Answers to Exam No. 1 on Topics from Chapters 2 & 3

I.1 TRUE. The variance measures how closely values are to the population mean, on average. If the *X*-values have a small sample variance, then OLS estimators $\hat{\alpha}$ and $\hat{\beta}$ will have larger variances and hence will be farther away, on average, to their expectations. This means that parameters are *less* precisely estimated.

I.2 PARTLY TRUE. Violation of Assumptions 3.5 and 3.6 only affects the BLUE property. Thus estimators are still unbiased and consistent but not BLUE.

II.

The estimated coefficient is $\hat{\beta} = \Sigma(X_t Y_t)/\Sigma(X_t^2)$. Substitute from the true model to obtain

$$\hat{\beta} = \frac{\Sigma X_t(\alpha + \beta X_t + u_t)}{\Sigma X_t^2} = \frac{\alpha \Sigma X_t}{\Sigma X_t^2} + \beta + \frac{\Sigma(X_t u_t)}{\Sigma X_t^2}$$

The expected value of the third term is zero because $E(u_t) = 0$. But $\hat{\beta}$ will be biased unless the first term is also zero. The required condition is therefore that $\Sigma X_t = 0$ or that the sample mean is zero.

III.1 The coefficient for income is $\Delta \hat{Y}_t / \Delta \hat{X}_t = 0.0556$. It is the marginal effect of income on travel expenses. For a one billion dollar increase in aggregate income, expenditure on travel is expected to increase, on average, by 0.0556 billions of dollars or 55.6 millions of dollars. This is quite reasonable.

III.2. For α, H_0:= 0, H_1: $\alpha \neq 0$. $t_c = 0.4981/0.5355 = 0.93$. Under the null hypothesis, t_c has the *t*-distribution with 49 (51-2) d.f. For a 5% level, critical t^* is in (2.000, 2.021). Since $t_c < t^*$, we cannot reject the null. For β, H_0: $\beta = 0$, H_1: $\beta \neq 0$. $t_c = 0.0556/0.0033 = 16.85$. Under the null hypothesis, t_c has the *t*-distribution with 49 d.f. For a 5% level, critical t^* is in (2.000, 2.021). Since $t_c > t^*$, we reject the null. Thus the conclusion is that α is not statistically different from zero but β is.

III.3

$$R^2 = 1 - \frac{ESS}{TSS} = 1 - \frac{417.11}{2841.33} = 1 - 0.147 = 0.853$$

III.4

Test statistic is

$$F_c = \frac{R^2(n-2)}{(1-R^2)} = \frac{0.853}{0.147} = 284.33$$

Under the null hypothesis that the correlation between expenses on travel and income is zero, F_c has *F*-distribution with one d.f. for the numerator and 49 d.f. for the denominator. Critical F^* for 1% level is in (7.08, 7.31). Since $F_c > F^*$, we conclude that correlation between travel expenses and income is significantly different from zero.

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III.5

Let X^* be the new income variable and Y^* be the new expenditure variable. Then $X^* = 1000^2$ and $Y^* = 1000^2\, Y$. The model is $Y = \alpha + \beta X + u$ and the estimated model is $\hat{Y} = \hat{\alpha} + \hat{\beta} X$.

$$Y^* / 1000^2 = \alpha + \beta X^* / 1000^2 + u \qquad \text{or} \qquad Y^* = 1000^2 \alpha + \beta X^* + 1000^2 u$$

We therefore have, $\hat{\alpha}^* = 1000^2\ \hat{\alpha} = 498120$, its standard error is 1000^2 0.535515 = 535515. R^2, $\hat{\beta}$, and its standard error, are unchanged. ESS* = 1000^4 ESS = 1000^4 417.110335 and TSS* = 1000^2 TSS = 1000^4 2841.33.