

Your name _____

Your Id No. (NOT Soc. Sec. no.) _____

Make sure that all pages (1 through 6) are there. The maximum number of points for the exam is 100. Read the questions carefully and make sure that you do not misunderstand them. If you get stuck somewhere, don't waste time but move on.

I.

Consider the following model of water consumption in San Diego (for simplicity, the t subscript is omitted):

$$\text{WATERCONS} = \beta_1 + \beta_2 \text{NHOUSE} + \beta_3 \text{POP} + \beta_4 \text{PCY} + \beta_5 \text{PRICE} + \beta_6 \text{RAIN} + u$$

where WATERCONS is total water consumption in million cubic meters, NHOUSE is the total number of housing units (in thousands), POP is total population (in thousands), PCY is per capita income in dollars, PRICE is the price of water in dollars per 100 cubic meters, RAIN is rainfall in inches and u is the unobserved random error term.

I.1. (10 points)

What signs would you expect for β_2 , β_3 , β_4 , β_5 , and β_6 ? Give your reasoning.

β_2 :

β_3 :

β_4 :

β_5 :

β_6 :

Using annual data for 15 years, the coefficients were estimated as follows (constant term is ignored):

	coefficient	standard error	Test statistic	Decision to reject or not
β_1	ignored	ignored		
β_2	0.305	0.339		
β_3	0.363	0.259		
β_4	- 0.005	0.008		
β_5	- 17.87	14.89		
β_6	- 1.123	1.404		

The unadjusted R^2 for the model is 0.958.

I.2. (3 points)

Do all the signs agree with your intuition stated earlier? If not, which ones are counterintuitive?

I.3. (15 points)

To carry out a test for the null hypothesis $\beta_1 = 0$ against a two-sided alternative, write down in the table above the numerical values of the test statistic for each of the above coefficients, excluding the constant term (5 points).

What is the statistical distribution of the test statistics, including the degrees of freedom? (5 points)

In each case, write down (in the table) the decision as reject or not reject the null at the 10 percent level (5 points).

I.4. (3 points)

Based on your test, identify variables that are prime candidates for omission from the model? [Note: you might find the result strange but you can't explain it yet.]

I.5. (20 points)

Test the model for overall significance at the 1 percent level by carrying out the following steps.

State the null and alternative hypotheses (3 points).

Write down an expression for the test statistic and compute its value (3 + 2 points).

State its distribution under the null including the d.f. (3 points).

State the criterion for rejection of the null (at the 1 percent level) and apply it (3 points).

What is your conclusion about the overall goodness of fit (3 points)?

Is there a conflict between your conclusion here and that in question I.4? (3 points). [Note: if there is, you NEED NOT explain the reason for such a contradictory result.]

I.6. (9 points)

Suppose the error term *does not* have the normal distribution and we don't know its distribution. Does this cause any problems in terms of estimating the model (3 points), the properties of unbiasedness, consistency, and efficiency of the parameters (3 points), and tests of hypotheses (3 points)? Carefully justify your answers in each case.

I.7. (20 points)

Describe how you would test the hypothesis $\beta_2 = \beta_3$ by carrying out the following steps.

Derive the restricted model (4 points).

Describe what regressions to run (3 points).

Describe how to compute the test statistic (4 points). Give numerical values where known.

Write down its distribution and the values of d.f. (4 points).

Write down the numerical value (or range) of the critical value for the test statistic (2 points).

Write down the criterion for rejection at 5 percent (3 points). [Note: you cannot actually carry out this test because you don't have enough information.]

L8 (6 points)

Suppose water consumption is measured in billions of cubic meters AND AT THE SAME TIME per capita income is measured in thousands of dollars. Write down below the new estimated model in the form

$$\text{WATERCONS}^* = \hat{\alpha}_1 + \hat{\alpha}_2 \text{NHOUSE} + \hat{\alpha}_3 \text{POP} + \hat{\alpha}_4 \text{PCY}^* + \hat{\alpha}_5 \text{PRICE} + \hat{\alpha}_6 \text{RAIN}$$

where the variables with * are the new ones and $\hat{\alpha}$ must be actual numerical values. Below each estimate write down the corresponding value of the new standard error.

II. (6 points) Show that if a regression coefficient is insignificant at the 10 percent level, then it will not be significant at a level lower than that.

III. (8 points) Show that the adjusted R^2 and the estimated residual variance $\hat{\sigma}^2 = ESS/(n-k)$ move in opposite directions; that is, if one goes up the other will go down and vice versa.