

Exam No. 1 on Topics from Chapter 2 & 3 (1.5 hours)

I.

In the simple regression model $Y_t = \mathbf{a} + \mathbf{b} X_t + u_t$, are the following statements correct? Fully explain why or why not?

I.1 (7 points) If the X -values have a small sample variance, then OLS estimators $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ will be less precisely estimated.

I.2 (8 points) If the errors u_t are serially correlated or heteroscedastic, then OLS estimators $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ will still be unbiased and consistent, but not efficient.

II.

Suppose you specified the regression model as $Y_t = \mathbf{b} X_t + u_t$ and estimated \mathbf{b} as $\hat{\mathbf{b}} = \Sigma(X_t Y_t) / \Sigma(X_t^2)$. However, the true model has a constant term so that Y_t is actually given by $Y_t = \mathbf{a} + \mathbf{b} X_t + u_t$, where u_t has zero expectation, X is given, and $\mathbf{a} \neq 0$.

II.1 (12 points) Carefully derive the true expected value of $\hat{\mathbf{b}}$ and show that it is biased.

II.2 (4 points) Derive the condition under which $\hat{\mathbf{b}}$ will be unbiased (it should not be $\mathbf{a} = 0$).

II.3 (4 points) What is the intuitive interpretation of the condition you just derived?

III.

Consider the simple regression model $Y_t = \mathbf{a} + \mathbf{b} X_t + u_t$ in which Y_t is total expenditure on travel and X_t is total income for the t th State. Including the District of Columbia, you have data for 51 observations. Both variables are measured in billions of dollars. The following is a partial computer output for the above data.

VARIABLE	COEFFICIENT	STDERROR
Constant	0.49812	0.535515
Income	0.055573	0.003293
Error Sum of Squares (ESS)	417.110335	
Total Sum of Squares (TSS)	2841.33	

III.1 (5 points) What is the econometric interpretation of the estimated coefficient for income? Does the numerical value appear reasonable?

III.2 (14 points) Test individually whether the coefficients for the constant term and income are significantly different from zero at the 5% level. Be sure to state the null and alternative hypotheses, the test statistic and its distribution, the critical value (or range), and the criterion. What is your conclusion?

III.3 (3 points) Compute the measure of goodness of fit.

III.4 (5 points) Test the model for goodness of fit at the 1 percent level of significance. Show all your derivations. What is your conclusion?

III.5 (8 points) Suppose the data on X and Y are converted to thousands and a new model is estimated as $Y^* = \mathbf{a}^* + \mathbf{b}^* X_t^* + u_t^*$, where the variables with asterisks are the transformed ones. In the table below, fill in the blanks, indicated by underlined items, that give the values for the transformed model. Show your derivations.

VARIABLE	COEFFICIENT	STDERROR
Constant*	_____	_____
Income*	_____	_____
Error Sum of Sq (ESS*)	_____	
Total sum of Squares (TSS*)	_____	