

# Take-Home Midterm EVEN-ODD-ODD

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## 1

Consider the general competitive equilibrium of a production economy with redistributive taxation of income from endowment. Half of each household's income from endowment (based on actual endowment, not net sales) is taxed away. The proceeds of the tax are then distributed equally to all households. We then have,

$$M^i(p) = p \cdot (.5r^i) + \sum_{j \in F} \alpha^{ij} p \cdot y^j + T,$$

where  $T$  is the transfer of tax revenues to the household,

$$T = (1/\#H) \sum_{h \in H} p \cdot (.5r^h).$$

Assume that Walras's Law holds as an equality.

1. Define a competitive equilibrium in this economy.
2. Does a competitive equilibrium generally exist in this economy? Explain.

## 2

Consider the following Edgeworth Box example.

Do all parts.

Superscripts are used to denote the name of the households *and unfortunately, raising the consumption to a squared value — we'll try to keep them straight*. Households are characterized by a utility function and an endowment vector. The possible consumption set is the nonnegative quadrant,  $R_+^2$ . There are two commodities,  $x$  and  $y$ .

Household  $A$  is characterized as  $u^A(x, y) = [x]^2 + [y]^2$  (where the terms in brackets are raised to the power 2), with endowment  $r^A = (5, 5)$ . Household  $A$ 's optimizing consumption subject to budget constraint will typically be a corner solution, so marginal equivalences will not be fulfilled as an equality.

Household  $B$  has the same preferences and endowment.  $u^B(x, y) = [x]^2 + [y]^2$ ,  $r^B = (5, 5)$ . Denote  $A$ 's demand as  $(x^A, y^A)$ ,  $B$ 's as  $(x^B, y^B)$ .

1. The utility functions in this Edgeworth Box example violate some of the usual assumptions C.I - C.V, C.VI(SC), C.VII. Which one(s) do they violate? Explain.
2. We claim there is however a competitive equilibrium in this Edgeworth Box. Find competitive equilibrium prices  $(p_x^*, p_y^*)$  and an equilibrium allocation.
3. Considering that the conditions of Theorem 14.1 have not been fulfilled, how is it possible that there is a competitive equilibrium? Is this a counterexample to Theorem 14.1 (that is, does it demonstrate that Theorem 14.1 is false?) ?

### 3

Consider an Edgeworth Box for two households. The two goods are denoted  $x, y$ . The expression  $\succ$  denotes strict preference; the expression  $\sim$  denotes indifference, equivalence in preference. The households have identical preferences:

$$\begin{aligned} (x, y) &\succ (x', y') \text{ if } 3x + y > 3x' + y', \text{ or} \\ (x, y) &\succ (x', y') \text{ if } 3x + y = 3x' + y' \text{ and } x > x'. \\ (x, y) &\sim (x', y') \text{ only if } (x, y) = (x', y'). \end{aligned}$$

They have identical endowments of  $(20, 20)$ . Demonstrate that there is no competitive equilibrium. Is this example a counterexample to Theorem 18.1 (does it demonstrate that Theorem 18.1 is false?) ? If so explain why Theorem 18.1 is false. If not, state which of Theorem 18.1's assumptions is not fulfilled and demonstrate that it is violated. (Hint: Consider demand in the neighborhood of price vector  $(p_x, p_y) = (.75, .25)$ .)