Answers to Exam No. 2 on Topics from Chapters 2 through 9

The basic model (R) is $LQ_t = a + b\,LY_t + g\,LP_t + d\,Lr_t + e\,LU_t + u_t$.

1. First define three dummy variables; SUMMER, which takes the value 1 for the Summer quarter and 0 elsewhere, SPRING, which has the value 1 for Spring quarter only, and FALL, with a value of 1 for the Fall quarter only. Winter quarter is the control period. Let $a = a_1 + a_2$SUMMER + a_3SPRING + a_4FALL, and similarly for the other Greek letters. Substituting these in Model R and grouping terms appropriately, we obtain the general model (U) as follows.

$$
\begin{aligned}
LQ_t = {} & a_1 + a_2\,\text{SUMMER} + a_3\,\text{SPRING} + a_4\,\text{FALL} \\
& + b_1 LY_t + b_2(LY_t \times \text{SUMMER}) + b_3(LY_t \times \text{SPRING}) + b_4(LY_t \times \text{FALL}) \\
& + g_1 LP_t + g_2(LP_t \times \text{SUMMER}) + g_3(LP_t \times \text{SPRING}) + g_4(LP_t \times \text{FALL}) \\
& + d_1 Lr_t + d_2(Lr_t \times \text{SUMMER}) + d_3(Lr_t \times \text{SPRING}) + d_4(Lr_t \times \text{FALL}) \\
& + e_1 LU_t + e_2(LU_t \times \text{SUMMER}) + e_3(LU_t \times \text{SPRING}) + e_4(LU_t \times \text{FALL}) \\
& + u_t
\end{aligned}
$$

The new variables to be generated are those in parentheses in the above equation.

2. The error structure is given by $u_t = r_1 u_{t-1} + r_2 u_{t-2} + r_3 u_{t-3} + r_4 u_{t-4} + e_t$. The null hypothesis is $r_i = 0$, for i = 1, … 4.

3. The test statistic is LM = $60 \times 0.557 = 33.42$. Under the null hypotheses, LM has the Chi-square distribution with 4 d.f. The critical value for a 1 percent level is 13.2767. Since LM is greater than this, we reject the null hypotheses and conclude that there is significant fourth-order autocorrelation.

4. Step 1: Regress LQ against a constant, LY, LP, Lr, LU, all the dummy variables, and all the interaction terms in Model U, and save the residuals as ut = $\hat{u}_t$.

Step 2: Next generate uti = ut(–i), for i = 1, .., 4. Then, using observations 5 through 64, regress ut against ut1, ut2, ut3, and ut4, with no constant term, to obtain $\hat{r}_i$ for i = 1, .., 4.

Step 3: Generate LQ* = $LQ_t - \hat{r}_1 LQ_{t-1} \ldots - \hat{r}_4 LQ_{t-4}$ and perform similar transformations on each of the right hand side variables in Model U.

Step 4: Regress LQ* against a constant and all the newly created variables in Step 3, to obtain the parameters in Model U.

Step 5: Using the parameters of Model U, obtained in Step 4, compute new residuals ut. Then go back to Step 2 and iterate until the following rule applies.

Step 6: Stop when the error sum of squares obtained in Step 4 by successive iterations do not change by more than some specified percent, say, 0.01.

5. The estimated models are given below.

WINTER: $L\hat{Q} = 6.5123 - 2.3187\text{ LP} + 2.7734\text{ LY}.$

SPRING: $L\hat{Q} = 6.5123 - 2.3187\text{ LP} + 2.7734\text{ LY} + 0.2214\text{ LY} - 0.1772\text{ Lr}$

$= 6.5123 - 2.3187\text{ LP} + 2.9948\text{ LY} - 0.1772\text{ Lr}.$

SUMMER: $L\hat{Q} = 6.5123 - 1.0481 - 2.3187\text{ LP} + 0.2961\text{ LP} + 2.7734\text{ LY} - 0.1197\text{ Lr}.$

$= 5.4642 - 2.0226\text{ LP} + 0.2961\text{ LP} + 2.7734\text{ LY} - 0.1197\text{ Lr}.$

FALL: $L\hat{Q} = 6.5123 - 2.3187\text{ LP} + 2.7734\text{ LY} - 0.1140\text{ Lr} + 0.1237\text{ LU}.$

6. As the price of a commodity increases we would expect demand to drop and hence the price elasticity will be negative. The income effect is expected to be positive. When borrowing costs go up, demand is likely to drop and hence we would expect the interest rate elasticity to be negative. Unemployment rate is a measure of the business cycle. A rise in the rate is a sign of weakness and demand is likely to drop. We note from the above estimates that all signs are as expected except for LU in the Fall which is counterintuitive. Unemployment rate effect is statistically insignificant in the other seasons.

Income and price effects are elastic, that is, have numerical values greater than 1. Interest rate and unemployment rate effects are inelastic.