PART II – HW II

ECON-120C (FALL 2003)

1 To get the reduced form equations we first substitute equation (2) into (1):

$$Y_t = \alpha_0 + \alpha_1\left(\beta_0 + \beta_1 Y_t + \beta_2 X_{t-1} + V_t\right) + \alpha_2 Y_{t-1} + U_t \Rightarrow$$

$$\Rightarrow Y_t - \alpha_1\beta_1 Y_t = \left(\alpha_0 + \alpha_1\beta_0\right) + \alpha_1\beta_2 X_{t-1} + \alpha_2 Y_{t-1} + \underbrace{U_t + \alpha_1 V_t}_{\text{new error term } (\varepsilon_{t1})}$$

Now, dividing both sides by $(1-\alpha_1\beta_1)$ we get:

$$Y_t = \frac{\alpha_0 + \alpha_1\beta_0}{1-\alpha_1\beta_1} + \frac{\alpha_1\beta_2}{1-\alpha_1\beta_1} X_{t-1} + \frac{\alpha_2}{1-\alpha_1\beta_1} Y_{t-1} + \varepsilon_{t1} \quad \text{or}$$

$$\boxed{Y_t = \pi_0 + \pi_1 X_{t-1} + \pi_2 Y_{t-1} + \varepsilon_{t1}} \text{ with } \boxed{\pi_0 = \frac{\alpha_0 + \alpha_1\beta_0}{1-\alpha_1\beta_1}},$$

$$\boxed{\pi_1 = \frac{\alpha_1\beta_2}{1-\alpha_1\beta_1}} \text{ and } \boxed{\pi_2 = \frac{\alpha_2}{1-\alpha_1\beta_1}}.$$

Similarly, by plugging equation (1) into (2), we obtain

$$X_t = \beta_0 + \beta_1\left(\alpha_0 + \alpha_1 X_t + \alpha_2 Y_{t-1} + U_t\right) + \beta_2 X_{t-1} + V_t \Rightarrow$$

$$\Rightarrow X_t = \frac{\beta_0 + \alpha_0\beta_1}{1-\alpha_1\beta_1} + \frac{\beta_2}{1-\alpha_1\beta_1} X_{t-1} + \frac{\alpha_2\beta_1}{1-\alpha_1\beta_1} Y_{t-1} + \varepsilon_{t2} \quad \text{or}$$

$$\boxed{X_t = \mu_0 + \mu_1 X_{t-1} + \mu_2 Y_{t-1} + \varepsilon_{t2}}$$ with

$$\mu_0 = \frac{\beta_0 + \alpha_0 \beta_1}{1-\alpha_1\beta_1}, \quad \mu_1 = \frac{\beta_2}{1-\alpha_1\beta_1} \text{ and } \mu_2 = \frac{\alpha_2\beta_1}{1-\alpha_1\beta_1}$$

$\boxed{2}$ Both equations are <u>exactly identified</u>, since they both <u>include one</u> endogenous variable (X_t in eq. (1), Y_t in eq. (2)), and <u>do not include one</u> predetermined variable (X_{t-1} in eq. (1), Y_{t-1} in eq. (2)). (Recall identification rule in page 550 (Property 13.2)).

In order to get expressions for the α's and β's we proceed by plugging the reduced form equation of X_t on the structural equation (1):

$$Y_t = \alpha_0 + \alpha_1(\mu_0 + \mu_1 X_{t-1} + \mu_2 Y_{t-1} + \varepsilon_{t2}) + \alpha_2 Y_{t-1} + u_t =$$

$$= \underbrace{(\alpha_0 + \alpha_1\mu_0)}_{\pi_0} + \underbrace{\alpha_1\mu_1}_{\pi_1} X_{t-1} + \underbrace{(\alpha_1\mu_2 + \alpha_2)}_{\pi_2} Y_{t-1} + \text{error}$$

Therefore, $\begin{cases} \pi_0 = \alpha_0 + \alpha_1\mu_0 \\ \pi_1 = \alpha_1\mu_1 \\ \pi_2 = \alpha_1\mu_2 + \alpha_2 \end{cases} \Rightarrow \boxed{\begin{cases} \alpha_0 = \pi_0 - \frac{\pi_1}{\mu_1}\mu_0 \\ \alpha_1 = \frac{\pi_1}{\mu_1} \\ \alpha_2 = \pi_2 - \frac{\pi_1}{\mu_1}\mu_2 \end{cases}}$

Similarly, we plug the reduced form for Y_t into eq. (2) to get expressions for the β's:

$$X_t = \beta_0 + \beta_1 \left(\pi_0 + \pi_1 X_{t-1} + \pi_2 Y_{t-1} + \varepsilon_{t1}\right) + \beta_2 X_{t-1} + u_t \Rightarrow$$

$$\Rightarrow \begin{cases} \mu_0 = \beta_0 + \beta_1 \pi_0 \\ \mu_1 = \beta_1 \pi_1 + \beta_2 \\ \mu_2 = \beta_1 \pi_2 \end{cases} \Rightarrow \boxed{\begin{cases} \beta_0 = \mu_0 - \dfrac{\mu_2}{\pi_2} \pi_0 \\ \beta_1 = \dfrac{\mu_2}{\pi_2} \\ \beta_2 = \mu_1 - \dfrac{\mu_2}{\pi_2} \pi_1 \end{cases}}$$