result in a higher real growth rate. The reason is that an increase in the liquidity of financial markets can cause a change in the composition of savings which favors the holding of financial assets (here claims to existing CIP) at the expense of the initiation of new capital investment. When this effect is large enough—which requires an alteration in the equilibrium choice of investment technologies—improvements in the liquidity of financial markets will be growth reducing. Conversely, if sufficiently large reductions in transactions costs lead to the adoption of longer maturity capital investments of sufficiently greater productivity, then capital market improvements are growth enhancing.

APPENDIX

Proof of Lemma 1. Parts (a)–(c) of the lemma follow immediately from the fact that, by definition,

$$F_j(x) \equiv (x^j - 1)/(x-1)\,x^j. \tag{A.1}$$

Part (d) of the lemma follows from applying L'Hopital's rule to (A.1).

For part (e), note that

$$F_j'(x)/F_j(x) = [\,j(x-1) - x(x^j - 1)\,]/x(x-1)(x^j - 1). \tag{A.2}$$

Now define

$$\eta_j(x) \equiv j(x-1) - x(x^j - 1). \tag{A.3}$$

Then $\eta_j(1) = 0$, and

$$\eta_j'(x) = (j+1)(1 - x^j). \tag{A.4}$$

Thus $\eta_j'(x) > 0$ holds $\forall x \in [0, 1)$, while $\eta_j'(x) < 0$ holds $\forall x > 1$. Therefore, $\eta_j(x) < 0$ $\forall x \neq 1$, which establishes that $F_j'(x)/F_j(x) < 0$ $\forall x \neq 1$. For $x = 1$, application of L'Hopital's rule to (A.2) yields that $F_j'(1)/F_j(1) = -(j+1)/2 < 0$. ∎

Proof of Proposition 3. The proof is by contradiction. Suppose, contrary to the proposition, that $j(z_2) > j(z_1)$. Then, by definition,

$$\begin{aligned}\gamma_{j(z_1)}(z_1) &\equiv [\,\rho \tilde{R}_{j(z_1)}(z_1)\,]^{1/j(z_1)} > \gamma_{j(z_2)}(z_1) \\ &\equiv [\,\rho \tilde{R}_{j(z_2)}(z_1)\,]^{1/j(z_2)},\end{aligned} \tag{A.5}$$

while at the same time

$$\gamma_{j(z_2)}(z_2) > \gamma_{j(z_1)}(z_2). \tag{A.6}$$

However, by definition, $\forall j = 1, \ldots, J$,

$$\gamma_j'(z)/\gamma_j(z) = \tilde{R}_j'(z)/j\tilde{R}_j(z). \tag{A.7}$$

Then $j(z_2) > j(z_1)$, (a.2), and (A.7) imply that

$$\gamma'_{j(z_2)}(z) \geqslant [\gamma_{j(z_2)}(z)/\gamma_{j(z_1)}(z)]\, \gamma'_{j(z_1)}(z).$$

Therefore, since $\gamma_{j(z_2)}(z_2) > \gamma_{j(z_1)}(z_2)$, it follows that $\gamma_{j(z_2)}(z) > \gamma_{j(z_1)}(z)$ for all $z \geqslant z_2$. But this contradicts $z_1 > z_2$ and (A.5), establishing the desired result. ∎

Proof of Proposition 6. Equation (27) can be rewritten as

$$[(\sigma^*)^j/\mu_j]\, f_j(\gamma_j/\sigma^*) = 1. \tag{A.8}$$

Moreover, from (30), (31), and the definition of γ_j,

$$F_j(\gamma_j/\sigma^*) = \psi\{[\rho\tilde{R}_j(z)]^{1/j}\}(1-\theta)/\theta. \tag{A.9}$$

Therefore,

$$d(\gamma_j/\sigma^*)/dz = \psi'(\gamma_j)\, \gamma_j(1-\theta)\, \tilde{R}'_j(z)/\theta_j \tilde{R}_j(z)\, F'_j(\gamma_j/\sigma^*) \leqslant 0. \tag{A.10}$$

Now from (A.8),

$$d[(\sigma^*)^j/\mu_j]/dz = -[d(\gamma_j/\sigma^*)/dz]\, f'_j(-)/f_j(-)^2. \tag{A.11}$$

It is easy to verify that $f'_j(-) > 0$, $\forall j > 1$. Therefore $d[(\sigma^*)^j/\mu_j]/dz \geqslant 0$, as claimed. ∎

Proof of Lemma 2. We show that $F_{j+1}(x) > F_j(x)$ holds, $\forall x > 0$, $\forall j \geqslant 1$. The result then follows by induction.

By definition, $F_j(x) \equiv (x^j - 1)/(x-1)\, x^j$. Therefore,

$$\begin{aligned} F_{j+1}(x) &\equiv (x^{j+1}-1)/(x-1)\, x^{j+1} \\ &= [(x^j-1)/(x-1)\, x^j][(x^{j+1}-1)/(x^{j+1}-x)] \\ &\equiv F_j(x)[(x^{j+1}-1)/(x^{j+1}-x)]. \end{aligned} \tag{A.12}$$

Moreover, it is easy to show that $(x^{j+1}-1)/(x^{j+1}-x) > 1$ holds $\forall x > 0$. This establishes the result. ∎

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