Lecture Notes for April 1, 2011

An elementary general equilibrium model – The Robinson Crusoe economy

Robinson Crusoe is endowed with 168 man-hours per week. On his island there is only one production activity, harvesting oysters from an oyster bed, and only one input to this production activity, Robinson's labor.

$$q = F(L), \tag{2.1}$$

where F is concave, L is the input of labor, and q is the output of oysters.

Denote Robinson's consumption of oysters by c and his consumption of leisure by R.

$$R = 168 - L, \tag{2.2}$$

$u(c, R)$

$$F'(\cdot) > 0, F''(\cdot) < 0, \frac{\partial u}{\partial R} > 0, \frac{\partial u}{\partial c} > 0, \frac{\partial^2 u}{\partial R^2} < 0, \frac{\partial^2 u}{\partial c^2} < 0, \frac{\partial^2 u}{\partial R \partial c} > 0,$$

and that $F'(0) = +\infty$.

Centralized allocation

$$u(c, R) = u(F(L), 168 - L). \tag{2.3}$$

We now seek to choose L to maximize u:

$$\max_L u(F(L), 168 - L). \tag{2.4}$$

The first-order condition for an extremum then is

$$\frac{d}{dL} u(F(L), 168 - L) = 0. \tag{2.5}$$

That is,

$$u_c F' - u_R = 0, \tag{2.6}$$

where u_c and u_R denote partial derivatives. Hence, at an optimum — a utility maximum subject to resource and technology constraint — we have

$$\frac{u_R}{u_c} = -\frac{dq}{dR} = F'. \tag{2.7}$$

Restating (2.7),

$$MRS_{R,c} = -\frac{\partial \mathrm{c}}{\partial \mathrm{R}}|_{u=constant} = \frac{u_R}{u_c} = -\frac{dq}{dR} = F' = MRT_{R,c}$$

Equations (2.5), (2.6), and (2.7) represent conditions evaluated at the optimizing allocation, fulfilling (2.4).

Pareto efficient : • the allocation makes technically efficient use of productive resources (labor) to produce output (that the input-output combination is on the production frontier)

• the mix of outputs (oysters and leisure) is the best possible among the achievable allocations in terms of achieving household utility. Equation (2.7), which shows the equality of slopes of the production function and the indifference curve, is the principal characterization of an efficient allocation.

$$MRS_{R,c} = MRT_{R,q}\ .$$

2.1 Decentralized allocation

Fix the price of oysters at 1

wage rate w is expressed in oysters per man-hour.

$$\Pi = F(L) - wL = q - wL, \tag{2.8}$$

where q is oyster supply and L is labor demanded.

$$Y = w \cdot 168 + \Pi. \tag{2.9}$$

$$Y = wR + c. \tag{2.10}$$

Robinson is a price-taker; he regards w parametrically (as a fixed value that he cannot affect by bargaining). As the passive owner of the oyster harvesting firm, he is also a profit-taker; he treats Π parametrically.

$$\Pi = q - wL \tag{2.11}$$

a line in L-q space. Rearranging terms for a fixed value of profits Π', the line

$$q = \Pi' + wL \tag{2.12}$$

isoprofit line; each point (L, q) on the line represents a mix of q and L consistent with the level of profit, Π'.

$$\Pi^o = q - wL = F(L) - wL \tag{2.13}$$

Maximum Π^o, q^o, L^o that

$$\frac{d\Pi}{dL} = F' - w = 0, \text{ and so } F'(L^o) = w, \tag{2.14}$$

$$\Pi' = q - wL = q - w(168 - R) = q + wR - w168 = \text{constant} .$$

Consumer then faces the budget constraint

$$wR + c = Y = \Pi^o + 168w. \tag{2.15}$$

The household faces the problem:

$$\text{Choose c, R to maximize } u(c, R) \text{ subject to } wR + c = Y. \tag{2.16}$$

We have then that $R = (Y - c)/w$. We can restate the household's problem as choosing c (and implicitly choosing R) to

$$\text{maximize } u\left(c, \frac{Y - c}{w}\right) \tag{2.17}$$

$$\frac{du}{dc} = \frac{\partial u}{\partial c} - \frac{1}{w}\frac{\partial u}{\partial R} = 0 \tag{2.18}$$

$$\frac{\frac{\partial u}{\partial R}}{\frac{\partial u}{\partial c}} = w. \tag{2.19}$$

We can restate (2.19) more completely as

$$-\frac{\partial \mathrm{c}}{\partial \mathrm{R}}|_{u=constant} = MRS_{R,c} = \frac{\mathrm{u_R}}{\mathrm{u_c}} = \frac{\frac{\partial u}{\partial R}}{\frac{\partial u}{\partial c}} = w.$$

$(R, q) = (0, \Pi^o + 168 \cdot w)$. Equation (2.9) at $\Pi = \Pi^o$ combined with (2.10) gives

$$wR + c = 168w + \Pi^o \tag{2.20}$$

or

$$c = w(168 - R) + \Pi^o, \tag{2.21}$$

which is the equation of the line with slope $-w$) through $(R, q) = (168, \Pi^o)$. Since $R = 168 - L$, this is the same line as derived by (2.8). This means that Robinson the consumer can afford to buy the oysters produced by the harvesting firm at any prevailing wage.

$$Y = w \cdot 168 + \Pi = 168w + q - wL = wR + c. \tag{2.22}$$

$$0 = w[R + L - 168] + (c - q), \tag{2.23}$$

where w is the wage rate in oysters per hour, L is labor demanded, R is leisure demanded, $q = F(L)$ is oyster supply, and c is oyster demand. This is Walras' Law (true both in and out of equilibrium).

Equilibrium in the market will be characterized by a wage rate w so that $c = q$ and $L = 168 - R$. When that happens, the separate household and firm decisions will be consistent with one another, the markets will clear, and equilibrium will be determined. In Figure 2.2, point M represents the equilibrium allocation. The wage rate w^o, chosen so that $-w^o$ is the slope of the budget/isoprofit line KMP, is the equilibrium wage rate. At M, the separate supply and demand decisions coincide.

KMP = equilibrium budget/isoprofit line,
OF = equilibrium oyster output/demand,
OB = equilibrium leisure demand,
DB = equilibrium labor demand,
UP = equilibrium wage bill, and
PD = equilibrium profit.

The idea of equilibrium becomes clearer when we consider the corresponding disequilibrium. Suppose we have not found an equilibrium wage rate, and we would like to try out the wage w' as a candidate. In Figure 2.2, let JSNQ represent the budget/isoprofit line at wage rate w'. Then we have

OG = planned supply of oysters,
OE = planned demand for oysters,
EG = excess supply of oysters,
DA = planned demand for labor,
DC = planned supply of labor,
AC = excess demand for labor,
DQ = planned profit of firm,
VQ = planned wage bill of firm, and
TQ = planned labor income of household.

Definition Market equilibrium. Market equilibrium consists of a wage rate w^o such that at $w^o, q{=}c$ and $L{=}168 - R$, where q and L are determined by firm profit maximizing decisions and c and R are determined by household utility maximization.

2.2 Pareto Efficiency of the Competitive Equilibrium Allocation: First Fundamental Theorem of Welfare Economics

Profit maximization for equilibrium wage rate w^o requires $w^o{=}F'(L^o)$. Utility maximization subject to budget constraint requires (at market-clearing w^o corresponding to leisure demand R^o)

$$\frac{u_R(c^o, R^o)}{u_c(c^o, R^o)} = w^o, \tag{2.24}$$

where R^o and c^o are utility optimizing leisure and consumption levels subject to budget constraint. However, at market-clearing, $R^o = 168 - L^o$ and $c^o = F(L^o)$. By (2.13), $F'(L^o) = w^o$. Hence,

$$F' = \frac{u_R}{u_c}, \tag{2.25}$$

which is the first-order condition for Pareto efficiency, equation (2.7), established above. First Fundamental Theorem of Welfare Economics: A competitive equilibrium allocation is Pareto efficient.

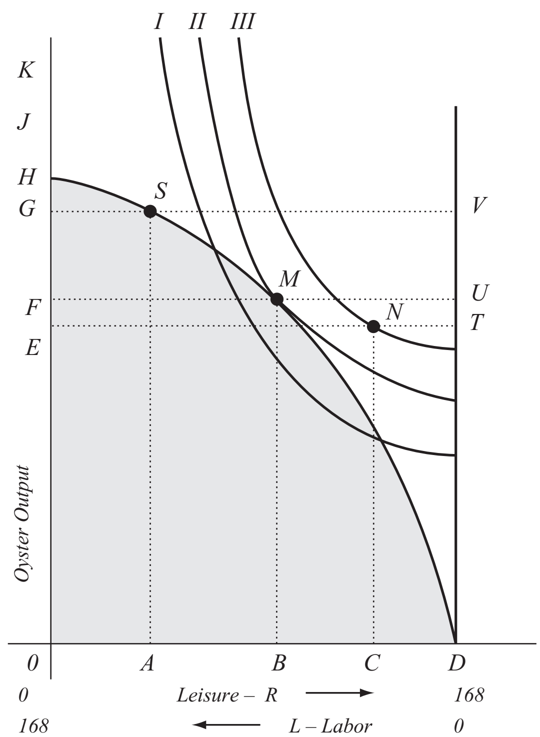

Figure I.1

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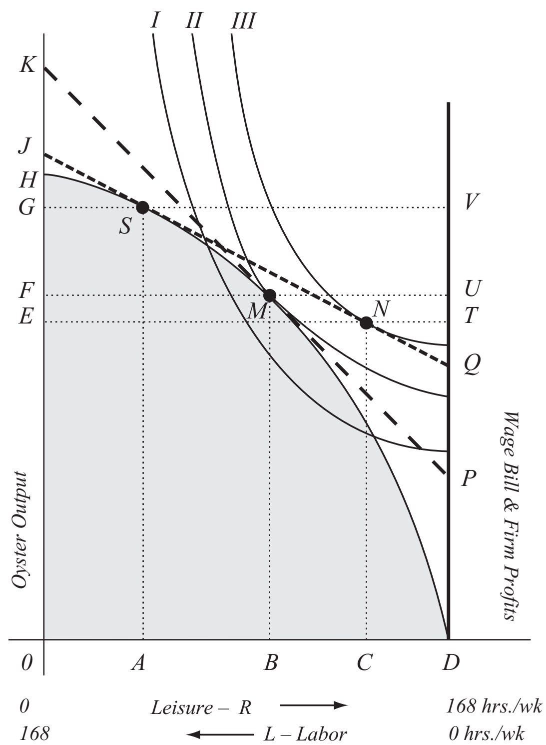

Figure I.2

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