Answers to Exam on Topics from Chapters 2 through 10

1. Solve for Y_t^* from the first equation to obtain $Y_t^* = (C_t - a)/b$. Next substitute this in the second equation. We get

$$\frac{C_t - a}{b} = l\,\frac{C_{t-1} - a}{b} + (1 - l)Y_{t-1}$$

Multiplying throughout by b we get $C_t - a = l\,(C_{t-1} - a) + b\,(1 - l)Y_{t-1}$. Rearranging terms and adding an error term, we obtain the model as

$$\begin{aligned} C_t &= a\,(1 - l) + l\,C_{t-1} + b\,(1 - l)Y_{t-1} + u_t \\ &= b_1 + b_2 C_{t-1} + b_3 Y_{t-1} + u_t \end{aligned}$$

From the equations $b_1 = a\,(1 - l)$, $b_2 = l$, and $b_3 = b\,(1 - l)$, we obtain estimates as follows.

$$\hat{l} = \hat{b}_2, \qquad \hat{a} = \frac{\hat{b}_1}{1 - \hat{b}_2}, \qquad \hat{b} = \frac{\hat{b}_3}{1 - \hat{b}_2}$$

2. If the error terms u_t satisfy Assumptions 3.2 through 3.8, then OLS estimators will still be biased but consistent and asymptotically efficient. Hypothesis tests will be valid for large samples but not for small samples. If u_t is serially correlated, even the consistency property is destroyed. Also, hypothesis tests are no longer valid even for large samples.

3. The estimated model is $\hat{C}_t = 29{,}686 + 0.726\ C_{t-1} - 0.439\ Y_{t-1}$

The estimates of the original model are $\hat{l} = 0.726$, $\hat{a} = 29686/(1–0.726) = 108{,}343$, and $\hat{b} = -0.439 / (1–0.726) = -1.602$. The estimates are nonsensical and unacceptable, especially the large negative value for the marginal propensity to consume out of expected income. For this reason the long run multiplier should not be calculated. Although, the question did not ask for possible reasons, one can suggest two; (1) strong multicollinearity between *C* and *Y* and (2) the fact that there might be a feedback effect from consumption to income that causes estimates to be biased (more on this in Chapter 13).

4.

Step 1: Generate the variables $X_t = \ln(C_t)$, $DX_t = X_t - X_{t-1}$, $X1_t = X_{t-1}$, and $DX1_t = DX_{t-1}$.

Step 2: Regress DX_t against a constant, time (t), $X1_t$, and $DX1_t$, after suppressing the first two observations.

Step 3: For the Dickey-Fuller test for the null hypothesis that the coefficient of X_{t-1} is zero, compute its *t*-statistic(t_c). Reject the null hypothesis if $t_c < t^*$ which is the negative critical value corresponding to the number of observations and the selected level of significance.

The procedure for $\ln(Y_t)$ is identical.

5.
<u>Step 1</u>: Generate $y_t = \ln(C_t)$, $x_t = \ln(Y_t)$, $Dy_t = y_t - y_{t-1}$, and $Dx_t = X_t - X_{t-1}$.
<u>Step 2</u>: Regress Dy_t against a constant, Dx_t, x_{t-1}, and y_{t-1}.

6.
First regress C_t against a constant and Y_t and compute the standard Durbin-Watson statistic d. Reject the null hypothesis of no cointegration if d exceeds the appropriate critical value from Table 10.7. For the Augmented Dickey-Fuller test, obtain the residuals from the above regression as $\hat{u}_t$ = $C_t - \hat{a} - \hat{b} Y_t$. Then estimate the Dickey-Fuller regression

$$\Delta \hat{u}_t = f \hat{u}_{t-1} + \sum_{i=1}^{p} b_i \Delta \hat{u}_{t-1} + e_t$$

where p is the pre-selected number of lags. The test statistic is the t-statistic for f. Reject the null hypothesis of no conintegration if the critical value is above the relevant one in Table 10.7.

7.
First regress C_t against a constant, C_{t-1}, C_{t-2}, C_{t-3}, C_{t-4}, Y_{t-1}, Y_{t-2}, Y_{t-3}, and Y_{t-4}, using observations from 5 onwards. Use the standard F-test to test that the coefficients for Y_{t-i} (i = 1, …, 4) are jointly zero. This is to test whether Y Granger-causes C. For the reverse, simply switch C_t and Y_t in the above test.