Econ 120C — **Ramu Ramanathan**

Fall 2003 — **Second exam answers**

I.

In an aggregate economy, let M* be the desired stock of money, Y per capita income, and r the interest rate. The long-run equilibrium relationship for the desired stock of money is given by the double log model (where LN stands for natural log)

(1) $$LNM_t^* = a + b\, LNY_t + g\, LNr_t + u_t$$

Because of market frictions, the desired stock of money can be attained only through dynamic adjustments. Consider the following partial adjustment mechanism that determines the actual stock of money (LNM_t).

(2) $$LNM_t = LNM_{t-1} + l\,(LNM_t^* - LNM_{t-1}) \qquad 0 < l < 1$$

Ia (8 points). Derive an estimable econometric relation of the form

(3) $$LNM_t = b_1 + b_2 LNM_{t-1} + b_3 LNY_t + b_4 LNr_t + u_t$$

in which the coefficients of equation (3) are expressed in terms of a, b, g, and l (show your derivation). Express each b_i in terms of a, b, g, and l.

$$LNM_t = LNM_{t-1} + l\,(LNM_t^* - LNM_{t-1}) = (1 - l)\,LNM_{t-1} + l\,LNM_t^*$$

$$= (1 - l)\,LNM_{t-1} + l\,(a + b\,LNY_t + g\,LNr_t + u_t)$$

$$= l\,a + (1 - l)\,LNM_{t-1} + l\,b\,LNY_t + l\,g\,LNr_t + error$$

Therefore, $b_1 = l\,a$, $b_2 = 1 - l$, $b_3 = l\,b$, *and* $b_4 = l\,g$.

Ib (8 points). With quarterly data for 89 observations, the following estimated relation was obtained by the OLS procedure.

$$\widehat{LNM_t} = -\,0.077 + 0.799\,LNM_{t-1} + 0.172\,LNY_t - 0.077\,LNr_t$$

Carefully calculate the estimates of a, b, g, and l.

$\hat{l}$ **)** $= 1 - \hat{b}_2 = 1 - 0.799 = 0.201$

$\hat{a}$ **)** $= \hat{b}_1 / \hat{l} = -\,0.077 / 0.201 = -\,0.383$

$\hat{b}$) $= \hat{b}_3 / \hat{l} = 0.172/0.201 = 0.856$

$\hat{g}$) $= \hat{b}_4 / \hat{l} = -0.077/0.201 = -0.383$

Ic (4 points). Compute the long-run elasticities of income and interest rate.

Income) *It is the same as* $\hat{b} = 0.856$

Interest rate) *It is the same as* $\hat{g} = -0.383$

Id (3 points). Because the data are quarterly, I suspected serial correlation of order 4. Write down the auxiliary equation for the error term and state the null hypothesis of no serial correlation.

$$u_t = r_1 u_{t-1} + r_2 u_{t-2} + r_3 u_{t-3} + r_4 u_{t-4} + e_t$$

H_0: $r_i = 0$ *for i=1, 2, 3, 4.*

Ie (12 points). The unadjusted R^2 for the auxiliary regression was 0.0946. Compute the LM test statistic, state its distribution under the null including d.f., and carry out test at the 5 percent level. Do you conclude that there is significant autocorrelation? In the light of your conclusion, was OLS acceptable or should some other procedure have been used? If the latter, state the name of the procedure that should be used (you need not describe it).

$LM = (n-p)R^2 = 85´\ 0.0946 = 8.041$
Under the null, LM has the chi-square distribution with 4 d.f.
LM = 9.48773. Because LM < LM*, we cannot reject* H_0.
The conclusion is that there is no significant serial correlation of order 4.
This implies that OLS procedure is acceptable.

II.
Consider the following simultaneous equation model.

(1) $\mathbf{Y_t} = a_0 + a_1 \mathbf{X_t} + \mathbf{u_t}$

(2) $\mathbf{X_t} = b_0 + b_1 \mathbf{X_{t-1}} + b_2 \mathbf{Y_{t-1}} + \mathbf{v_t}$

u_t and v_t are the random error terms. The reduced form equation for Y_t is of the form

(3) $\mathbf{Y_t} = p_0 + p_1 \mathbf{X_{t-1}} + p_2 \mathbf{Y_{t-1}} + \textbf{error}$

IIa (10 points). Explicitly derive the reduced form equation for Y_t and express the p s in terms of the a s and b s. You need not do the same for X_t because it is already in reduced form (since it does not contain Y_t).

Substitute (2) in (1) for X_t, to get $Y_t = a_0 + a_1(b_0 + b_1 X_{t-1} + b_2 Y_{t-1} + v_t) + u_t$

Grouping terms, $Y_t = (a_0 + a_1 b_0) + a_1 b_1 X_{t-1} + a_1 b_2 Y_{t-1} + error$

We thus have, $p_0 = (a_0 + a_1 b_0)$, $p_1 = a_1 b_1$, *and* $p_2 = a_1 b_2$

IIb (5 points). Suppose you have estimates of the reduced form equations for X_t and Y_t, that is, you have the $\hat{p}$ s and the $\hat{b}$ s. Using the relationships you derived in IIa, obtain the expressions for $\hat{a}_0$ and $\hat{a}_1$ in terms of the $\hat{p}$ s and the $\hat{b}$ s. Is Equation (1) exactly identified, under identified or over identified?

Solving backwards we get, $\hat{a}_1 = \hat{p}_1 / \hat{b}_1$, $\hat{a}_1 = \hat{p}_2 / \hat{b}_2$, *and* $\hat{a}_0 = \hat{p}_0 - \hat{a}_1 \hat{b}_0$.

Because there are two estimators for a_1, the model is over-identified.