Answers to Exam No. 1 on Topics from Chapters 2 through 6

1.
Taking logarithms of both sides of the Cobb-Douglas production function, we get

$$\text{(Model A)} \qquad \ln Q_t = a + b \ln K_t + g \ln L_t + u_t$$

2.
Let $a = a_1 + a_2 t$, $b = b_1 + b_2 t$, and $g = g_1 + g_2 t$. Substituting these in the basic model and regrouping terms, we obtain the unrestricted model.

$$\text{(Model B)} \qquad \ln Q_t = a_1 + a_2 t + b_1 \ln K_t + b_2 (t \ln K_t) + g_1 \ln L_t + g_2 (t \ln L_t) + v_t$$

The variables to be generated are: $\ln Q_t$, $\ln K_t$, $\ln L_t$, $(t \ln K_t)$, and $(t \ln L_t)$.

3.
To identify Multicollinearity (MC), compute the correlation coefficients between every pair of variables in the list, t, $\ln K_t$, $\ln L_t$, $(t \ln K_t)$, and $(t \ln L_t)$. If MC is present, we would notice large values for these correlations. A second way to identify MC is to omit the variable that has the least significant coefficient. If the results for the remaining coefficients change drastically, MC is surely present. Estimates are still unbiased, consistent, and BLUE (that is, most efficient among linear unbiased estimators). However, standard errors and confidence intervals are likely to be larger with lower values for t-statistics making more coefficient insignificant. Hypothesis tests are, however, valid.

4.
The null hypothesis is $a_2 = b_2 = g_2 = 0$. The alternative is that at least one of these coefficients is not zero. First regress $\ln Q_t$ against a constant, $\ln K_t$, and $\ln L_t$, and save the error sum of squares as *ESSA*. Next regress $\ln Q_t$ against a constant, $\ln K_t$, $\ln L_t$, $(t \ln K_t)$, and $(t \ln L_t)$ and save the error sum of squares as *ESSB*. The test statistic is

$$F_c = \frac{(ESSA - ESSB)/3}{ESSB/(n-6)}$$

where n is the number of observations. Under the null, F_c has the F-distribution with 3 d.f. for the numerator and $n-6$ d.f. for the denominator. From the F-table, look at the critical value $F^*_{3,n-6}$ corresponding to the level of significance and reject the null hypothesis if $F_c > F^*$.