Answers to Exam on Topics from Chapters 2 through 11

I.1 Let t refer to the time trend variable that takes the value 1 for the first observations, 2 for the second observation, and so on until n for the last observation. The models are

(A) Straight Line: $Y_t = b_1 + b_2 t + u_t$

(B) Quadratic: $Y_t = b_1 + b_2 t + b_3 t^2 + u_t$

(C) Cubic: $Y_t = b_1 + b_2 t + b_3 t^2 + b_4 t^3 + u_t$

(D) Linear-log: $Y_t = b_1 + b_2 \ln(t) + u_t$

(E) Reciprocal: $Y_t = b_1 + b_2 (1/t) + u_t$

(F) Log-linear: $\ln(Y_t) = b_1 + b_2 t + u_t$ $\quad Y_t > 0$

(G) Double-log: $\ln(Y_t) = b_1 + b_2 \ln(t) + u_t$ $\quad Y_t > 0$

(H) Logistic: $\ln\left[\frac{Y_t}{1-Y_t}\right] = b_1 + b_2 t + u_t$ $\quad 0 < Y_t < 1$

I.2

First estimate each of the above models using the first 90 percent of observations and generate forecasts (call them $\hat{Y}_t$) for the last 10 percent. Compute the forecast error sum of squares ESS = $\Sigma(Y_t - \hat{Y}_t)^2$. Use ESS to compute the model selection statistics described in Chapter 4. The model with the lowest selection statistics would be judged the "best" according to these criteria. Also regress Y_t against a constant and $\hat{Y}_t$ for each model forecasts. Ideally we would want the intercept term to be zero and the slope term to be 1. Use these and the previous criteria to select one or more models.

I.3

Regress Y_t against a constant and the $\hat{Y}_t$ values from each of the three models. Use this regression to generate forecasts. They are the "best" in terms minimizing the error sum of squares and having a mean error of zero.

II.

The model is $C_t = b_1 + b_2 C_{t-1} + b_3 Y_t + b_4 Y_{t-1} + u_t$

with $u_t = r_1 u_{t-1} + r_2 u_{t-2} + r_3 u_{t-3} + r_4 u_{t-4} + e_t$

A one-step ahead forecast for $t = n+1$ is (denoting forecasts of Y_t made independently as $\hat{Y}_t$),

$$\hat{C}_{n+1} = \hat{b}_1 + \hat{b}_2 C_n + \hat{b}_3 \hat{Y}_{n+1} + \hat{b}_4 Y_n + \hat{u}_{n+1}$$

where $\hat{u}_{n+1} = \hat{r}_1 \hat{u}_n + \hat{r}_2 \hat{u}_{n-1} + \hat{r}_3 \hat{u}_{n-2} + \hat{r}_4 \hat{u}_{n-3}$

For higher steps, repeat this procedure with $\hat{C}_t$ instead of C_t and $\hat{Y}_t$ for Y_t.