Answers to Exam on Chapter 2 Topics

I..
We know that subtracting the mean and dividing by the standard deviation results in a distribution with mean 0 and standard deviation 1. Thus, the area between 375 and 400 in a normal distribution with mean 352 and standard deviation 31 is equivalent to the area between $(375-352)/31$ = 0.74 and $(400-352)/31$ = 1.55. From Table A.1, this is the area from 0 to 1.55 minus the area from 0 to 0.74, which is, $0.4394-0.2704$ = 0.169.

II.
$E(Y)=\sum_{i=1}^{n}(a+bx_i)f(x_i)=a\sum_{i-1}^{n}f(x_i)+b\sum_{i=1}^{n}x_i f(x_i)$ $= a + b\,m$, because probabilities sum to 1 and the second summation is $E(X)= m$. By proceeding similarly, it is easy to verify that $E(Y^2)$ = $\sum_{i=1}^{n}(a+bx_i)^2 f(x_i)$ $= a^2 + 2\,ab\,E(X) + b^2\,E(X^2) = a^2 + 2\,ab\,m + b^2\,E(X^2)$. Therefore, Var($Y$) = $E(Y^2)$ $[E(Y)]^2$ = $a^2 + 2\,ab\,E(X) + b^2\,E(X^2)$ $-(a+bm)^2 = b^2\,[E(X^2)-m^2] = b^2\,s^2$.

III.

First location: E(Profit) = $\frac{1}{2}$ 20000 - $\frac{1}{2}$ 2000 = 9,000

Second location: E(Profit) = $\frac{1}{2}$ 25000 - $\frac{1}{2}$ 5000 = 10,000

Because the expected profit is higher in the second location, the company should locate there.

IV.
$E(\hat{m}_1) = 0.2\,m + 0.3\,m + 0.5\,m = m$. $E(\hat{m}_2) = 0.4\,m + 0.2\,m + 0.4\,m = m$. $E(\hat{m}_3) = 0.3\,m$ + $0.3\,m + 0.3\,m = 0.9\,m$. It is evident that $\hat{m}_3$ is biased. Var($\hat{m}_1$) = $0.2^2 s^2 + 0.3^2 s^2 + 0.5^2 s^2$ = $0.38\,s^2$. Var($\hat{m}_2$) = $0.4^2\,s^2 + 0.2^2 s^2 + 0.4^2\,s^2 = 0.36\,s^2$. Because $\hat{m}_2$ has a smaller variance, it is more efficient.

V.
In this binomial example, n = 18 and p = 0.6. Let E = 9 or fewer of customers prefer Brand A. We want $P(E)$ when p = 0.6. The expression for the binomial probability is given in Section 2.1. We
Have,

$$P(E) = \sum_{i=0}^{9} \binom{n}{i} p^i (1-p)^{n-i} = \sum_{i=0}^{9} \binom{18}{i} 0.6^i 0.4^{18-i}$$

Calculating this is obviously cumbersome. However, standard statistical tables provide tabulations of the values of the cumulative binomial distribution for different values of n, i, and p. Appendix Table A.6 has

$$P(X \geq x'-1) = \sum_{x=x'}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

for different values of n and $x¢$ and selected values of $p \leq 0.5$. But this is not exactly what we want (our p is greater than 0.5). The trick is to resort to the symmetry of the binomial distribution to get what we want. For example, let $x = 18 - i$. Then, noting that $\binom{n}{x} = \binom{n}{n-x}$, we have,

$$\binom{18}{i} 0.6^{i} 0.4^{18-i} = \binom{18}{18-x} 0.6^{18-x} 0.4^{x} = \binom{18}{x} 0.4^{x} 0.6^{18-x}$$

Hence,

$$P(E) = \sum_{x=10}^{18} \binom{18}{x} 0.4^{x} 0.6^{18-x}$$

This has the form of the binomial with $p = 0.4$, which is tabulated in Table A.6. By referring to it we find that $P(E) = 0.1347$. Thus there is a 13.47% chance of erroneously concluding that Brand A is not preferred.