Econ 120B
Winter 1998

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First Midterm (20%)

Your name ______________________________ **Your Student Id. __________________**

DO NOT TURN THE PAGE UNTIL EVERYONE HAS RECEIVED THE EXAM AND YOU ARE GIVEN THE SIGNAL TO START. ALSO, YOU MUST STOP WRITING WHEN YOU ARE ASKED TO DO SO (YOU WILL BE GIVEN A 2 MINUTE WARNING). TEN POINTS WILL BE DEDUCTED FOR EACH MINUTE OF EXTRA TIME IT TAKES YOU TO STOP WRITING.

If you use a pencil, you forfeit the right to complain about grading UNLESS YOU PICK UP THE EXAM FROM THE TA FROM HIS/HER OFFICE AND LOOK AT THE GRADING BEFORE LEAVING THE OFFICE.

Make sure that all pages (1 through 4) are there. The maximum number of points for the exam is 100. Read the questions carefully and make sure that you do not misunderstand them. If you get stuck somewhere, don't waste time but move on.

I CONSIDER CHEATING AS A VERY SERIOUS MATTER AND WILL GIVE AN F IN THE COURSE TO ANY ONE CHEATING AND ALSO REFER HIM/HER TO THE DEAN FOR DISCIPLINARY ACTION.

I.

a. **(12 points) The continuous random variable X has the uniform distribution with the constant density function $f(x) = 1$, $0 \le x \le 1$. Using the fact that $\int x^n \, dx = x^{n+1} / (n+1)$ and the definition of expected values, derive $E(x)$, $E(x^2)$, $E(x^3)$, and $E(x^4)$. Note that the integrals go from 0 to 1.**

b. **(14 points) Use the above derivations and compute Var(X), Var(Y), and Cov(X,Y) where $Y = X^2$.**

c. **(16 points) Next compute the coefficient of correlation between X and Y (use the fact that $\sqrt{15} = 3.87$) and show that it is not equal to 1 even though there is an exact relation between X and Y.**

II. (16 points)
Let X and Y be two random variables with $E(X) = E(Y) = 0$, $Var(X) = \sigma_x^2$, $Var(Y) = \sigma_y^2$, and $Cov(X,Y) = \sigma_{xy}$. Now make the transformations $U = X + Y$ and $V = X - Y$. Derive Cov(U,V) and the condition under which U and V will be uncorrelated.

III.
Suppose the true model is $Y_t = \beta X_t + u_t$, that is, $\alpha = 0$. I construct an alternative estimator of β as $\tilde{\beta} = \bar{Y} / \bar{X}$, where $\bar{Y} = (1/n)\Sigma Y_t$ and similarly for $\bar{X}$.

a. **(20 points) Compute the expected value of $\tilde{\beta}$ and check whether it is unbiased or not. Be sure to state assumptions needed for your proof (you will lose points if you state unnecessary assumptions).**

b. (10 points) Without any derivations explain why $\tilde{\beta}$ is inferior to the OLS estimator of β, clearly defining what you mean by "inferior."

IV.

In the simple linear regression model, u_t is the random error term with $E(u_t) = 0$. The subscript t refers to a typical observation and n is the size of the sample. For each of the following equations state whether it is correct or not. Explain why or why not (note: the explanations are fairly simple).

a. (6 points) $\sum_{t=1}^{n} u_t = 0$.

b. (6 points) $\sum_{t=1}^{n} \hat{u}_t = 0$, where $\hat{u}_t$ is the sample residual $Y_t - \hat{\alpha} - \hat{\beta} X_t$.