

Answers to Exam No. 1 on Topics from Chapters 2 through 7

1. Let $\mathbf{a} = \mathbf{a}_0 + \mathbf{a}_1 \text{dn} + \mathbf{a}_2 \text{di} + \mathbf{a}_3 \text{doecd}$, $\mathbf{b} = \mathbf{b}_0 + \mathbf{b}_1 \text{dn} + \mathbf{b}_2 \text{di} + \mathbf{b}_3 \text{doecd}$,
 $\mathbf{g} = \mathbf{g}_0 + \mathbf{g}_1 \text{dn} + \mathbf{g}_2 \text{di} + \mathbf{g}_3 \text{doecd}$, $\mathbf{d} = \mathbf{d}_0 + \mathbf{d}_1 \text{dn} + \mathbf{d}_2 \text{di} + \mathbf{d}_3 \text{doecd}$, and
 $\mathbf{e} = \mathbf{e}_0 + \mathbf{e}_1 \text{dn} + \mathbf{e}_2 \text{di} + \mathbf{e}_3 \text{doecd}$

Substituting these in the original model we obtain the following unrestricted model.

$$\begin{aligned}
 (\text{Model U}) \quad \text{grth} &= \mathbf{a}_0 + \mathbf{a}_1 \text{dn} + \mathbf{a}_2 \text{di} + \mathbf{a}_3 \text{doecd} \\
 &+ y60 (\mathbf{b}_0 + \mathbf{b}_1 \text{dn} + \mathbf{b}_2 \text{di} + \mathbf{b}_3 \text{doecd}) \\
 &+ \text{inv} (\mathbf{g}_0 + \mathbf{g}_1 \text{dn} + \mathbf{g}_2 \text{di} + \mathbf{g}_3 \text{doecd}) \\
 &+ \text{school} (\mathbf{d}_0 + \mathbf{d}_1 \text{dn} + \mathbf{d}_2 \text{di} + \mathbf{d}_3 \text{doecd}) \\
 &+ \text{pop} (\mathbf{e}_0 + \mathbf{e}_1 \text{dn} + \mathbf{e}_2 \text{di} + \mathbf{e}_3 \text{doecd}) + v \\
 &= \mathbf{a}_0 + \mathbf{a}_1 \text{dn} + \mathbf{a}_2 \text{di} + \mathbf{a}_3 \text{doecd} \\
 &+ \mathbf{b}_0 y60 + \mathbf{b}_1 \text{dn} \times y60 + \mathbf{b}_2 \text{di} \times y60 + \mathbf{b}_3 \text{doecd} \times y60 \\
 &+ \mathbf{g}_0 \text{inv} + \mathbf{g}_1 \text{dn} \times \text{inv} + \mathbf{g}_2 \text{di} \times \text{inv} + \mathbf{g}_3 \text{doecd} \times \text{inv} \\
 &+ \mathbf{d}_0 \text{school} + \mathbf{d}_1 \text{dn} \times \text{school} + \mathbf{d}_2 \text{di} \times \text{school} + \mathbf{d}_3 \text{doecd} \times \text{school} \\
 &+ \mathbf{e}_0 \text{pop} + \mathbf{e}_1 \text{dn} \times \text{pop} + \mathbf{e}_2 \text{di} \times \text{pop} + \mathbf{e}_3 \text{doecd} \times \text{pop} + v
 \end{aligned}$$

The null hypothesis is that $\mathbf{a}_i = \mathbf{b}_i = \mathbf{g}_i = \mathbf{d}_i = \mathbf{e}_i = 0$ for $i = 1, 2$, and 3 . This gives 15 restrictions.

2. Regress grth against a constant, y60, inv, school, and pop, and save the error sum of squares as *ESSR*. Next regress grth against a constant, y60, inv, school, pop, plus all the interaction terms with the dummies listed in Model U, and save the error sum of squares as *ESSU*. Then compute the *F*-statistic

$$F_c = \frac{(ESSR - ESSU) / 15}{ESSU / (104 - 20)}$$

Reject the null hypothesis if $F_c > F_{15,84}^*(0.05)$, where F^* is the point on the *F*-distribution with 15 d.f. for the numerator and 84 d.f. for the denominator such that the area to the right is 0.05. From the *F*-table, F^* is between 1.75 and 1.84 and can be interpolated to be approximately 1.81.

3. Regress grth against a constant, y60, inv, school, and pop, and save the residuals as \hat{u} . Next regress grth \hat{u} against a constant, y60, inv, school, pop, plus all the interaction terms with the dummies listed in Model U. Then compute the test statistic $LM = 104 R^2$. Reject the null hypothesis if $LM > LM^*$, the point on the Chi-square distribution with 15

d.f. From the Chi-square distribution we see that LM* is between 101.879 and 113.145, and can be interpolated to be 106.385.

4.

$\Delta \text{grth} / \Delta y60 = -0.408$. This implies that if income rose by 10 percent then the growth rate will decrease, on average, by 4.08 percent (note that since grth is in logs and y60 is log income, b_0 is the income elasticity of growth). In other words, there is diminishing marginal effect with respect to income.

$\Delta \text{grth} / \Delta \text{pop} = -0.0961$ doecd. Thus, this marginal effect is only for OECD countries. For such a country, a 10 percent increase in the population is expected to decrease the growth rate by 0.961 percent.

$\Delta \text{grth} / \Delta \text{school} = 0.3102$ di. School population has a significant effect only for industrialized countries. For such countries a one percent increase in the school population increases income growth, on average, by 0.3102 percent. This is an unfortunate result because one would hope that education would significantly increase a non-industrialized country's income growth too.

$\Delta \text{grth} / \Delta \text{inv} = 0.7933 - 0.3059$ dn. This means that the marginal effect of investment on income growth depends, not surprisingly, on whether the country is an oil-producing one. If no, the marginal effect of investment is 0.4874, otherwise it is 0.7933.