Exam on Chapter 2 Topics (2 hours)

I. (10 points)
Suppose that the number of phone calls made in a day by a company can be approximated by the normal distribution with $m = 352$ and $s = 31$. Compute the probability that the number of phone calls on a particular day is between 375 and 400.

II. (10 points)
.Let X be a random variable with mean m and variance s. Show that the random variable $Y = a + bX$ has the mean $a + b\,m$ and variance $b^2\,s^{\,2}$ (no need for integrals).

III. (15 points)
A company operating a chain of drug stores plans to open a new store in one of two locations. The management of the company figures that in the first location the store will show an annual profit of $20,000 if it is successful and an annual loss of $2,000 if it is not. So far as the second location is concerned, the store will show an annual profit of $25,000 if it is successful and an annual loss of $5,000 if it is not. If the probability of success is one-half for each location, where should the company open the new store so as to maximize expected profit. Show all your work in arriving at your conclusion.

IV. (20 points)
Let X be a random variable with mean m and variance $s^{\,2}$. Three observations are drawn from this distribution; x_1, x_2, and x_3. Consider the three different estimates of m: $\hat{m}_1 = 0.2\,x_1 + 0.3\,x_2 + 0.5\,x_3$; $\hat{m}_2 = 0.4\,x_1 + 0.2\,x_2 + 0.4\,x_3$; and $\hat{m}_3 = 0.3x_1 + 0.3x_2, + 0.3x_3$. Prove that the first two are unbiased estimates but the last one is biased (9 points). Express the variances of the first two estimates (ignore the third) in terms of (8 points). Which one is more efficient and why (3 points)?

V. (15 points)
A market research organization wants to test the claim that 60 percent of all housewives in a certain area prefer Brand A cleanser to all competing brands. It is decided to take a random sample of 18 housewives and reject the claim if 9 or fewer of them prefer Brand A over all other brands. Carefully derive an expression for the probability that the market research organization will make the error of rejecting the claim even though it is correct, that is, even though 60 percent of housewives truly prefer Brand A. Be sure to define all your symbols. Use the binomial table and write down the numerical answer.
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